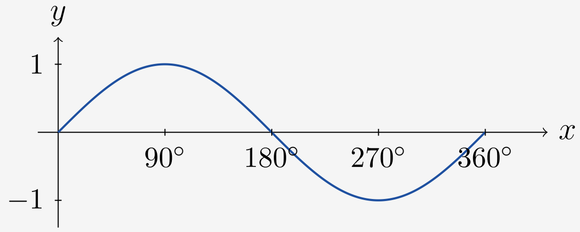
Question 1 (OCR 4752, Jan 2005, Q3)

Worked Solution

Sketch $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$, then solve $\sin x = -0.2$ for $0^\circ \leq x \leq 360^\circ$.

Sketch of $y = \sin x$:



Solving $\sin x = -0.2$:

The principal value is:

$$x = \arcsin(-0.2) = -11.537\ldots^\circ$$

Since this is outside $[0^\circ, 360^\circ]$, we use the symmetry of the sine curve.

In the range $[0^\circ, 360^\circ]$, sine is negative in the third and fourth quadrants:

$$x_1 = 180^\circ + 11.537\ldots^\circ = 191.5^\circ \quad \text{(to 1 d.p.)}$$

$$x_2 = 360^\circ - 11.537\ldots^\circ = 348.5^\circ \quad \text{(to 1 d.p.)}$$

$x = 191.5^\circ$ and $x = 348.5^\circ$

Question 2 (OCR 4752, Jun 2005, Q8)

Worked Solution

Part (i): Solve $\cos x = 0.4$ for $0° \leq x \leq 360°$.

The principal value is:

$$x = \arccos(0.4) = 66.4° \quad \text{(to 1 d.p.)}$$

Cosine is positive in the first and fourth quadrants, so the second solution is:

$$x_2 = 360° - 66.4° = 293.6° \quad \text{(to 1 d.p.)}$$

$x = 66.4°$ and $x = 293.6°$

Part (ii): Describe the transformation mapping $y = \cos x$ onto $y = \cos 2x$.

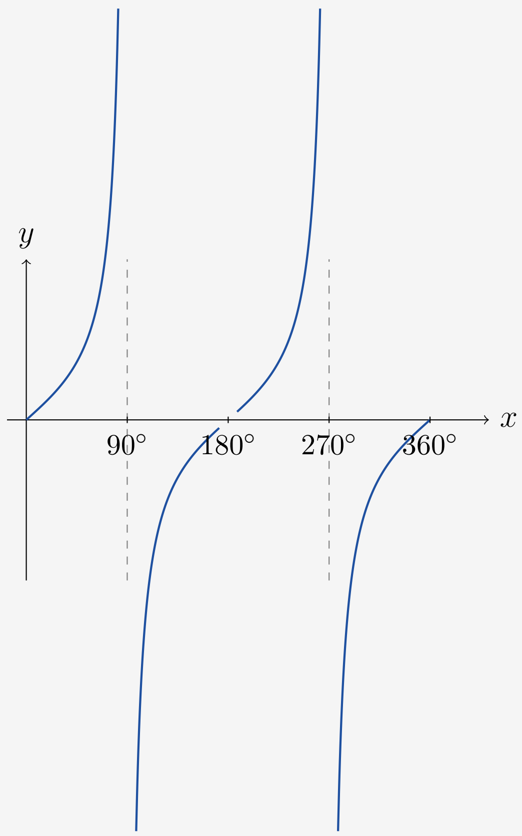
Replacing x by $2x$ compresses the graph horizontally by a scale factor of $\frac{1}{2}$.

Stretch (one way) parallel to the x-axis, scale factor $\frac{1}{2}$.

Question 3 (OCR 4752, Jan 2006, Q5)

Worked Solution

Part (i): Sketch $y = \tan x$ for $0° \le x \le 360°$.



Part (ii): Solve $4\sin x = 3\cos x$ for $0° \le x \le 360°$.

