Question 1

Worked Solution

Part (a) — Find the value of $16^{-\frac{1}{4}}$:

A negative fractional power means take the positive root and then invert:

$$16^{-\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}} = \frac{1}{\sqrt[4]{16}} = \frac{1}{2}$$

$$16^{-\frac{1}{4}} = \frac{1}{2}$$

Part (b) — Simplify $x\left(2x^{-\frac{1}{4}}\right)^4$:

First expand the bracket by applying the power 4 to both factors inside:

$$\left(2x^{-\frac{1}{4}}\right)^4 = 2^4 \cdot x^{-\frac{1}{4}\times 4} = 16x^{-1}$$

Now multiply by x:

$$x \cdot 16x^{-1} = 16x^{1+(-1)} = 16x^0 = 16$$

$$x\left(2x^{-\frac{1}{4}}\right)^4 = 16$$

Question 2

Worked Solution

Given that $32\sqrt{2} = 2^a$, find the value of a.

Express each factor as a power of 2:

$$32 = 2^5 \qquad \sqrt{2} = 2^{\frac{1}{2}}$$

So:

$$32\sqrt{2} = 2^5 \times 2^{\frac{1}{2}} = 2^{5+\frac{1}{2}} = 2^{\frac{11}{2}}$$

$$a = \frac{11}{2} = 5.5$$

Question 3

Worked Solution

Express 8^{2x+3} in the form 2^y, stating y in terms of x.

Write $8 = 2^3$:

$$8^{2x+3} = \left(2^3\right)^{2x+3} = 2^{3(2x+3)} = 2^{6x+9}$$

$$8^{2x+3} = 2^y \quad \text{where} \quad y = 6x + 9$$

Question 4

Worked Solution

Part (a) — Solve $2^y = 8$:

$$2^y = 8 = 2^3 \implies y = 3$$

$$y = 3$$

Part (b) — Solve $2^x \times 4^{x+1} = 8$:

Convert everything to powers of 2. Note $4 = 2^2$ and $8 = 2^3$:

$$4^{x+1} = \left(2^2\right)^{x+1} = 2^{2(x+1)} = 2^{2x+2}$$

So the equation becomes:

$$2^x \times 2^{2x+2} = 2^3$$

Add the powers on the left:

$$2^{x+2x+2} = 2^3 \implies 2^{3x+2} = 2^3$$

Equate exponents:

$$3x + 2 = 3 \implies 3x = 1$$

$$x = \frac{1}{3}$$

Question 5

Worked Solution

Given $y = 2^x$.

Part (a) — Express 4^x in terms of y:

$$4^x = \left(2^2\right)^x = 2^{2x} = (2^x)^2 = y^2$$

$$4^x = y^2$$

Part (b) — Solve $8(4^x) - 9(2^x) + 1 = 0$:

Substitute $4^x = y^2$ and $2^x = y$:

$$8y^2 - 9y + 1 = 0$$

Factorise:

$$(8y - 1)(y - 1) = 0$$

So $y = \frac{1}{8}$ or $y = 1$.

Case 1: $2^x = \frac{1}{8} = 2^{-3}$

$$x = -3$$

Case 2: $2^x = 1 = 2^0$

$$x = 0$$

$$x = -3 \quad \text{or} \quad x = 0$$

Question 6

Worked Solution

Express 9^{3x+1} in the form 3^y, giving y in the form $ax+b$.

Write $9 = 3^2$:

$$9^{3x+1} = \left(3^2\right)^{3x+1} = 3^{2(3x+1)} = 3^{6x+2}$$

$$9^{3x+1} = 3^y \quad \text{where} \quad y = 6x+2 \quad (a=6,\ b=2)$$

Question 7

Worked Solution

Part (a) — Show that $2^{2x+1} - 17(2^x) + 8 = 0$ can be written as $2y^2 - 17y + 8 = 0$:

Given $y = 2^x$, rewrite 2^{2x+1} using the addition law of indices:

$$2^{2x+1} = 2^{2x} \times 2^1 = 2 \times 2^{2x} = 2 \times (2^x)^2 = 2y^2$$

Also $2^x = y$, so substituting into the equation:

$$2^{2x+1} - 17(2^x) + 8 = 2y^2 - 17y + 8 = 0 \square$$

Part (b) — Hence solve $2^{2x+1} - 17(2^x) + 8 = 0$:

Solve $2y^2 - 17y + 8 = 0$ by factorising:

$$(2y - 1)(y - 8) = 0$$

So $y = \frac{1}{2}$ or $y = 8$.

Case 1: $2^x = \frac{1}{2} = 2^{-1}$

$$x = -1$$

Case 2: $2^x = 8 = 2^3$

$$x = 3$$

$$x = -1 \quad \text{or} \quad x = 3$$

End of Worked Solutions — ALevelMathsRevision.com