Finding the Equation of a Curve Given The Differential Mark Scheme

Q1.

Question Number	Scheme		Marks
(a)	$(3-x^2)^2 = 9-6x^2+x^4$	An attempt to expand the numerator obtaining an expression of the form $9 \pm px^2 \pm qx^4$, $p,q \neq 0$	M1
	$9x^{-2}+x^2$	Must come from $\dfrac{9+x^4}{x^2}$	A1
	-6	Must come from $\dfrac{-6x^2}{x^2}$	A1
	Alternative 1: Writes $\dfrac{(3-x^2)^2}{x^2}$ as $(3x^{-1}-x)^2$ and attempts to expand = M1 then A1A1 as in the scheme.		
	Alternative 2: Sets $(3-x^2)^2 = 9+Ax^2+Bx^4$, expands $(3-x^2)^2$ and compares coefficients = M1 then A1A1 as in the scheme.		
			(3)
	$(f'(x) = 9x^{-2}-6+x^2)$		
(b)	$-18x^{-3}+2x$	M1: $x^n \to x^{n-1}$ on separate terms at least once. Do not award for $A \to 0$ (Integrating is M0) A1ft: $-18x^{-3}+2$"B"x with a numerical B and no extra terms. (A may have been incorrect or even zero)	M1 A1ft
			(2)
(c)	$f(x) = -9x^{-1}-6x+\dfrac{x^3}{3}(+c)$	M1: $x^n \to x^{n+1}$ on separate terms at least once. (Differentiating is M0) A1ft: $-9x^{-1}+Ax+\dfrac{Bx^3}{3}(+c)$ with numerical A and B, $A, B \neq 0$	M1A1ft
	$10 = \dfrac{-9}{-3}-6(-3)+\dfrac{(-3)^3}{3}+c$ so c = ...	Uses x = -3 and y =10 in what they think is f(x) (They may have differentiated here) but it must be a changed function i.e. not the original $f'(x)$, to form a linear equation in c and attempts to find c. No $+c$ gets M0 and A0 unless their method implies that they are correctly finding a constant.	M1
	c = -2	cso	A1
	$(f(x) =) -9x^{-1}-6x+\dfrac{x^3}{3}+$ their c	Follow through their c in an otherwise (possibly un-simplified) **correct expression**. Allow $-\dfrac{9}{x}$ for $-9x^{-1}$ or even $\dfrac{9x^{-1}}{-1}$.	A1ft
	Note that if they integrate in (b), no marks there but if they then go on to use their integration in (c), the marks for integration are available.		
			(5)
			[10]

Q2.

Question Number	Scheme		Marks
	$(\frac{dy}{dx}=)\quad -x^3+"2"x^{-2}-"\left(\frac{5}{2}\right)"x^{-3}$		M1
	$(y=)\quad -\frac{1}{4}x^4+\frac{"2"x^{-1}}{(-1)}-"\left(\frac{5}{2}\right)"\frac{x^{-2}}{(-2)}(+c)$	Raises power correctly on any one term. Any two follow through terms correct.	M1 A1ft
	$(y=)\quad -\frac{1}{4}x^4+\frac{2x^{-1}}{(-1)}-\frac{5}{2}\frac{x^{-2}}{(-2)}(+c)$	This is not follow through – must be correct	A1
	Given that $y=7$, at $x=1$, then $7=-\frac{1}{4}-2+\frac{5}{4}+c \Rightarrow c=$		M1
	So, $(y=)\quad -\frac{1}{4}x^4-2x^{-1}+\frac{5}{4}x^{-2}+c,\ c=8$ or $(y=)-\frac{1}{4}x^4-2x^{-1}+\frac{5}{4}x^{-2}+8$		A1
			[6]
			6 marks
	Notes		
	M1: Expresses as three term polynomial with powers 3, -2 and –3. Allow slips in coefficients. This may be implied by later integration having all three powers 4, -1 and -2. **M1:** An attempt to integrate at least one term so $x^n \to x^{n+1}$ (not a term in the numerator or denominator) **A1ft:** Any two integrations are correct – coefficients may be unsimplified (follow through errors in **coefficients only here**) so should have two of the powers 4, -1 and –2 after integration – depends on 2nd method mark only. There should be a maximum of three terms here. **A1:** Correct three terms – coefficients **may be unsimplified**- do not need constant for this mark Depends on both Method marks **M1: Need constant for this mark.** Uses $y=7$ and $x=1$ in their changed expression in order to find c, and attempt to find c. *This mark is available even after there is suggestion of differentiation.* **A1:** Need **all four** correct terms to be **simplified** and need $c=8$ here.		

