Inequalities Exam Questions Sheet 2

Q1.

Question	Scheme	Marks	AOs
	Finds critical values $x^2 - x > 20 \Rightarrow x^2 - x - 20 > 0 \Rightarrow x = (5, -4)$	M1	1.1b
	Chooses outside region for their values Eg. $x > 5, x < -4$	M1	1.1b
	Presents solution in set notation $\{x : x < -4\} \cup \{x : x > 5\}$ oe	A1	2.5
		(3)	
			(3 marks)

Notes

M1: Attempts to find the critical values using an algebraic method. Condone slips but an allowable method should be used and two critical values should be found

M1: Chooses the outside region for their critical values. This may appear in incorrect inequalities such as $5 < x < -4$

A1: Presents in set notation as required $\{x : x < -4\} \cup \{x : x > 5\}$ Accept $\{x < -4 \cup x > 5\}$.

Do not accept $\{x < -4, x > 5\}$

Note: If there is a contradiction of their solution on different lines of working do not penalise intermediate working and mark what appears to be their final answer.

Q2.

Question Number	Scheme		Marks
(a)	$6x+x > 1-8$	Attempts to expand the bracket and collect x terms on one side and constant terms on the other. Condone sign errors and allow one error in expanding the bracket. Allow $<, \leq, \geq, =$ instead of $>$.	M1
	$x > -1$	Cao	A1
	Do not isw here, mark their final answer.		
			(2)
(b)	$(x+3)(3x-1)[=0]$ $\Rightarrow x=-3$ and $\frac{1}{3}$	M1: Attempt to solve the quadratic to obtain two critical values A1: $x=-3$ **and** $\frac{1}{3}$ (may be implied by their inequality). Allow all equivalent fractions for -3 and 1/3. (Allow 0.333 for 1/3)	M1A1
	$-3 < x < \frac{1}{3}$	M1: Chooses "inside" region (The letter x does not need to be used here) A1ft: Allow $x < \frac{1}{3}$ **and** $x > -3$ or $\left(-3, \frac{1}{3}\right)$ or $x < \frac{1}{3} \cap x > -3$. Follow through their critical values. (must be in terms of x here) Allow all equivalent fractions for -3 and 1/3. Both ($x < \frac{1}{3}$ **or** $x > -3$) and ($x < \frac{1}{3}$, $x > -3$) as a final answer score A0.	M1A1ft
			(4)
			[6]
	Note that use of $\leq$ or $\geq$ appearing in an otherwise correct answer in (a) or (b) should only be penalised once, the first time it occurs.		

Q3.

Question	Scheme	Marks
(a)	$5x > 20$	M1
	$\underline{x > 4}$	A1 (2)
(b)	$x^2 - 4x - 12 = 0$	
	$(x+2)(x-6)[=0]$	M1
	$x = 6, -2$	A1
	$x < -2\ ,\ x > 6$	M1, A1ft
		(4)
		6 marks

	Notes
(a)	M1 for reducing to the form $px > q$ with one of p or q correct Using $px = q$ is M0 unless $>$ appears later on A1 $x > 4$ only
(b)	1st M1 for multiplying out and attempting to solve a 3TQ with at least $\pm 4x$ or ± 12 See **General Principles** for definitions of "attempt to solve" 1st A1 for 6 and -2 seen. Allow $x > 6$, $x > -2$ etc to score this mark. Values may be on a sketch. 2nd M1 for choosing the "outside region" for their critical values. Do not award simply for a diagram or table – they must have chosen their "outside" regions 2nd A1ft follow through their 2 distinct critical values. Allow "," "or" or a "blank" between answers. Use of "and" is M1A0 i.e. loses the final A1 $-2 > x > 6$ scores M1A0 i.e. loses the final A1 but apply ISW if $x > 6$, $x < -2$ has been seen Accept $(-\infty,-2)\cup(6,\infty)$ (o.e) Use of $\le$ instead of $<$ (or $\ge$ instead of $>$) loses the final A mark in (b) unless A mark was lost in (a) for $x \ge 4$ in which case allow it here.