Divide both sides by $\cos x$:

$$4\tan x = 3 \implies \tan x = \frac{3}{4}$$

Principal value:

$$x = \arctan\left(\tfrac{3}{4}\right) = 36.87\ldots°$$

Tangent has period 180°, so the second solution in range is:

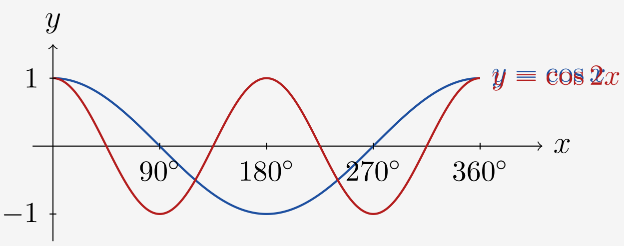
$$x_2 = 36.87\ldots° + 180° = 216.87\ldots°$$

$x = 36.9°$ and $x = 216.9°$ (to 1 d.p.)

Question 4 (OCR 4752, Jun 2006, Q7)

Worked Solution

Part (i): Sketch $y = \cos x$ and $y = \cos 2x$ on the same axes for $0° \leq x \leq 360°$.



$y = \cos x$ completes one full cycle; $y = \cos 2x$ completes two full cycles (period halved, amplitude unchanged).

Part (ii): Solve $\cos 2x = 0.5$ for $0° \leq x \leq 360°$.

Let $u = 2x$, so $u \in [0°, 720°]$.

$\cos u = 0.5 \implies u = 60°$

All solutions for u in $[0°, 720°]$:

$$u = 60°,\ 300°,\ 420°,\ 660°$$

Dividing by 2:

$x = 30°,\ 150°,\ 210°,\ 330°$

Question 5 (OCR 4722, Jun 2013, Q2)

Worked Solution

Part (i): Solve $\sin \frac{1}{2}x = 0.8$ **for** $0° \leq x \leq 360°$**.**

Let $u = \frac{1}{2}x$, so $u \in [0°, 180°]$.

$\sin u = 0.8 \implies u = \arcsin(0.8) = 53.1°$ (to 1 d.p.)

Second solution in $[0°, 180°]$:

$$u = 180° - 53.1° = 126.9°$$

Multiplying by 2:

$x = 106.2°$ and $x = 253.8°$ (allow 106°, 254°)

Wait — checking: $u \in [0°, 180°]$, so $x = 2u$ gives $x = 106.2°$ and $x = 253.8°$. Both are within $[0°, 360°]$. ✓

Part (ii): Solve $\sin x = 3\cos x$ **for** $0° \leq x \leq 360°$**.**

Divide both sides by $\cos x$:

$$\tan x = 3$$

Principal value:

$$x = \arctan 3 = 71.6° \quad \text{(to 1 d.p.)}$$

Second solution (tangent period 180°):

$$x = 71.6° + 180° = 251.6°$$

$x = 71.6°$ and $x = 251.6°$ (to 1 d.p.)

Question 6 (OCR 4722, Jun 2016, Q9i,ii)

Worked Solution

A curve has equation $y = \sin(ax)$, where a is a positive constant and x is in radians.

Part (i): State the period of $y = \sin(ax)$.

The standard sine function $y = \sin x$ has period 2π. Replacing x with ax compresses the period by factor a.

$$\text{Period} = \frac{2\pi}{a}$$

Part (ii): Given that $x = \frac{1}{5}\pi$ and $x = \frac{2}{5}\pi$ are the two smallest positive solutions of $\sin(ax) = k$, find a and k.

The two smallest positive solutions of $\sin\theta = k$ are symmetric about $\theta = \frac{\pi}{2}$:

$$\frac{1}{5}\pi a + \frac{2}{5}\pi a = \pi \quad \Longrightarrow \quad \frac{3}{5}\pi a = \pi \quad \Longrightarrow \quad a = \frac{5}{3}$$

Now find k: substitute $x = \frac{1}{5}\pi$:

$$k = \sin\left(\frac{5}{3} \cdot \frac{\pi}{5}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$a = \frac{5}{3}, \quad k = \frac{\sqrt{3}}{2}$$

End of Worked Solutions