Indices Sheet 2 Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$16^{\frac{1}{4}} = 2$ or $\dfrac{1}{16^{\frac{1}{4}}}$ or better	M1
	$\left(16^{-\frac{1}{4}} =\right)$ $\dfrac{1}{2}$ or 0.5 (ignore $\pm$)	A1
		(2)
(b)	$\left(2x^{-\frac{1}{4}}\right)^4 = 2^4 x^{-\frac{4}{4}}$ or $\dfrac{2^4}{x^{\frac{4}{4}}}$ or equivalent	M1
	$x\left(2x^{-\frac{1}{4}}\right)^4 = 2^4$ or 16	A1 cao
		(2) **4**
	Notes	
(a)	M1 for a correct statement dealing with the $\frac{1}{4}$ or the $-$ power This may be awarded if 2 is seen or for reciprocal of their $16^{\frac{1}{4}}$ s.c ¼ is M1 A0 , also 2^{-1} is M1 A0 $\pm\frac{1}{2}$ is not penalised so M1 A1	
(b)	M1 for correct use of the power 4 on both the 2 and the x terms A1 for cancelling the x and simplifying to one of these two forms. Correct answers with no working get full marks	

Q2.

Question Number	Scheme	Marks
Q	$32 = 2^5$ or $2048 = 2^{11}$, $\sqrt{2} = 2^{1/2}$ or $\sqrt{2048} = (2048)^{\frac{1}{2}}$ $a = \frac{11}{2}$ $\left(\text{or } 5\frac{1}{2} \text{ or } 5.5\right)$	B1, B1 B1 [3]
	1st B1 for $32 = 2^5$ or $2048 = 2^{11}$ This should be explicitly seen: $32\sqrt{2} = 2^a$ followed by $2^5\sqrt{2} = 2^a$ is OK Even writing $32 \times 2 = 2^5 \times 2 \left(= 2^6\right)$ is OK but simply writing $32 \times 2 = 2^6$ is NOT 2nd B1 for $2^{\frac{1}{2}}$ or $(2048)^{\frac{1}{2}}$ seen. This mark may be implied 3rd B1 for answer as written. **Need** $a =$ … so $2^{\frac{11}{2}}$ is B0 $a = \frac{11}{2}$ $\left(\text{or } 5\frac{1}{2} \text{ or } 5.5\right)$ with no working scores full marks. If $a = 5.5$ seen then award 3/3 unless it is clear that the value follows from totally incorrect work. Part solutions: e.g. $2^5\sqrt{2}$ scores the first B1. <u>Special case</u>: If $\sqrt{2} = 2^{1/2}$ is not explicitly seen, but the final answer includes $\frac{1}{2}$, e.g. $a = 2\frac{1}{2}$, $a = 4\frac{1}{2}$, the second B1 is given by implication.	

Q3.

Question Number	Scheme	Marks
	$(8^{2x+3} = (2^3)^{2x+3}) = 2^{3(2x+3)}$ or 2^{ax+b} with $a=6$ or $b=9$	M1
	$= 2^{6x+9}$ $or = 2^{3(2x+3)}$ as **final** answer with no errors or $(y=)6x+9$ or $3(2x+3)$	A1
		[2]
		2 marks
	Notes	
	M1: Uses $8=2^3$, and **multiplies** powers $3(2x+3)$. Does **not** add powers. (Just $8=2^3$ or $8^{\frac{1}{3}}=2$ is M0)	
	A1: Either 2^{6x+9} $or = 2^{3(2x+3)}$ or $(y=)6x+9$ or $3(2x+3)$	
	Note: Examples: 2^{6x+3} scores M1A0	
	: $8^{2x+3} = (2^3)^{2x+3} = 2^{3+2x+3}$ gets M0A0	
	Special case: : $= 2^{6x}2^9$ without seeing as single power M1A0	
	Alternative method using logs: $8^{2x+3} = 2^y \Rightarrow (2x+3)\log 8 = y\log 2 \Rightarrow y = \dfrac{(2x+3)\log 8}{\log 2}$	**M1**
	So $(y=)6x+9$ or $3(2x+3)$	**A1 [2]**

Q4.

