Question 1

Worked Solution

Part (i) — Evaluate $(0.2)^{-2}$:

A negative exponent means take the reciprocal, and $0.2 = \frac{1}{5}$:

$$(0.2)^{-2} = \left(\frac{1}{5}\right)^{-2} = 5^2 = 25$$

$$(0.2)^{-2} = 25$$

Part (ii) — Simplify $(16a^{12})^{\frac{3}{4}}$:

Apply the power $\frac{3}{4}$ to each factor separately:

$$\left(16a^{12}\right)^{\frac{3}{4}} = 16^{\frac{3}{4}} \times \left(a^{12}\right)^{\frac{3}{4}}$$

Coefficient: $16^{\frac{3}{4}} = \left(16^{\frac{1}{4}}\right)^3 = 2^3 = 8$

Variable: $(a^{12})^{\frac{3}{4}} = a^{12\times\frac{3}{4}} = a^9$

$$\left(16a^{12}\right)^{\frac{3}{4}} = 8a^9$$

Question 2

Worked Solution

Part (i) — Find $\left(\frac{5}{3}\right)^{-2}$:

A negative exponent inverts the fraction:

$$\left(\frac{5}{3}\right)^{-2} = \left(\frac{3}{5}\right)^{2} = \frac{9}{25}$$

$$\left(\frac{5}{3}\right)^{-2} = \frac{9}{25} = 0.36$$

Part (ii) — Find $81^{\frac{3}{4}}$:

$$81^{\frac{3}{4}} = \left(81^{\frac{1}{4}}\right)^{3} = 3^{3} = 27$$

since $81^{\frac{1}{4}} = \sqrt[4]{81} = 3$.

$$81^{\frac{3}{4}} = 27$$

Question 3

Worked Solution

Simplify $\dfrac{(4x^5y)^3}{(2xy^2)\times(8x^{10}y^4)}$**:**

Expand the numerator:

$$(4x^5y)^3 = 4^3\cdot x^{15}\cdot y^3 = 64x^{15}y^3$$

Expand the denominator:

$$(2xy^2)\times(8x^{10}y^4) = 16x^{11}y^6$$

Divide:

$$\frac{64x^{15}y^3}{16x^{11}y^6} = 4\cdot x^{15-11}\cdot y^{3-6} = 4x^4y^{-3}$$

$$\frac{(4x^5y)^3}{(2xy^2)\times(8x^{10}y^4)} = 4x^4y^{-3} = \frac{4x^4}{y^3}$$

Question 4

Worked Solution

Part (i) — Evaluate $9^{-\frac{1}{2}}$:

$$9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

$$9^{-\frac{1}{2}} = \frac{1}{3}$$

Part (ii) — Simplify $\dfrac{(4x^4)^3\,y^2}{2x^2y^5}$:

Expand the numerator:

$$(4x^4)^3 \cdot y^2 = 64x^{12} \cdot y^2$$

Divide:

$$\frac{64x^{12}y^2}{2x^2y^5} = 32 \cdot x^{12-2} \cdot y^{2-5} = 32x^{10}y^{-3}$$

$$\frac{(4x^4)^3\,y^2}{2x^2y^5} = 32x^{10}y^{-3} = \frac{32x^{10}}{y^3}$$

Question 5

Worked Solution

Express each of the following in the form 2^p.

Part (i) — $(2^5 \div 2^7)^3$**:**

$$(2^5 \div 2^7)^3 = (2^{5-7})^3 = (2^{-2})^3 = 2^{-6}$$

$$(2^5 \div 2^7)^3 = 2^{-6}$$

Part (ii) — $5 \times 4^{\frac{2}{3}} + 3 \times 16^{\frac{1}{3}}$**:**

Express everything as a power of 2. Note $4 = 2^2$ and $16 = 2^4$:

$$5 \times 4^{\frac{2}{3}} = 5 \times (2^2)^{\frac{2}{3}} = 5 \times 2^{\frac{4}{3}}$$

$$3 \times 16^{\frac{1}{3}} = 3 \times (2^4)^{\frac{1}{3}} = 3 \times 2^{\frac{4}{3}}$$

Adding:

$$5 \times 2^{\frac{4}{3}} + 3 \times 2^{\frac{4}{3}} = 8 \times 2^{\frac{4}{3}} = 2^3 \times 2^{\frac{4}{3}} = 2^{3+\frac{4}{3}} = 2^{\frac{13}{3}}$$

$$5 \times 4^{\frac{2}{3}} + 3 \times 16^{\frac{1}{3}} = 2^{\frac{13}{3}}$$

Question 6

Worked Solution

Express each of the following in the form 5^k.

Part (i) — 25^4**:**

$$25^4 = (5^2)^4 = 5^8$$

$$25^4 = 5^8$$

Part (ii) — $\dfrac{1}{\sqrt[4]{5}}$**:**

$$\frac{1}{\sqrt[4]{5}} = \frac{1}{5^{\frac{1}{4}}} = 5^{-\frac{1}{4}}$$

$$\frac{1}{\sqrt[4]{5}} = 5^{-\frac{1}{4}}$$

Part (iii) — $(5\sqrt{5})^3$**:**

$$(5\sqrt{5})^3 = (5^1 \cdot 5^{\frac{1}{2}})^3 = (5^{\frac{3}{2}})^3 = 5^{\frac{9}{2}}$$

$$(5\sqrt{5})^3 = 5^{\frac{9}{2}}$$

Question 7

Worked Solution

Part (i) — Solve $3^n = 1$:

Any base raised to the power 0 equals 1:

$$3^n = 3^0 \implies n = 0$$

$$n = 0$$

Part (ii) — Solve $t^{-3} = 64$:

$$t^{-3} = 64 \implies \frac{1}{t^3} = 64 \implies t^3 = \frac{1}{64}$$

$$t = \left(\frac{1}{64}\right)^{\frac{1}{3}} = \frac{1}{4}$$

$$t = \frac{1}{4}$$

Part (iii) — Solve $(8p^6)^{\frac{1}{3}} = 8$:

Apply the power $\frac{1}{3}$ to each factor:

$$8^{\frac{1}{3}} \cdot \left(p^6\right)^{\frac{1}{3}} = 8$$

$$2 \cdot p^2 = 8$$

$$p^2 = 4 \implies p = \pm 2$$

$$p = 2 \quad \text{or} \quad p = -2$$

Question 8

Worked Solution

Express each of the following in the form 7^k.

Part (i) — $\sqrt[4]{7}$:

$$\sqrt[4]{7} = 7^{\frac{1}{4}}$$

$$\sqrt[4]{7} = 7^{\frac{1}{4}}$$

Part (ii) — $\dfrac{1}{7\sqrt{7}}$:

$$\frac{1}{7\sqrt{7}} = \frac{1}{7^1 \cdot 7^{\frac{1}{2}}} = \frac{1}{7^{\frac{3}{2}}} = 7^{-\frac{3}{2}}$$

$$\frac{1}{7\sqrt{7}} = 7^{-\frac{3}{2}}$$

Part (iii) — $7^4 \times 49^{10}$:

Write $49 = 7^2$:

$$7^4 \times 49^{10} = 7^4 \times (7^2)^{10} = 7^4 \times 7^{20} = 7^{24}$$

$$7^4 \times 49^{10} = 7^{24}$$

End of Worked Solutions — ALevelMathsRevision.com