Q3.

Question Number	Scheme	Marks
(a)	$(x^2+3)^2=x^4+3x^2+3x^2+3^2$	M1
	$\frac{(x^2+3)^2}{x^2}=\frac{x^4+6x^2+9}{x^2}=x^2+6+9x^{-2}\quad (*)$	A1 cso (2)
(b)	$y=\frac{x^3}{3}+6x+\frac{9}{-1}x^{-1}(+c)$	M1 A1 A1
	$20=\frac{27}{3}+6\times 3-\frac{9}{3}+c$	M1
	$c=-4$	A1
	$[y=]\frac{x^3}{3}+6x-9x^{-1}-4$	A1 ft (6)
		(8 marks)

Q4.

Question Number	Scheme	Marks
	$\frac{dy}{dx} = 6x^{-\frac{1}{2}} + x\sqrt{x}$	
	$x\sqrt{x} = x^{\frac{3}{2}}$	B1
	$x^n \to x^{n+1}$	M1
	$y = \frac{6}{\frac{1}{2}}x^{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}}(+c)$	A1, A1
	Use $x=4, y=37$ to give equation in c. $37 = 12\sqrt{4} + \frac{2}{5}(\sqrt{4})^5 + c$	M1
	$\Rightarrow c = \frac{1}{5}$ or equivalent eg. 0.2	A1
	$(y) = 12x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{5}$	A1
		(7 marks)

B1 $x\sqrt{x} = x^{\frac{3}{2}}$. This may be implied by $+\frac{x^{\frac{5}{2}}}{\frac{5}{2}}$ oe in the subsequent work.

M1 $x^n \to x^{n+1}$ in at least one case so see either $x^{\frac{1}{2}}$ or $x^{\frac{5}{2}}$ or both

A1 One term integrated correctly. It does not have to be simplified Eg. $\frac{6}{\frac{1}{2}}x^{\frac{1}{2}}$ or $+\frac{x^{\frac{5}{2}}}{\frac{5}{2}}$.

No need for $+c$

A1 Other term integrated correctly. See above. No need to simplify nor for $+c$. Need to see

$\frac{6}{\frac{1}{2}}x^{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}}$ or a simplified correct version

M1 Substitute $x = 4, y = 37$ to produce an equation in c.

A1 Correctly calculates $c = \frac{1}{5}$ or equivalent e.g. 0.2

A1 cso $y = 12x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{5}$. Allow $5y = 60x^{\frac{1}{2}} + 2x^{\frac{5}{2}} + 1$ and accept fully simplified equivalents.

e.g. $y = \frac{1}{5}(60x^{\frac{1}{2}} + 2x^{\frac{5}{2}} + 1)$, $y = 12\sqrt{x} + \frac{2}{5}\sqrt{x^5} + \frac{1}{5}$

Q5.

Question Number	Scheme	Marks
(a)	$p=\frac{1}{2},\ q=2$ or $6x^{\frac{1}{2}},\ 3x^2$	B1, B1 (2)
(b)	$\frac{6x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+\frac{3x^3}{3}\quad\left(=4x^{\frac{3}{2}}+x^3\right)$	M1 A1ft
	$x=4,\ y=90:\ 32+64+C=90\ \Rightarrow\ C=-6$	M1 A1
	$y=4x^{\frac{3}{2}}+x^3+\text{"their} -6\text{"}$	A1 (5) 7
	Notes	
	(a) Accept any equivalent answers, e.g. $p=0.5,\ q=4/2$ (b) 1st M: Attempt to integrate $x^n\rightarrow x^{n+1}$ (for either term) 1st A: ft their p and q, but terms need not be simplified ($+C$ not required for this mark) 2nd M: Using $x=4$ <u>and</u> $y=90$ to form an equation in C. 2nd A: cao 3rd A: answer as shown with simplified correct coefficients and powers – but follow through their value for C If there is a 'restart' in part (b) it can be marked independently of part (a), but marks for part (a) cannot be scored for work seen in (b). <u>Numerator and denominator integrated separately</u>: First M mark **cannot** be awarded so only mark available is second M mark. So 1 out of 5 marks.	