Question Number	Scheme	Notes	Marks
(a)	$2^y = 8 \Rightarrow y = 3$	Cao (Can be implied i.e. by 2^3)	B1
	(Alternative: Takes logs base 2: $\log_2 2^y = \log_2 8 \Rightarrow y\log_2 2 = 3\log_2 2 \Rightarrow y = 3$)		
			(1)
(b)	$8 = 2^3$	Replaces 8 by 2^3 (May be implied)	M1
	$4^{x+1} = (2^2)^{x+1}$ or $(2^{x+1})^2$	Replaces 4 by 2^2 **correctly**.	M1
	$2^{3x+2} = 2^3 \Rightarrow 3x+2=3 \Rightarrow x=\frac{1}{3}$	M1: Adds their powers of 2 on the lhs and puts this equal to 3 leading to a solution for x.	M1A1
		A1: $x=\frac{1}{3}$ or $x=0.\dot{3}$ or awrt 0.333	
			(4)
(b) Way 2	$4^{x+1} = 4\times 4^x$	Obtains 4^{x+1} in terms of 4^x **correctly**	M1
	$2^x \times 4^x = 8^x$	Combines their 2^x and 4^x **correctly**	M1
	$4\times 8^x = 8 \Rightarrow 8^x = 2 \Rightarrow x = \frac{1}{3}$	M1: Solves $8^x = k$ leading to a solution for x.	M1A1
		A1: $x=\frac{1}{3}$ or $x=0.\dot{3}$ or awrt 0.333	
			[5]

Q5.

Question Number	Scheme		Marks
(a)	$(4^x =)y^2$	Allow y^2 or $y \times y$ or "y squared" "$4^x =$" not required	B1
	Must be seen in part (a)		
			(1)
(b)	$8y^2 - 9y + 1 = (8y-1)(y-1) = 0 \Rightarrow y = ...$ or $(8(2^x)-1)((2^x)-1) = 0 \Rightarrow 2^x = ...$	For attempting to solve the given equation as a **3 term quadratic** in y or as a **3 term quadratic** in 2^x leading to a value of y or 2^x (Apply usual rules for solving the quadratic – see general guidance) Allow x (or any other letter) instead of y for this mark e.g. an attempt to solve $8x^2 - 9x + 1 = 0$	M1
	$2^x (\text{or } y) = \frac{1}{8},\ 1$	Both correct answers of $\frac{1}{8}$ (oe) and 1 for 2^x or y or their letter but **<u>not x</u>** unless $2^x (\text{or } y)$ is implied later	A1
	$x = -3 \quad x = 0$	M1: A correct attempt to find one **numerical** value of x from their $2^x (\text{or } y)$ **which must have come from a 3 term quadratic equation**. If logs are used then they must be evaluated. A1: Both $x = -3$ and/or $x = 0$ May be implied by e.g. $2^{-3} = \frac{1}{8}$ and $2^0 = 1$ **and no extra values.**	M1A1
			(4)
			(5 marks)

Q6.

Question Number	Scheme	Notes	Marks
	$9^{3x+1}=$ for example $3^{2(3x+1)}$ or $\left(3^2\right)^{3x+1}$ or $\left(3^{(3x+1)}\right)^2$ or $3^{3x+1}\times 3^{3x+1}$ or $(3\times 3)^{3x+1}$ or $3^2\times\left(3^2\right)^{3x}$ or $\left(9^{\frac{1}{2}}\right)^y$ or $9^{\frac{1}{2}y}$ or $y=2(3x+1)$	Expresses 9^{3x+1} correctly as a power of 3 or expresses 3^y correctly as a power of 9 or expresses y correctly in terms of x **(This mark is <u>not</u> for just $3^2=9$)**	M1
	$=3^{6x+2}$ or $y=6x+2$ or $a=6$, $b=2$	Cao (isw if necessary)	A1
	Providing there is no incorrect work, allow sight of $6x+2$ to score both marks Correct answer only implies both marks **Special case:** 3^{6x+1} **only** scores M1A0		
			[2]
	Alternative using logs		
	$9^{3x+1}=3^y \Rightarrow \log 9^{3x+1}=\log 3^y$		
	$(3x+1)\log 9 = y\log 3$	Use power law correctly on both sides	M1
	$y=\dfrac{\log 9}{\log 3}(3x+1)$		
	$y=6x+2$	cao	A1
			2 marks

Q7.

