Modelling With Exponentials Exam Questions Sheet 2 Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	(£)18 000	B1	3.4
		(1)	
(b)	(i) $\frac{dV}{dt} = -3925e^{-0.25t}$	M1 A1	3.1b 1.1b
	Sets $-3925e^{-0.25T} = -500 \Rightarrow 3925e^{-0.25T} = 500$ * cso	A1*	3.4
	(ii) $e^{-0.25T} = 0.127... \Rightarrow -0.25T = \ln 0.127...$	M1	1.1b
	$T = 8.24$ (awrt)	A1	1.1b
	8 years 3 months	A1	3.2a
		(6)	
(c)	2 300	B1	1.1b
		(1)	
(d)	Any suitable reason such as • Other factors affect price such as condition/mileage • If the car has had an accident it will be worth less than the model predicts • The price may go up in the long term as it becomes rare • £2300 is too large a value for a car's scrap price. Most cars scrap for around £400	B1	3.5b
		(1)	
			(9 marks)

Notes

(a)

B1: £18 000 There is no requirement to have the units

(b)(i)

M1: Award for making the link between gradient and rate of change.

Score for attempting to differentiate V to $\frac{dV}{dt} = ke^{-0.25t}$ An attempt at both sides are required.

For the left hand side you may condone attempts such as $\frac{dy}{dx}$

A1: Achieves $\frac{dV}{dt} = -3925e^{-0.25t}$ or $\frac{dV}{dt} = 15700 \times -0.25e^{-0.25t}$ with both sides correct

A1*: Sets $-3925e^{-0.25T} = -500$ oe and proceeds to $3925e^{-0.25T} = 500$

This is a given answer and to achieve this mark, all aspects must be seen and be correct.

t must be changed to T at some point even if just at the end of their solution/proof

SC: Award SC 110 candidates who simply write

$-3925e^{-0.25T} = -500 \Rightarrow 3925e^{-0.25T} = 500$ without any mention or reference to $\frac{dV}{dt}$

Or $15700 \times -0.25e^{-0.25t} = -500 \Rightarrow 3925e^{-0.25T} = 500$ without any mention or reference to $\frac{dV}{dt}$

(b)(ii)

M1: Proceeds from $e^{-0.25T} = A, A > 0$ using ln's to $\pm 0.25T =$..

Alternatively takes lns first $3925e^{-0.25T} = 500 \Rightarrow \ln 3925 - 0.25T = \ln 500 \Rightarrow \pm 0.25T = ...$

but $3925e^{-0.25T} = 500 \Rightarrow \ln 3925 \times -0.25T = \ln 500 \Rightarrow \pm 0.25T = ...$ is M0

A1: $T =$ awrt 8.24 or $-\frac{1}{0.25}\ln\left(\frac{20}{157}\right)$ Allow $t =$ awrt 8.24

A1: 8 years 3 months. Correct answer and solution only

Answers obtained numerically score 0 marks. The M mark must be scored.

(c)

B1: 2 300 but condone £ 2 300

(d)

B1: Any suitable reason. See scheme

Accept "Scrappage" schemes may pay more (or less) than £ 2 300.

Do not accept "does not take into account inflation"

It asks for a limitation of the model so candidates cannot score marks by suggesting other suitable models " the value may fall by the same amount each year"

Q2.

Question Number	Scheme	Marks
(a)	$(\theta =)20$	B1 **(1)**
(b)	Sub $t = 40, \theta = 70 \Rightarrow 70 = 120 - 100e^{-40\lambda}$	
	$\Rightarrow e^{-40\lambda} = 0.5$	M1A1
	$\Rightarrow \lambda = \frac{\ln 2}{40}$	M1A1
		(4)
(c)	$\theta = 100 \Rightarrow T = \frac{\ln 0.2}{-\text{their } '\lambda'}$	M1
	$T =$ awrt 93	A1
		(2)
		(7 marks)
Alt (b)	Sub $t = 40, \theta = 70 \Rightarrow 100e^{-40\lambda} = 50$	
	$\Rightarrow \ln 100 - 40\lambda = \ln 50$	M1A1
	$\Rightarrow \lambda = \frac{\ln 100 - \ln 50}{40} = \frac{\ln 2}{40}$	M1A1
		(4)

