Question 1 (Jun 2009, Q10)

Illustrate on one graph: $y \geq x - 1$, $x \geq 2$, $2x + y \geq 8$. Shade the **excluded** region. Then write down the minimum value of y in the feasible region.

Worked Solution

Method: which side of the line to shade.
Rearrange each inequality to make y the subject. Then:

- $y \geq f(x)$ means keep **above** the line. Shade **below** as excluded.
- $y \leq f(x)$ means keep **below** the line. Shade **above** as excluded.

For a **vertical** line $x = c$: read the inequality directly. $x \geq c$ means keep the right side; shade the left as excluded.

Boundary lines and which side to keep:

(1) $y \geq x - 1$. Already in the form $y \geq f(x)$. Boundary: $y = x - 1$, through $(0, -1)$ and $(1, 0)$. Inequality says $y \geq$, so keep **above** the line. Shade **below** as excluded.

(2) $x \geq 2$. Vertical line $x = 2$. Inequality says $x \geq$, so keep **right** of the line. Shade $x < 2$ as excluded.

(3) $2x + y \geq 8 \Rightarrow y \geq 8 - 2x$. Boundary: $y = 8 - 2x$, through $(0, 8)$ and $(4, 0)$. Inequality says $y \geq$, so keep **above** the line. Shade **below** as excluded.

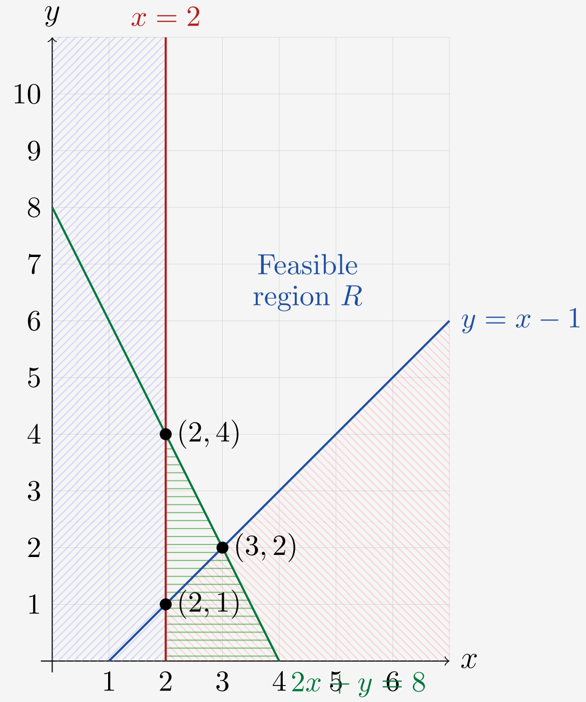
Key intersections:

$y = x - 1$ and $x = 2$: $x = 2$, $y = 1 \Rightarrow$ vertex $(2, 1)$.

$y = x - 1$ and $2x + y = 8$: $2x + (x - 1) = 8 \Rightarrow 3x = 9 \Rightarrow x = 3$, $y = 2 \Rightarrow$ vertex $(3, 2)$.

$x = 2$ and $2x + y = 8$: $4 + y = 8 \Rightarrow y = 4 \Rightarrow$ vertex $(2, 4)$.

The feasible region is the triangle with vertices $(2, 1)$, $(3, 2)$, $(2, 4)$ and extending upward/rightward from there (it is unbounded).



Part (ii): Minimum value of y in the feasible region.

The feasible region is bounded below-left by the vertex $(2, 1)$ (intersection of $x = 2$ and $y = x - 1$). This is the point with the smallest y-value in the region.

Minimum value of $y = 2$ (at vertex $(3, 2)$, the lowest point of the feasible region not on $x = 2$)

Note: Checking both lower vertices: $(2, 1)$ and $(3, 2)$. At $(2, 1)$, all three inequalities are satisfied: $1 \geq 2 - 1 = 1$ ✓, $2 \geq 2$ ✓, $4 + 1 = 5 \geq 8$? **No** — $(2, 1)$ does not satisfy $2x + y \geq 8$. So the minimum y in the feasible region occurs at $(3, 2)$.

Minimum value of $y = 2$

Question 2 (Jun 2011, Q8i)

Indicate the region satisfying all four inequalities. Shade the region that is **not** required.

$$5x + 3y \geq 30, \qquad 3x + y \geq 12, \qquad y \geq 0, \qquad x \geq 0$$

Worked Solution

Boundary lines and which side to keep:

(1) $5x + 3y \geq 30 \;\Rightarrow\; y \geq \dfrac{30 - 5x}{3}$. Boundary through $(6, 0)$ and $(0, 10)$. Inequality says $y \geq$, so keep **above** the line. Shade **below** as excluded.

(2) $3x + y \geq 12 \;\Rightarrow\; y \geq 12 - 3x$. Boundary through $(4, 0)$ and $(0, 12)$. Inequality says $y \geq$, so keep **above** the line. Shade **below** as excluded.

(3) $y \geq 0$: keep **above** the x-axis. Shade $y < 0$ as excluded.

