Question 1

Worked Solution

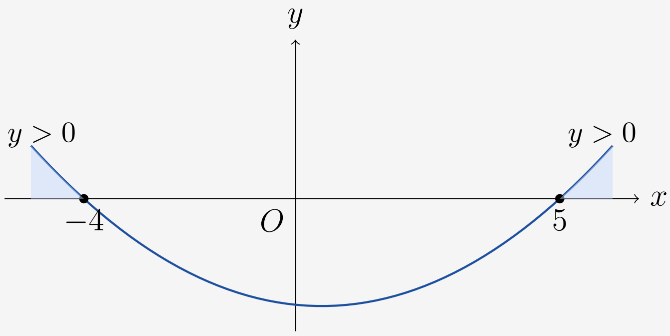
Solve $x^2 - x > 20$, giving the answer in set notation.

Rearrange:

$$x^2 - x - 20 > 0$$

Critical values: $(x-5)(x+4) = 0 \implies x = 5$ or $x = -4$.

Sketch:



U-shaped parabola, above x-axis **outside** the roots.

$$\{x : x < -4\} \cup \{x : x > 5\}$$

Question 2

Worked Solution

Part (a) — Solve $2(3x+4) > 1 - x$:

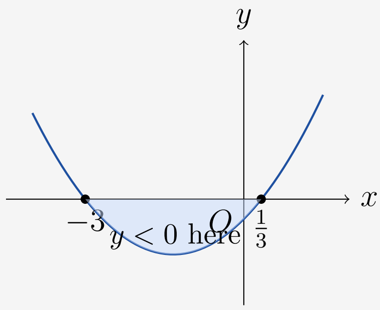
$$6x + 8 > 1 - x \implies 7x > -7$$

$$x > -1$$

Part (b) — Solve $3x^2 + 8x - 3 < 0$:

Critical values: $(x+3)(3x-1) = 0 \implies x = -3$ or $x = \frac{1}{3}$.

Sketch:



Between the roots:

$$-3 < x < \frac{1}{3}$$

Question 3

Worked Solution

Part (a) — Solve $4x - 5 > 15 - x$:

$$5x > 20 \implies x > 4$$

$$x > 4$$

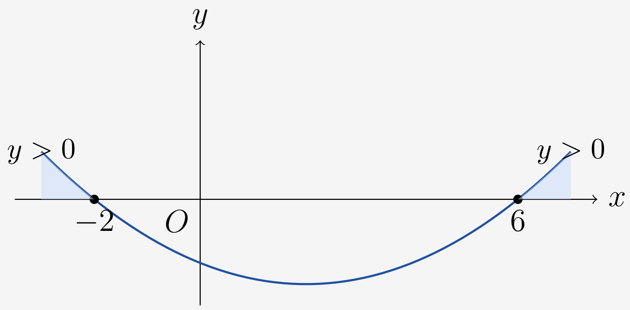
Part (b) — Solve $x(x-4) > 12$:

Expand and rearrange:

$$x^2 - 4x - 12 > 0$$

Critical values: $(x+2)(x-6) = 0 \implies x = -2$ or $x = 6$.

Sketch:



Outside the roots:

$$x < -2 \quad \text{or} \quad x > 6$$

Question 4

Worked Solution

Part (a) — Solve $4x - 3 > 7 - x$:

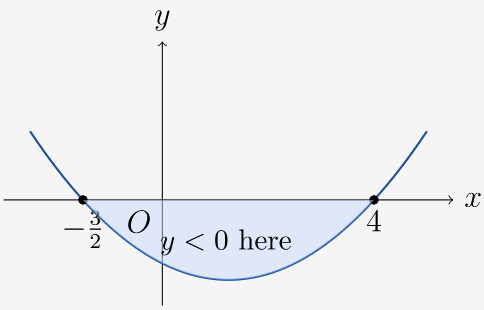
$$5x > 10 \implies x > 2$$

$$x > 2$$

Part (b) — Solve $2x^2 - 5x - 12 < 0$:

Critical values: $(2x+3)(x-4) = 0 \implies x = -\frac{3}{2}$ or $x = 4$.

Sketch:



Between the roots:

$$-\frac{3}{2} < x < 4$$

Part (c) — Both inequalities satisfied:

From (a): $x > 2$. From (b): $-\frac{3}{2} < x < 4$. Intersection:

$$2 < x < 4$$

Question 5

Worked Solution

Part (a) — Solve $3(x-2) < 8-2x$:

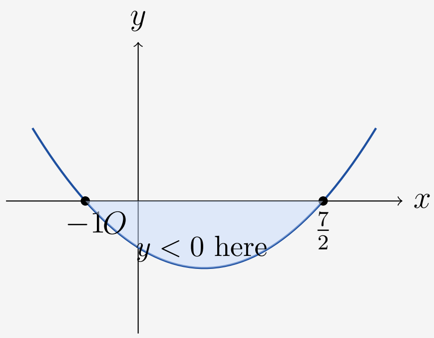
$$3x-6 < 8-2x \implies 5x < 14$$

$$x < 2.8$$

Part (b) — Solve $(2x-7)(1+x) < 0$:

Critical values: $2x-7=0 \Rightarrow x=\frac{7}{2}$ and $1+x=0 \Rightarrow x=-1$.