(a)

B1 Sight of $(\theta =)20$

(b)

M1 Sub $t = 40, \theta = 70 \Rightarrow 70 = 120 - 100e^{-40\lambda}$ and proceed to $e^{\pm 40\lambda} = A$ where A is a constant. Allow sign slips and copying errors.

A1 $e^{-40\lambda} = 0.5$ or $e^{40\lambda} = 2$ or exact equivalent

M1 For undoing the e's by taking ln's and proceeding to $\lambda = ..$

May be implied by the correct decimal answer awrt 0.017 or $\lambda = \frac{\ln 0.5}{-40}$

A1 cso $\lambda = \frac{\ln 2}{40}$

Accept equivalents in the form $\frac{\ln a}{b}$, $a, b \in \mathbb{Z}$ such as $\lambda = \frac{\ln 4}{80}$

(c)

M1 Substitutes $\theta = 100$ and their numerical value of λ into $\theta = 120 - 100e^{-\lambda t}$ and proceed to $T = \pm\frac{\ln 0.2}{\text{their } '\lambda'}$ or $T = \pm\frac{\ln 5}{\text{their } '\lambda'}$ Allow inequalities here.

A1 awrt $T = 93$

Watch for candidates who lose the minus sign in (b) and use $\lambda = \frac{\ln 1/2}{40}$ in (c). Many then reach $T = -93$ and ignore the minus. This is M1 A0

Q3.

Question Number	Scheme	Marks
(a)	$P=\frac{800e^0}{1+3e^0},=\frac{800}{1+3}=200$	M1,A1 (2)
(b)	$250=\frac{800e^{0.1t}}{1+3e^{0.1t}}$ $250(1+3e^{0.1t})=800e^{0.1t}\Rightarrow 50e^{0.1t}=250,\ \Rightarrow e^{0.1t}=5$ $t=\frac{1}{0.1}\ln(5)$ $t=10\ln(5)$	M1,A1 M1 A1 (4)
(d)	$P=\frac{800e^{0.1t}}{1+3e^{0.1t}}=\frac{800}{e^{-0.1t}+3}\Rightarrow P_{max}=\frac{800}{3}=266$. Hence P cannot be 270	B1 (1) **(11 marks)**

(a)

M1 Sub $t=0$ into P **and** use $e^0=1$ in at least one of the two cases. Accept $P=\frac{800}{1+3}$ as evidence

A1 200. Accept this for both marks as long as no incorrect working is seen.

(b)

M1 Sub P=250 into $P=\frac{800e^{0.1t}}{1+3e^{0.1t}}$, cross multiply, collect terms in $e^{0.1t}$ **and** proceed to $Ae^{0.1t}=B$

Condone bracketing issues and slips in arithmetic.

If they divide terms by $e^{0.1t}$ you should expect to see $Ce^{-0.1t}=D$

A1 $e^{0.1t}=5$ or $e^{-0.1t}=0.2$

M1 Dependent upon gaining $e^{0.1t}=E$, for taking ln's of both sides and proceeding to t=…

Accept $e^{0.1t}=E\Rightarrow 0.1t=\ln E\Rightarrow t=...$It could be implied by $t=$ awrt 16.1

A1 $t=10\ln(5)$

Accept exact equivalents of this as long as a and b are integers. Eg. $t=5\ln(25)$ is fine.

Accept solutions where it implies the max value is 266.6 or 267. For example accept sight of $\frac{800}{3}$, with a comment 'so it cannot reach 270', or a large value of t ($t > 99$) being substituted in to get 266.6 or 267 with a similar statement, or a graph drawn with an asymptote marked at 266.6 or 267

Do not accept exp's cannot be negative or you cannot ln a negative number without numerical evidence.

