

Finding the Equation of a Curve Given the Differential (From OCR 4722)

Q1, (Jan 2007, Q3)

(i) $\int(4x - 5)dx = 2x^2 - 5x + c$

(ii) $y = 2x^2 - 5x + c$
 $7 = 2 \times 3^2 - 5 \times 3 + c \Rightarrow c = 4$
 So equation is $y = 2x^2 - 5x + 4$

M1		Obtain at least one correct term
A1	2	Obtain at least $2x^2 - 5x$
B1√		State or imply $y =$ their integral from (i)
M1		Use (3,7) to evaluate c
A1	3	Correct final equation
5		

Q2, (Jan 2008, Q5)

$\int 12x^{\frac{1}{2}} dx = 8x^{\frac{3}{2}}$

$y = 8x^{\frac{3}{2}} + c \Rightarrow 50 = 8 \times 4^{\frac{3}{2}} + c$

$\Rightarrow c = -14$

Hence $y = 8x^{\frac{3}{2}} - 14$

M1		Attempt to integrate
A1√		Obtain correct, unsimplified, integral following their $f(x)$
A1		Obtain $8x^{\frac{3}{2}}$, with or without $+ c$
M1		Use (4, 50) to find c
A1√		Obtain $c = -14$, following $kx^{\frac{3}{2}}$ only
A1	6	State $y = 8x^{\frac{3}{2}} - 14$ aef, as long as single power of x
6		

Q3, (Jun 2011, Q2)

(i) $\int (6x^{\frac{1}{2}} - 1) dx = 4x^{\frac{3}{2}} - x + c$

- M1** Obtain $kx^{\frac{3}{2}}$ Any k , as long as numerical.
Allow both M1 and A1 for equiv eg $x\sqrt{x}$
- A1** Obtain $4x^{\frac{3}{2}}$ Allow for unsimplified coefficient as well (ie $6/_{1.5}$).
- B1** 3 Obtain $-x$ (don't penalise lack of $+c$) Allow $-1x$.

Maximum of 2 marks if \int or dx still present in final answer.
Maximum of 2 marks if not given as one expression – eg the two terms are integrated separately and never combined.

(ii) $y = 4x^{\frac{3}{2}} - x + c$
 $17 = 32 - 4 + c \Rightarrow c = -11$
 hence $y = 4x^{\frac{3}{2}} - x - 11$

- M1*** State or imply $y =$ their integral from (i) Must have come from integration attempt ie increase in power by 1 for at least one term, but allow if -1 disappeared in part (i) ie at least one of the M1 and the B1 must have been awarded in part (i).
Can still get this M1 if no $+c$.
The y does not have to be explicit – it could be implied by eg $17 = F(4)$.
M0 if they start with $y =$ their integral from (i), but then attempt to use $y - 17 = m(x - 4)$. This is a re-start and gains no credit.
- M1d*** Attempt to find c using (4, 17) M0 if no $+c$.
M0 if using $x = 17, y = 4$.
- A1** 3 Obtain $y = 4x^{\frac{3}{2}} - x - 11$ Coefficients now need to be simplified, so $-1x$ is A0.
Allow A1 for equiv eg $x\sqrt{x}$
Must be an equation ie $y = \dots$, so A0 for 'equation = ...' or 'f(x) = ...'

Q4, (Jun 2012, Q2)

(i)	$\int (x^2 - 2x + 5) dx = \frac{1}{3}x^3 - x^2 + 5x + c$	<p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p>	<p>Attempt integration</p> <p>Obtain two correct (algebraic) terms</p> <p>Obtain fully correct expression (allow no + c)</p>	<p>An increase in power by 1 for at least 2 terms. Allow if the +5 disappears.</p> <p>Allow if the coefficient of x^2 isn't yet simplified.</p> <p>Allow if the coefficient of x^2 isn't yet simplified. A0 if integral sign or dx still present in their answer (but allow $\int = \dots$). A0 if a list of terms rather than an expression.</p>
(ii)	$y = \frac{1}{3}x^3 - x^2 + 5x + c$ $11 = 9 - 9 + 15 + c \Rightarrow c = -4$ <p>hence $y = \frac{1}{3}x^3 - x^2 + 5x - 4$</p>	<p>M1*</p> <p>M1d*</p> <p>A1</p> <p>[3]</p>	<p>State or imply $y =$ their integral from (i)</p> <p>Attempt to find c using (3, 11)</p> <p>Obtain $y = \frac{1}{3}x^3 - x^2 + 5x - 4$</p>	<p>Must have come from integration attempt ie the M1 must have been gained in part (i). Allow slips when transferring expression from (i). Can still get this M1 if no + c. The y does not have to be explicit - it could be implied by eg $11 = F(3)$ (but not by $3 = F(11)$). Using definite integration with limits of 3 & 11 is M0. M0 if they start with $y =$ their integral from (i), but then attempt to use $y - 11 = m(x - 3)$. This is a re-start and gains no credit.</p> <p>Need to get as far as attempting c. M1 could be implied by eg $11 = 9 - 9 + 15$ and then an attempt to include a constant to balance the eqn, even though + c never actually seen. M0 if no + c seen or implied. M0 if using $x = 11, y = 3$.</p> <p>Coeff of x^2 now needs to be simplified (A0 for $-1x^2$). Must be an equation ie $y = \dots$, so A0 for 'f(x) = ...' or 'equation = ...' Allow aef, such as $3y = x^3 - 3x^2 + 15x - 12$.</p>