Q6.

Question Number	Scheme	Marks
	$(\text{f}(x)=)\frac{12x^3}{3}-\frac{8x^2}{2}+x(+c)$	M1 A1 A1
	$(\text{f}(-1)=0\Rightarrow)\ \ 0=4\times(-1)-4\times1-1+c$	M1
	$c=\underline{9}$	A1
	$\left[\text{f}(x)=4x^3-4x^2+x+9\right]$	
		5
	Notes	
	1st M1 for an attempt to integrate $x^n\rightarrow x^{n+1}$ 1st A1 for at least 2 terms in x correct - needn't be simplified, ignore $+c$ 2nd A1 for all the terms in x correct but they need not be simplified. No need for $+c$ 2nd M1 for using $x=-1$ and $y=0$ to form a linear equation in c. No $+c$ gets M0A0 3rd A1 for $c=9$. Final form of f(x) is not required.	

Q7.

Question	Scheme	Marks
	$[\text{f}(x)=]\frac{3x^3}{3}-\frac{3x^2}{2}+5x[+c]$ or $\left\{x^3-\frac{3}{2}x^2+5x(+c)\right\}$	M1A1
	$10=8-6+10+c$	M1
	$c=-2$	A1
	$\text{f}(1)=1-\frac{3}{2}+5\ \ \text{"}-2\text{"}=\underline{\frac{5}{2}}$ (o.e.)	A1ft (5)
		5 marks
	Notes	
	1st M1 for attempt to integrate $x^n\rightarrow x^{n+1}$ 1st A1 all correct, possibly unsimplified. Ignore $+c$ here. 2nd M1 for using $x=2$ <u>and</u> f(2) = 10 to form a linear equation in c. Allow sign errors. They should be substituting into a <u>changed</u> expression 2nd A1 for $c=-2$ 3rd A1ft for $\frac{9}{2}+c$ Follow through their <u>numerical</u> c ($\neq 0$) This mark is dependent on 1st M1 and 1st A1 only.	

Q8.

Question Number	Scheme	Marks
(a)	$(y=)\frac{3x^2}{2}-\frac{5x^{\frac{1}{2}}}{\frac{1}{2}}-2x\ \ (+c)$	M1A1A1
	$f(4)=5\ \Rightarrow 5=\frac{3}{2}\times 16-10\times 2-8+c$	M1
	$\underline{c=9}$	A1 (5)
	$\left[f(x)=\frac{3}{2}x^2-10x^{\frac{1}{2}}-2x+9\right]$	
(b)	$m=3\times 4-\frac{5}{2}-2\ \left(=7.5 \text{ or } \frac{15}{2}\right)$	M1
	Equation is: $y-5=\frac{15}{2}(x-4)$	M1A1
	$\underline{2y-15x+50=0}$ o.e.	A1 (4) **(9marks)**
(a)	1st M1 for an attempt to integrate $x^n \to x^{n+1}$ 1st A1 for at least 2 correct terms in x (unsimplified) 2nd A1 for all 3 terms in x correct (condone missing $+c$ at this point). Needn't be simplified 2nd M1 for using the point (4, 5) to form a linear equation for c. Must use $x=4$ and $y=5$ and have no x term and the function must have "changed". 3rd A1 for $c=9$. The final expression is not required.	
(b)	1st M1 for an attempt to evaluate $f'(4)$. Some correct use of $x=4$ in $f'(x)$ but condone slips. They must therefore have at least 3×4 or $-\frac{5}{2}$ and clearly be using $f'(x)$ with $x=4$. Award this mark wherever it is seen. 2nd M1 for using their value of m [or their $-\frac{1}{m}$] (provided it clearly comes from using $x=4$ in $f'(x)$) to form an equation of the line through (4,5)). Allow this mark for an attempt at a normal or tangent. Their m must be numerical. Use of $y=mx+c$ scores this mark when c is found. 1st A1 for any correct expression for the equation of the line 2nd A1 for any correct equation with integer coefficients. An "=" is required. e.g. $2y=15x-50$ etc as long as the equation is correct and has integer coefficients.	
Normal	Attempt at normal can score both M marks in (b) but A0A0	