Q4.

Question Number	Scheme	Marks
Q (a)	$5x > 10$, $x > 2$ [Condone $x > \frac{10}{5} = 2$ for M1A1]	M1, A1 (2)
(b)	$(2x+3)(x-4) = 0$, 'Critical values' are $-\frac{3}{2}$ and 4	M1, A1
	$-\frac{3}{2} < x < 4$	M1 A1ft (4)
(c)	$2 < x < 4$	B1ft (1) [7]
(a)	M1 for attempt to collect like terms on each side leading to $ax > b$, or $ax < b$, or $ax = b$ Must have a or b correct so eg $3x > 4$ scores M0	
(b)	1st M1 for an attempt to factorize or solve to find critical values. Method must potentially give 2 critical values 1st A1 for $-\frac{3}{2}$ and 4 seen. They may write $x < -\frac{3}{2}$, $x < 4$ and still get this A1 2nd M1 for choosing the "inside region" for their critical values 2nd A1ft follow through their 2 distinct critical values Allow $x > -\frac{3}{2}$ with "or" "," "$\cup$" " " $x < 4$ to score M1A0 but "and" or "$\cap$" score M1A1 $x \in (-\frac{3}{2}, 4)$ is M1A1 but $x \in [-\frac{3}{2}, 4]$ is M1A0. Score M0A0 for a number line or graph only	
(c)	B1ft Allow if **a correct answer is seen** or follow through their answer to (a) and their answer to (b) but their answers to (a) and (b) <u>must be regions</u>. Do not follow through single values. If their follow through answer is the empty set accept $\varnothing$ or {} or equivalent in words If (a) or (b) are not given then score this mark for cao NB You may see $x<4$ (with anything or nothing in-between) $x < -1.5$ in (b) and empty set in (c) for B1ft **Do not award marks for part (b) if only seen in part (c)** Use of $\leq$ instead of $<$ (or $\geq$ instead of $>$) loses one accuracy mark only, at first occurrence.	

Q5.

Question Number	Scheme	Marks	
(a)	$3x-6<8-2x \rightarrow 5x<14$ (Accept $5x-14<0$ (o.e.))	M1	
	$x<2.8$ or $\frac{14}{5}$ or $2\frac{4}{5}$ (condone $\le$)	A1	(2)
(b)	Critical values are $x=\frac{7}{2}$ and -1	B1	
	Choosing "inside" $-1<x<\frac{7}{2}$	M1 A1	(3)
(c)	$-1<x<2.8$	B1ft	(1)
	Accept any exact equivalents to -1, 2.8, 3.5		6

	Notes
(a)	M1 for attempt to rearrange to $kx<m$ (o.e.) Either $k=5$ or $m=14$ should be correct Allow $5x=14$ or even $5x>14$
(b)	B1 for both correct critical values. (May be implied by a correct inequality) M1 ft their values and choose the "inside" region A1 for fully correct inequality (Must be in part (b): do not give marks if only seen in (c)) Condone seeing $x<-1$ in working provided $-1<x$ is in the final answer. e.g. $x>-1,\ x<\frac{7}{2}$ <u>or</u> $x>-1$ "or" $x<\frac{7}{2}$ <u>or</u> $x>-1$ "blank space" $x<\frac{7}{2}$ score M1A0 BUT allow $x>-1$ and $x<\frac{7}{2}$ to score M1A1 (the "and" must be seen) Also $\left(-1,\frac{7}{2}\right)$ will score M1A1 NB $x<-1, x<\frac{7}{2}$ is of course M0A0 and a number line even with "open" ends is M0A0 Allow 3.5 instead of $\frac{7}{2}$
(c)	B1ft for $-1<x<2.8$ (ignoring their previous answers) <u>or</u> ft their answers to part (a) and part (b) provided both answers were regions and not single values. Allow use of "and" between inequalities as in part (b) If their set is empty allow a suitable description in words or the symbol $\varnothing$. <u>Common error:</u> If (a) is correct and in (b) they simply leave their answer as $x<-1$, $x<3.5$ then in (c) $x<-1$ would get B1ft as this is a correct follow through of these 3 inequalities. Penalise use of $\le$ only on the A1 in part (b). [i.e. condone in part (a)]

Q6.