Sketch:



Between the roots:

$$-1 < x < \frac{7}{2}$$

Part (c) — Both satisfied:

From (a): $x < 2.8$. From (b): $-1 < x < \frac{7}{2}$. Intersection:

$$-1 < x < 2.8$$

Question 6

Worked Solution

Part (a) — Solve $3x - 7 > 3 - x$:

$$4x > 10 \implies x > 2.5$$

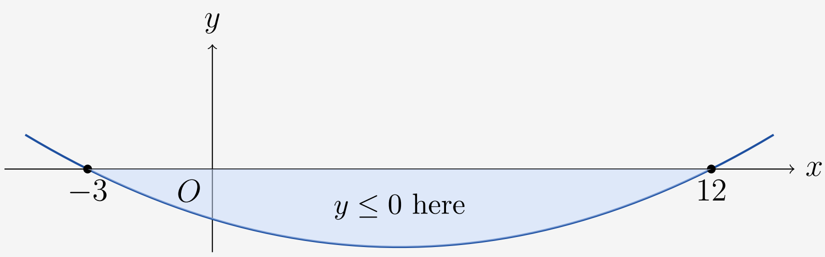
$$x > 2.5$$

Part (b) — Solve $x^2 - 9x \leq 36$:

Rearrange: $x^2 - 9x - 36 \leq 0$

Critical values: $(x-12)(x+3) = 0 \implies x = 12$ or $x = -3$.

Sketch:



Between the roots:

$$-3 \leq x \leq 12$$

Part (c) — Both satisfied:

From (a): $x > 2.5$. From (b): $-3 \leq x \leq 12$. Intersection:

$$2.5 < x \leq 12$$

Question 7

Worked Solution

Width $= x$ m, length $= x + 4$ m.

Part (a) — Show $x > 2.8$:

Perimeter $= 2(x + x + 4) = 4x + 8$. Setting > 19.2:

$$4x + 8 > 19.2 \implies 4x > 11.2 \implies x > 2.8\square$$

Part (b)(i) — Area inequality:

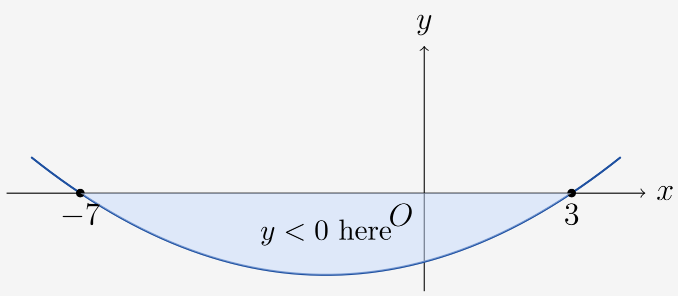
$$x(x + 4) < 21$$

Part (b)(ii) — Solve $x(x + 4) < 21$:

Expand: $x^2 + 4x - 21 < 0$

Critical values: $(x + 7)(x - 3) = 0 \implies x = -7$ or $x = 3$.

Sketch:



Between the roots: $-7 < x < 3$. Since $x > 0$: $0 < x < 3$.

Part (c) — Combined range:

From (a): $x > 2.8$. From (b): $0 < x < 3$. Intersection:

$$2.8 < x < 3$$

Question 8

Worked Solution

The garden has dimensions: full width $= 6x + 3$, full height $= 4x$, with a notch of width $2x + 1$ and depth $2x$ cut from the top left.

Part (a) — Show $x > 1.7$:

The perimeter consists of 6 edges. Working around the shape:

$$P = (6x + 3) + 4x + (6x + 3 - (2x + 1)) + 2x + (2x + 1) + (4x - 2x)$$

Using the mark scheme: $P = 2x + 1 + 2x + 4x + 2 + 2x + 6x + 3 + 4x = 20x + 6$

Set $P > 40$:

$$20x + 6 > 40 \implies 20x > 34 \implies x > 1.7\square$$

Part (b) — Area < 120 m^2:

Area of full rectangle minus the notch:

$$A = (6x + 3)(4x) - (2x + 1)(2x) = 24x^2 + 12x - 4x^2 - 2x = 20x^2 + 10x$$

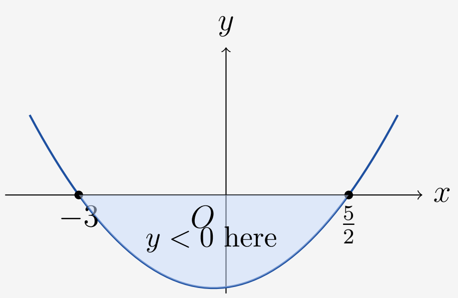
Wait — using the mark scheme method, area $= 2x(2x + 1) + 2x(6x + 3) = 4x^2 + 2x + 12x^2 + 6x = 16x^2 + 8x$.

Set < 120:

$$16x^2 + 8x < 120 \implies 16x^2 + 8x - 120 < 0 \implies 2x^2 + x - 15 < 0$$

Critical values: $(2x - 5)(x + 3) = 0 \implies x = \frac{5}{2}$ or $x = -3$.

Sketch:



Between the roots: $-3 < x < \frac{5}{2}$. Since $x > 0$: $0 < x < \frac{5}{2}$.

Part (c) — Combined range:

From (a): $x > 1.7$. From (b): $0 < x < \frac{5}{2}$. Intersection:

$$1.7 < x < \frac{5}{2}$$

End of Worked Solutions — ALevelMathsRevision.com