Question Number	Scheme		Marks
(a)	Replaces 2^{2x+1} with $2^{2x}\times 2$ or states $2^{2x+1}=2^{2x}\times 2$ or states $\left(2^x\right)^2=2^{2x}$	Uses the addition **or** power law of indices on 2^{2x} or 2^{2x+1}. E.g. $2^x\times 2^x=2^{2x}$ or $\left(2^x\right)^2=2^{2x}$ or $2^{2x+1}=2\times 2^{2x}$ or $2^{x+0.5}=2^x\times\sqrt{2}$ or $2^{2x+1}=\left(2^{x+0.5}\right)^2$.	M1
	$2^{2x+1}-17\times 2^x+8=0$ $\Rightarrow 2y^2-17y+8=0$*	Cso. Complete proof that includes explicit statements for the addition **and** power law of indices on 2^{2x+1} with no errors. The equation needs to be as printed including the "= 0". If they work backwards, they do not need to write down the printed answer first but must end with the version in 2^x including '= 0'.	A1*

	The following are examples of acceptable proofs.	
	$2^{2x+1}=\left(2^{x+0.5}\right)^2=\left(2^x\sqrt{2}\right)^2=\left(y\sqrt{2}\right)^2=2y^2$ $\Rightarrow 2^{2x+1}-17\left(2^x\right)+8=2y^2-17y+8=0$	
	$2y^2=2\times2^x\times2^x=2^{2x+1}$ $\Rightarrow 2^{2x+1}-17\left(2^x\right)+8=2y^2-17y+8=0$	
	$2y^2-17y+8=0\Rightarrow 2\left(2^x\right)^2-17\left(2^x\right)+8=0$ $\Rightarrow 2\times2^{2x}-17\left(2^x\right)+8=0\Rightarrow 2^{2x+1}-17\left(2^x\right)+8=0$	
	$2^{2x+1}=2\times2^{2x}\Rightarrow 2\times2^{2x}-17\left(2^x\right)+8=0$ $\Rightarrow 2y^2-17y+8=0$ Scores **M1A0** as $2^{2x}=\left(2^x\right)^2$ has not been shown explicitly	
	Special Case: $2^{2x+1}=2^1\times\left(2^x\right)^2$ or $2^{2x+1}=\left(2^x\right)^2\times2^1$ **With or without the multiplication signs and with no subsequent explicit evidence of the power law scores M1A0**	
	Example of insufficient working: $2^{2x+1}=2\left(2^x\right)^2=2y^2$ **scores no marks as neither rule has been shown explicitly.**	
		(2)

(b)	$2y^2-17y+8=0\Rightarrow(2y-1)(y-8)(=0)\Rightarrow y=\ldots$ or $2\left(2^x\right)^2-17\left(2^x\right)+8=0\Rightarrow\left(2\left(2^x\right)-1\right)\left(\left(2^x\right)-8\right)(=0)\Rightarrow 2^x=\ldots$ Solves the **given** quadratic either in terms of y or in terms of 2^x See General Principles for solving a 3 term quadratic Note that completing the square on e.g. $y^2-\frac{17}{2}y+4=0$ requires $\left(y\pm\frac{17}{4}\right)^2\pm q\pm4=0\Rightarrow y=\ldots$		M1
	$(y=)\frac{1}{2},8$ or $\left(2^x=\right)\frac{1}{2},8$	Correct values	A1
	$\Rightarrow 2^x=\frac{1}{2},8\Rightarrow x=-1,3$	M1: Either finds one correct value of x for their 2^x or obtains a correct numerical expression in terms of logs e.g. for $k>0$ $2^x=k\Rightarrow x=\log_2 k$ or $\frac{\log k}{\log 2}$ A1: $x=-1,3$ only. Must be values of x.	M1 A1
			(4)
			(6 marks)