

Question 1 (Jan 2007, Q3)**Worked Solution**

(i) Find $\int (4x - 5) dx$.

$$\int (4x - 5) dx = 2x^2 - 5x + c$$

$$2x^2 - 5x + c$$

(ii) The gradient of a curve is $\frac{dy}{dx} = 4x - 5$. The curve passes through $(3, 7)$. Find the equation.

From (i): $y = 2x^2 - 5x + c$

Use $(3, 7)$:

$$7 = 2(9) - 5(3) + c = 18 - 15 + c = 3 + c \implies c = 4$$

$$y = 2x^2 - 5x + 4$$

Question 2 (Jan 2008, Q5)**Worked Solution**

$\frac{dy}{dx} = 12\sqrt{x} = 12x^{\frac{1}{2}}$. Curve passes through (4, 50).

Integrate:

$$y = \frac{12x^{\frac{3}{2}}}{\frac{3}{2}} + c = 8x^{\frac{3}{2}} + c$$

Use (4, 50):

$$50 = 8(4)^{\frac{3}{2}} + c = 8(8) + c = 64 + c \implies c = -14$$

$$y = 8x^{\frac{3}{2}} - 14$$

Question 3 (Jun 2011, Q2)

Worked Solution

(i) Find $\int (6x^{\frac{1}{2}} - 1) dx$.

$$\int (6x^{\frac{1}{2}} - 1) dx = \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} - x + c = 4x^{\frac{3}{2}} - x + c$$

$$4x^{\frac{3}{2}} - x + c$$

(ii) The curve has $\frac{dy}{dx} = 6x^{\frac{1}{2}} - 1$ and passes through (4, 17).

From (i): $y = 4x^{\frac{3}{2}} - x + c$

Use (4, 17):

$$17 = 4(8) - 4 + c = 32 - 4 + c = 28 + c \implies c = -11$$

$$y = 4x^{\frac{3}{2}} - x - 11$$

Question 4 (Jun 2012, Q2)**Worked Solution**

(i) Find $\int (x^2 - 2x + 5) dx$.

$$\int (x^2 - 2x + 5) dx = \frac{x^3}{3} - x^2 + 5x + c$$

$$\frac{x^3}{3} - x^2 + 5x + c$$

(ii) Curve has $\frac{dy}{dx} = x^2 - 2x + 5$ and passes through (3, 11).

From (i): $y = \frac{x^3}{3} - x^2 + 5x + c$

Use (3, 11):

$$11 = \frac{27}{3} - 9 + 15 + c = 9 - 9 + 15 + c = 15 + c \implies c = -4$$

$$y = \frac{x^3}{3} - x^2 + 5x - 4$$

Question 5 (Jun 2015, Q5)**Worked Solution**

$\frac{d^2y}{dx^2} = 3x^{-\frac{1}{2}}$. Point $P(4, 1)$ lies on the curve; gradient at P is 5.

First integration to find $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} + c_1 = 6x^{\frac{1}{2}} + c_1$$

Use gradient = 5 at $x = 4$:

$$5 = 6\sqrt{4} + c_1 = 12 + c_1 \implies c_1 = -7$$

So $\frac{dy}{dx} = 6x^{\frac{1}{2}} - 7$.

Second integration:

$$y = \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} - 7x + c_2 = 4x^{\frac{3}{2}} - 7x + c_2$$

Use $y = 1$ at $x = 4$:

$$1 = 4(8) - 7(4) + c_2 = 32 - 28 + c_2 = 4 + c_2 \implies c_2 = -3$$

$$y = 4x^{\frac{3}{2}} - 7x - 3$$

Question 6 (Jun 2018, Q8)

Worked Solution

$\frac{dy}{dx} = 2x + \frac{a}{x^3} + 3$. Curve passes through (1, 8).

Integrate:

$$y = x^2 - \frac{a}{2x^2} + 3x + c$$

Use (1, 8):

$$8 = 1 - \frac{a}{2} + 3 + c = 4 - \frac{a}{2} + c \implies c = 4 + \frac{a}{2}$$

The area enclosed by the curve, x -axis, $x = 1$ and $x = 3$ is 30 square units:

$$\int_1^3 \left(x^2 - \frac{a}{2x^2} + 3x + 4 + \frac{a}{2} \right) dx = 30$$

$$\int_1^3 \left(x^2 + \frac{a}{2} - \frac{a}{2}x^{-2} + 3x + 4 \right) dx = \left[\frac{x^3}{3} + \frac{a}{2}x + \frac{a}{2x} + \frac{3x^2}{2} + 4x \right]_1^3$$

Upper limit ($x = 3$):

$$9 + \frac{3a}{2} + \frac{a}{6} + \frac{27}{2} + 12 = 21 + \frac{27}{2} + \frac{3a}{2} + \frac{a}{6} = \frac{69}{2} + \frac{10a}{6}$$

Lower limit ($x = 1$):

$$\frac{1}{3} + \frac{a}{2} + \frac{a}{2} + \frac{3}{2} + 4 = \frac{1}{3} + a + \frac{11}{2} = \frac{35}{6} + a$$

Setting the difference to 30:

$$\frac{69}{2} + \frac{10a}{6} - \frac{35}{6} - a = 30$$

$$\frac{69}{2} - \frac{35}{6} + \frac{10a}{6} - a = 30$$

$$\frac{207}{6} - \frac{35}{6} + \frac{10a - 6a}{6} = 30$$

$$\frac{172}{6} + \frac{4a}{6} = 30 \implies 172 + 4a = 180 \implies 4a = 8$$

$a = 2$