Question Number	Scheme	Marks
	(a) $3x-7>3-x$	
	$4x>10$	M1
	$x>2.5,\quad x>\frac{5}{2},\quad \frac{5}{2}<x$ o.e.	A1
		(2)
	(b) Obtain $x^2-9x-36$ and attempt to solve $x^2-9x-36=0$	
	e.g. $(x-12)(x+3)=0$ so $x=$, or $x=\frac{9\pm\sqrt{81+144}}{2}$	M1
	12, -3	A1
	$-3\le x\le 12$	M1A1
		(4)
	(c) $2.5<x\le 12$	A1cso
		(1)
		(7 marks)

Notes

(a) M1 Reaching $px>q$ with one or both of p or q correct. Also give for $-4x<-10$

A1 Cao $x>2.5$ o.e. Accept alternatives to 2.5 like $2\frac{1}{2}$ and $\frac{5}{2}$ even allow $\frac{10}{4}$ and allow $\frac{5}{2}<x$ o.e. This answer must occur and be credited as part (a) A correct answer implies M1A1

Mark parts (b) and (c) together.

(b) M1 Rearrange 3TQ ≤ 0 or 3TQ $=0$ or even 3TQ >0 Do not worry about the inequality at this stage AND attempt to solve by factorising, formula or completion of the square with the usual rules (see notes)

A1 12 and -3 seen as critical values
M1 Inside region for their critical values – must be stated – not just a table or a graph
A1 $-3\le x\le 12$ Accept $x\ge -3$ **and** $x\le 12$ or $[-3, 12]$
For the A mark: Do not accept $x\ge -3$ **or** $x\le 12$ nor $-3<x<12$ nor $(-3, 12)$ nor $x\ge -3, x\le 12$
However allow recovery if they follow these statements by a correct statement, either in (b) or as they start part (c)
N.B. $-3\le 0\le 12$ and $x\ge -3, x\le 12$ are poor notation and get M1A0 here.

(c) A1 cso $2.5<x\le 12$ Accept $x>2.5$ and $x\le 12$ Allow $\frac{10}{4}$ Do not accept $x>2.5$ **or** $x\le 12$

Accept (2.5, 12] A graph or table is not sufficient. **Must follow correct earlier work** – except for special case

Special case (c) $x>2.5, x\le 12$; $2.5<0\le 12$ are poor notation – but if this poor notation has been penalised in (b) then allow A1 here. Any other errors are penalised in both (b) and (c).

Q7.