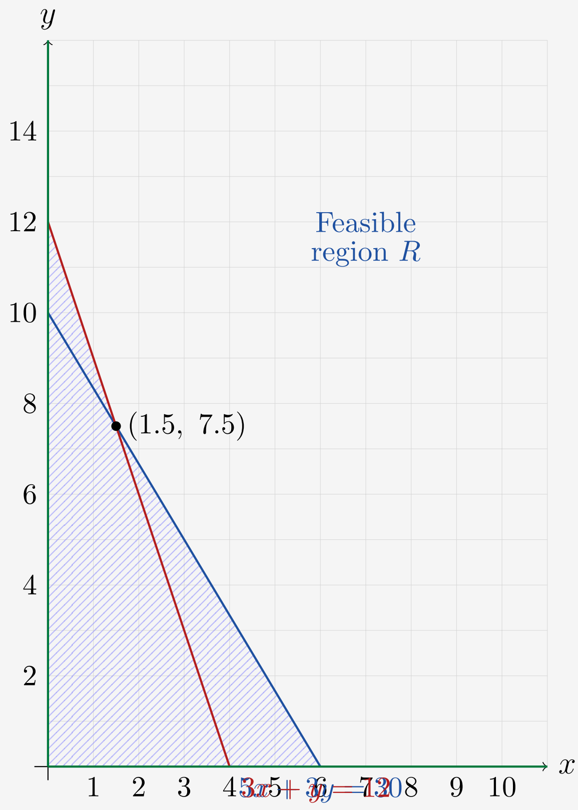
(4) $x \geq 0$: vertical line at $x = 0$. Keep **right** of the y-axis. Shade $x < 0$ as excluded.

Key intersection: $5x + 3y = 30$ and $3x + y = 12$:

From (2): $y = 12 - 3x$. Sub into (1): $5x + 3(12 - 3x) = 30 \Rightarrow 5x + 36 - 9x = 30 \Rightarrow -4x = -6 \Rightarrow x = 1.5$, $y = 7.5$.

So the two lines meet at $(1.5,\ 7.5)$.

The feasible region is the area *above and to the right* of both lines, in the first quadrant. It is unbounded (extends to large x and y).



The hatched region is excluded. The unshaded area (top-right of both lines, in the first quadrant) is where all four inequalities hold simultaneously.

Question 3 (Jun 2016, Q10i)

Indicate the region satisfying all inequalities. Shade the region **not** satisfied.

$$4x + 3y \leq 30, \qquad y \geq 2x, \qquad x \geq 1$$

Worked Solution

Boundary lines and which side to keep:

(1) $4x + 3y \leq 30 \Rightarrow y \leq \frac{30 - 4x}{3}$. Boundary through $(7.5, 0)$ and $(0, 10)$. Inequality says $y \leq$, so keep **below** the line. Shade **above** as excluded.

(2) $y \geq 2x$. Already in the form $y \geq f(x)$. Boundary: $y = 2x$, through $(0, 0)$ and $(5, 10)$. Inequality says $y \geq$, so keep **above** the line. Shade **below** as excluded.

(3) $x \geq 1$. Vertical line $x = 1$. Inequality says $x \geq$, so keep **right** of the line. Shade $x < 1$ as excluded.

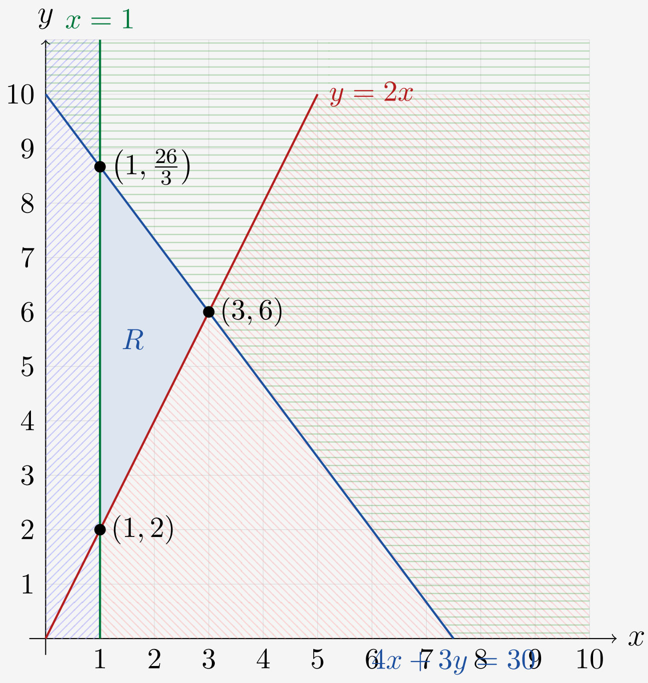
Key intersections:

$y = 2x$ and $x = 1$: $y = 2 \Rightarrow$ vertex $(1, 2)$.

$y = 2x$ and $4x + 3y = 30$: $4x + 6x = 30 \Rightarrow 10x = 30 \Rightarrow x = 3$, $y = 6 \Rightarrow$ vertex $(3, 6)$.

$4x + 3y = 30$ and $x = 1$: $4 + 3y = 30 \Rightarrow y = \frac{26}{3} \approx 8.67 \Rightarrow$ vertex $(1, \frac{26}{3})$.

The feasible region is bounded by these three vertices and is a triangle (since the three constraints together create a closed region).



Reading the diagram: Three different hatch patterns show the three excluded zones. The unshaded triangle R with vertices $(1, 2)$, $(3, 6)$, $(1, \frac{26}{3})$ is the feasible region where all three inequalities hold simultaneously.

End of Worked Solutions