Look for both a statement and a comment

Q4.

Question	Scheme	Marks
(a)	Subs $D=15$ and $t=4$ $\quad x=15e^{-0.2\times4}=6.740\ (mg)$	M1A1 (2)
(b)	$15e^{-0.2\times7}+15e^{-0.2\times2}=13.754(mg)$	M1A1* (2)
(c)	$15e^{-0.2\times T}+15e^{-0.2\times(T+5)}=7.5$ $15e^{-0.2\times T}+15e^{-0.2\times T}e^{-1}=7.5$ $15e^{-0.2\times T}(1+e^{-1})=7.5\Rightarrow e^{-0.2\times T}=\dfrac{7.5}{15(1+e^{-1})}$ $T=-5\ln\left(\dfrac{7.5}{15(1+e^{-1})}\right)=5\ln\left(2+\dfrac{2}{e}\right)$	M1 dM1 A1, A1 (4)
		(8 marks)

(a)

M1 Attempts to substitute both $D=15$ and $t=4$ in $x=De^{-0.2t}$
It can be implied by sight of $15e^{-0.8}$, $15e^{-0.2\times4}$ or awrt 6.7
Condone slips on the power. Eg you may see -0.02

A1 CAO 6.740 (mg) Note that 6.74 (mg) is A0

(b)

M1 Attempt to find the sum of two expressions with $D=15$ in both terms with t values of 2 and 7
Evidence would be $15e^{-0.2\times7}+15e^{-0.2\times2}$ or similar expressions such as $(15e^{-1}+15)e^{-0.2\times2}$
Award for the sight of the two numbers awrt **3.70** and awrt **10.05**, followed by their total awrt **13.75**
Alternatively finds the amount after 5 hours, $15e^{-1}$ = awrt **5.52** adds the second dose = **15** to get a total of awrt **20.52** then multiplies this by $e^{-0.4}$ to get awrt **13.75**.
Sight of 5.52+15=20.52→ 13.75 is fine.

A1* cso so both the expression $15e^{-0.2\times7}+15e^{-0.2\times2}$ and $13.754(mg)$ are required
Alternatively both the expression $(15e^{-0.2\times5}+15)\times e^{-0.2\times2}$ and $13.754(mg)$ are required.
Sight of just the numbers is not enough for the A1*

(c)

M1 Attempts to write down a correct equation involving T or t. Accept with or without correct bracketing
Eg. accept $15e^{-0.2\times T}+15e^{-0.2\times(T\pm5)}=7.5$ or similar equations $(15e^{-1}+15)e^{-0.2\times T}=7.5$

dM1 Attempts to solve their equation, dependent upon the previous mark, by proceeding to $e^{-0.2\times T}=\ldots$
An attempt should involve an attempt at the index law $x^{m+n}=x^m\times x^n$ and taking out a factor of $e^{-0.2\times T}$ Also score for candidates who make $e^{+0.2\times T}$ the subject using the same criteria

A1 Any correct form of the answer, for example, $-5\ln\left(\dfrac{7.5}{15(1+e^{-1})}\right)$

A1 CSO $T=5\ln\left(2+\dfrac{2}{e}\right)$ Condone t appearing for T throughout this question.

Alt (c) using lns

(c)	$15e^{-0.2\times T}+15e^{-0.2\times(T+5)}=7.5$ $15e^{-0.2\times T}+15e^{-0.2\times T}e^{-1}=7.5$ $e^{-0.2\times T}(1+e^{-1})=0.5\Rightarrow -0.2\times T+\ln(1+e^{-1})=\ln 0.5$ $\Rightarrow T=\frac{\ln 0.5-\ln(1+e^{-1})}{-0.2},\Rightarrow T=5\ln\left(2+\frac{2}{e}\right)$	M1 dM1 A1, A1 **(4)** **(8 marks)**

You may see numerical attempts at part (c).

Such an attempt can score a maximum of two marks.