Question Number	Scheme	Notes	Marks
	Ignore any references to the units in this question		
(a)	length is '$x+4$'	May be implied	B1
	$x+x+x+4+x+4>19.2 \Rightarrow x>..$	$2x+2(x\pm4)>19.2$ and proceeds to $x>$ ..... (Accept 'invisible' brackets) Attempts 2 widths + 2 lengths > 19.2 leading to $x>$	M1
	E.g. $x+x+4x+4x>19.2 \Rightarrow x>1.92$ scores B0M1A0		
	$x>2.8$ *	Achieves $x>2.8$ with no errors	A1(*)
			(3)
	Mark parts (b) and (c) together		
(b)(i)	$x(x+4)<21$	Cao	B1
b(ii)	$x^2+4x-21<0$ $(x+7)(x-3)<0 \Rightarrow x=...$	Multiply out lhs, produce 3TQ = 0 and attempt to solve leading to $x=$... according to general guidelines	M1
	Either $-7<x<3$ or $0<x<3$	M1: Attempts the 'inside' for their critical values (may be from a 2TQ here) A1: Accept either $-7<x<3$ or $0<x<3$ or ($x>-7$ **and** $x<3$) or ($x>0$ **and** $x<3$) but not e.g. ($x>-7$, $x<3$) or ($x>-7$ **or** $x<3$) (There is no specific need for them to realise $x>0$)	M1A1
	Note that <u>many</u> candidates stop here		
			(4)
(c)	$2.8<x<3$	Follow through their answers to (a) and (b) Provided "their 3" > 2.8	B1ft
			(1)
			[8]
	Examples		
	$x(x-4)<21 \Rightarrow x^2-4x-21<0$ $(x-7)(x+3)<0,\ x=7, x=-3$ $-3<x<7$ *or* $0<x<7$ $2.8<x<7$ Scores B0M1M1A0B1ft	$x\times4x<21 \Rightarrow 4x^2-21<0$ $(2x-\sqrt{21})(2x+\sqrt{21})<0,\ x=\pm\frac{\sqrt{21}}{2}$ $-\frac{\sqrt{21}}{2}<x<\frac{\sqrt{21}}{2}$ *or* $0<x<\frac{\sqrt{21}}{2}$ $2.8<x<\frac{\sqrt{21}}{2}$ Scores B0M0M1A0B0	

Q8.

Question Number	Scheme	Marks
(a).	$P = 20x + 6$ o.e	B1
	$20x + 6 > 40 \Rightarrow x >$	M1
	$x > 1.7$	A1*
		(3)
	Mark parts (b) and (c) together	
(b)	$A = 2x(2x+1) + 2x(6x+3) = 16x^2 + 8x$	B1
	$16x^2 + 8x - 120 < 0$	M1
	Try to solve their $2x^2 + x - 15 = 0$ e.g. $(2x-5)(x+3) = 0$ so $x =$	M1
	Choose inside region	M1
	$-3 < x < \frac{5}{2}$ or $0 < x < \frac{5}{2}$ (as x is a length)	A1
(c)		(5)
	$1.7 < x < \frac{5}{2}$	B1cao
		(1)
		(9 marks)

(a) B1 Correct expression for perimeter but may not be simplified so accept $2x + 1 + 2x + 4x + 2 + 2x + 6x + 3 + 4x$ or $2(10x + 3)$ or any equivalent

M1: Set $P > 40$ with their linear expression for P (this may not be correct but should be a sum of sides) and manipulate to get $x >$…

A1* cao $x > 1.7$. This is a given answer, there must not be any errors, but accept $1.7 < x$

(b) Marks parts (b) and (c) together

B1 Writes a correct statement in x for the area. It need not be simplified. You may isw Amongst numerous possibilities are.
$2x(2x+1) + 2x(6x+3)$, $16x^2 + 8x$, $4x(6x+3) - 2x(4x+2)$, $4x(2x+1)+2x(4x+2)$

M1 Sets their quadratic expression < 120 and collects on one side of the inequality

M1 For an attempt to solve a 3 Term quadratic equation producing two solutions by factorising, formula or completion of the square with the usual rules (see notes)

M1 For choosing the 'inside' region. Can follow through from their critical values – must be stated – not just a table or a graph. Can also be implied by $0 < x <$ upper value

A1 $-3 < x < \frac{5}{2}$. Accept $x > -3$ **and** $x < 2.5$ or (-3, 2.5)

As x is a width, accept $0 < x < \frac{5}{2}$ Also accept $\frac{10}{4}$ or 2.5 instead of $\frac{5}{2}$. $\leq$ would be M1A0

Allow this final answer to part (b) to appear as answer to part (c) This would score final M1A1 in (b) then B0 in (c)

(c) B1cao $1.7 < x < \frac{5}{2}$. Must be correct. [This does not imply final M1 in (b)]