Q5, (Jun 2015, Q5)

$$\frac{dy}{dx} = 6x^{0.5} + c$$

$$5 = 12 + c$$

$$c = -7$$

$$y = 4x^{1.5} - 7x + k$$

$$1 = 32 - 28 + k, \text{ hence } k = -3$$

$$y = 4x^{1.5} - 7x - 3$$

M1*	Attempt integration	Must be of form $px^{0.5}$, any (non-zero) numerical p , and no other algebraic terms
A1	Obtain $6x^{0.5}$ (allow no $+c$)	Allow unsimplified coeff ie $3/_{0.5}$, even if subsequently incorrect No need to see $\frac{dy}{dx} =$, and ignore if incorrect (eg $y = \dots$)
M1d*	Attempt to use $x = 4$, gradient = 5	Must follow attempt at integration M0 if no $+c$ Condone incorrect notation (eg $y = \dots$) as long as 5 used correctly Attempt to use $x = 4$, $\frac{dy}{dx} = 5$ – allow slip as long as intention clear
A1	Rearrange to obtain $c = -7$	No need to see explicit expression for $\frac{dy}{dx}$
M1 dd*	Attempt second integration	Must be of form $qx^{1.5} + rx$, any (non-zero) numerical q, r , and no other algebraic terms Dependent on at least M1 M1 awarded
M1 ddd*	Attempt to find k using (4, 1)	Condone notation for the constant of integration being the same as previously used Dependent on all previous M marks Attempt to use $x = 4, y = 1$
A1	Obtain $y = 4x^{1.5} - 7x - 3$	Coefficients must now be simplified Must be an equation, ie $y = \dots$, so A0 for 'f(x) = ...' or 'equation = ...'
[7]		

Q6, (Jun 2018, Q8)

$$y = x^2 - \frac{1}{2}ax^2 + 3x + c$$

$$8 = 1 - \frac{1}{2}a + 3 + c$$

$$c = 4 + \frac{1}{2}a$$

M1*	Attempt integration	At least two terms to increase in power by 1
A1	Obtain correct integral	Condone no + c Condone no 'y =' Allow unsimplified coefficients
M1	Attempt to find equation in a and c using (1, 8)	Must follow attempt at integration ie two terms increasing in power by 1 M1 could be implied by eg $8 = 1 - \frac{1}{2}a + 3$ followed by an attempt to balance the equation M0 if no + c seen or implied M0 for $x = 8, y = 1$ Allow a slip when substituting, as long as it is clear that use of $x = 1, y = 8$ is intended
A1	Obtain correct equation in c and a	Could be explicit or as part of the correct $y = f(x)$ equation

$$\frac{1}{3}x^3 + \frac{1}{2}ax^{-1} + \frac{3}{2}x^2 + 4x + \frac{1}{2}ax$$

OR

$$\frac{1}{3}x^3 + \frac{1}{2}ax^{-1} + \frac{3}{2}x^2 + cx$$

OR

$$\frac{1}{3}x^3 + (c-4)x^{-1} + \frac{3}{2}x^2 + cx$$

$$(9 + \frac{1}{6}a + \frac{27}{2} + 12 + \frac{3}{2}a)$$

$$-(\frac{1}{3} + \frac{1}{2}a + \frac{3}{2} + 4 + \frac{1}{2}a)$$

$$\frac{86}{3} + \frac{2}{3}a = 30$$

$$\frac{2}{3}a = \frac{4}{3}$$

$$a = 2$$

M1d*

Attempt integration

A1

Obtain correct integral

M1**

Attempt correct use of limits and equate to 30

M1d**

Attempt to solve for either a or c

A1

Obtain $a = 2$

[9]

Must follow first attempt at integration, and include either c or an attempt at c in terms of a (and possibly even have a in terms of c)

At least three terms to increase in power by 1

Any correct integral in terms of a and/or c

Correct order and subtraction

Must be using limits of 1 and 3

Dependent on M1M1 for the two integration attempts

Solving for c gives $c = 5$

Another valid method is to find an equation involving a and c from use of limits ($6c - a = 28$) and solve simultaneously with their $c = 4 + \frac{1}{2}a$