This can be achieved either by

Method One

1st Mark (Method): $15e^{-0.2\times T}+\text{awrt } 5.52e^{-0.2\times T}=7.5\Rightarrow e^{-0.2\times T}=\text{awrt } 0.37$

2nd Mark (Accuracy): T=-5ln(awrt 0.37) or awrt 5.03 or $T=-5\ln\left(\frac{7.5}{\text{awrt } 20.52}\right)$

Method Two

1st Mark (Method): $13.754e^{-0.2\times T}=7.5\Rightarrow T=-5\ln\left(\frac{7.5}{13.754}\right)$ or equivalent such as 3.03

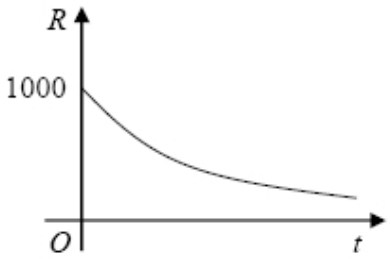
2nd Mark (Accuracy): 3.03+2=5.03 Allow $-5\ln\left(\frac{7.5}{13.754}\right)+2$

Method Three (by trial and improvement)

1st Mark (Method): $15e^{-0.2\times 5}+15e^{-0.2\times 10}=7.55$ or $15e^{-0.2\times 5.1}+15e^{-0.2\times 10.1}=7.40$ or any value between

2nd Mark (Accuracy): Answer $T=5.03$.

Q5.

Question Number	Scheme		Marks	
	(a) 1000		B1	(1)
	(b) $1000e^{-5730c} = 500$		M1	
	$e^{-5730c} = \frac{1}{2}$		A1	
	$-5730c = \ln\frac{1}{2}$		M1	
	$c = 0.000121$	cao	A1	(4)
	(c) $R = 1000e^{-22920c} = 62.5$	Accept 62-63	M1 A1	(2)
	(d) 	Shape 1000	B1 B1	(2)
				[9]

Q6.

Question Number	Scheme	Marks
(a)	$p=7.5$	B1 (1)
(b)	$2.5 = 7.5e^{-4k}$	M1
	$e^{-4k} = \frac{1}{3}$	M1
	$-4k = \ln(\frac{1}{3})$	dM1
	$-4k = -\ln(3)$	
	$k = \frac{1}{4}\ln(3)$	A1*
	See notes for additional correct solutions and the last A1	(4)
(c)	$\frac{dm}{dt} = -kpe^{-kt}$ ft on their p and k	M1A1ft
	$-\frac{1}{4}ln3 \times 7.5e^{-\frac{1}{4}(ln3)t} = -0.6ln3$	
	$e^{-\frac{1}{4}(ln3)t} = \frac{2.4}{7.5} = (0.32)$	M1A1
	$-\frac{1}{4}(ln3)t = \ln(0.32)$	dM1
	t=4.1486…. 4.15 or awrt 4.1	A1
		(6)
		11Marks

Q7

Question	Scheme	Marks	AOs
(a)	(£)18 000	B1	3.4
		(1)	
(b)	(i) $\frac{dV}{dt} = -3925e^{-0.25t}$	M1 A1	3.1b 1.1b
	Sets $-3925e^{-0.25T} = -500 \Rightarrow 3925e^{-0.25T} = 500$ * cso	A1*	3.4
	(ii) $e^{-0.25T} = 0.127... \Rightarrow -0.25T = \ln 0.127...$	M1	1.1b
	$T = 8.24$ (awrt)	A1	1.1b
	8 years 3 months	A1	3.2a
		(6)	
(c)	2 300	B1	1.1b
		(1)	
(d)	Any suitable reason such as • Other factors affect price such as condition/mileage • If the car has had an accident it will be worth less than the model predicts • The price may go up in the long term as it becomes rare • £2300 is too large a value for a car's scrap price. Most cars scrap for around £400	B1	3.5b
		(1)	
			(9 marks)