Question 1 (OCR 4722, Jun 2009, Q3)

Worked Solution

Solve $7^x = 2^{x+1}$, giving x correct to 3 s.f.

Step 1: Take logarithms of both sides.

$$x \log 7 = (x+1) \log 2.$$

Step 2: Expand and collect x terms.

$$x \log 7 = x \log 2 + \log 2 \implies x(\log 7 - \log 2) = \log 2 \implies x = \frac{\log 2}{\log 7 - \log 2}.$$

$$x = \frac{\log 2}{\log 7 - \log 2} \approx 0.553$$

Question 2 (OCR 4722, Jan 2010, Q8)

Worked Solution

Part (a): Solve $5^{3w-1} = 4^{250}$, giving w correct to 3 s.f.

Step 1: Take logarithms and drop the powers.

$$(3w-1)\log 5 = 250\log 4.$$

Step 2: Solve for w.

$$3w - 1 = \frac{250\log 4}{\log 5} \implies w = \frac{1}{3}\left(\frac{250\log 4}{\log 5} + 1\right) \approx 72.1.$$

$$w \approx 72.1$$

Part (b): Given $\log_x(5y+1) - \log_x 3 = 4$, express y in terms of x.

Step 1: Combine logarithms.

$$\log_x\left(\frac{5y+1}{3}\right) = 4.$$

Step 2: Use the inverse of $\log_x$, i.e. raise x to the power of both sides.

$$\frac{5y+1}{3} = x^4.$$

Step 3: Solve for y.

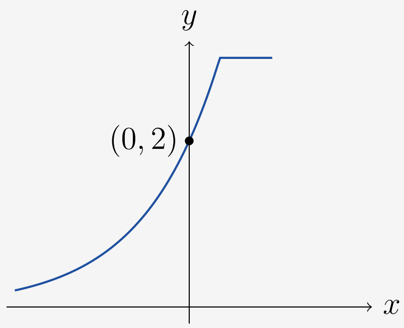
$$5y + 1 = 3x^4 \implies y = \frac{3x^4 - 1}{5}.$$

$$y = \frac{3x^4 - 1}{5}$$

Question 3 (OCR 4722, Jun 2008, Q8)

Worked Solution

Part (i): Sketch $y = 2 \times 3^x$, stating axis intersections.



y-intercept at $(0, 2)$; curve does not cross the x-axis.

Part (ii): Show the x-coordinate of the intersection of $y = 2 \times 3^x$ and $y = 8^x$ is $\dfrac{1}{3 - \log_2 3}$.

Set $8^x = 2 \times 3^x$. Take $\log_2$ of both sides:

$$\log_2 8^x = \log_2(2 \times 3^x) \implies x \log_2 8 = \log_2 2 + x \log_2 3.$$

Since $\log_2 8 = 3$:

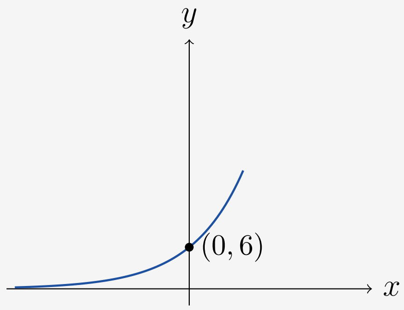
$$3x = 1 + x \log_2 3 \implies x(3 - \log_2 3) = 1 \implies x = \frac{1}{3 - \log_2 3}. \quad \square$$

$$x = \frac{1}{3 - \log_2 3}$$

Question 4 (OCR 4722, Jan 2010, Q9)

Worked Solution

Part (i): Sketch $y = 6 \times 5^x$, stating axis intersections.



y-intercept $(0, 6)$; no x-intercept.

Part (ii): On $y = 9^x$, find x when $y = 150$.

$9^x = 150 \Rightarrow x \log 9 = \log 150$.

$$x = \frac{\log 150}{\log 9} \approx 2.28$$

Part (iii): Show the intersection of $y = 6 \times 5^x$ and $y = 9^x$ has x-coordinate $\dfrac{1 + \log_3 2}{2 - \log_3 5}$.

Set $6 \times 5^x = 9^x$. Take $\log_3$ throughout:

$$\log_3 6 + x \log_3 5 = x \log_3 9 = 2x.$$

Since $\log_3 6 = \log_3 3 + \log_3 2 = 1 + \log_3 2$:

$$1 + \log_3 2 = x(2 - \log_3 5) \implies x = \frac{1 + \log_3 2}{2 - \log_3 5}. \quad \square$$

$$x = \frac{1 + \log_3 2}{2 - \log_3 5}$$

Question 5 (OCR 4722, Jan 2009, Q8)

Worked Solution

Given $\log_a x = p$ and $\log_a y = q$.

Part (a)(i): $\log_a(xy)$.

Using $\log_a(xy) = \log_a x + \log_a y$:

$$\log_a(xy) = p + q$$

Part (a)(ii): $\log_a\left(\frac{a^2x^3}{y}\right)$.

$$\log_a\left(\frac{a^2x^3}{y}\right) = \log_a a^2 + \log_a x^3 - \log_a y = 2 + 3p - q.$$

$$\log_a\left(\frac{a^2x^3}{y}\right) = 2 + 3p - q$$

Part (b)(i): Express $\log_{10}(x^2 - 10) - \log_{10} x$ as a single logarithm.

$$\log_{10}\left(\frac{x^2 - 10}{x}\right)$$

Part (b)(ii): Solve $\log_{10}(x^2 - 10) - \log_{10} x = 2\log_{10} 3$.

From (b)(i), the LHS is $\log_{10}\left(\frac{x^2 - 10}{x}\right)$. The RHS is $\log_{10} 9$.

Since the logs are equal:

$$\frac{x^2 - 10}{x} = 9 \implies x^2 - 10 = 9x \implies x^2 - 9x - 10 = 0.$$

Factorise:

$$(x - 10)(x + 1) = 0 \implies x = 10 \text{ or } x = -1.$$

Reject $x = -1$ (logarithm of negative number undefined).

$$x = 10$$

Question 6 (OCR 4722, Jan 2013, Q8)

Worked Solution

The diagram shows $y = \log_2 x$ and $y = \log_2(x-3)$.

Part (i): Transformation from $y = \log_2 x$ to $y = \log_2(x-3)$.

Replacing x by $x - 3$ translates the curve 3 units to the right.

Translation of 3 units in the positive x-direction.

Part (ii): $y = \log_2 x$ passes through $(a, 3)$. Find a.

$\log_2 a = 3 \implies a = 2^3 = 8.$

$a = 8$

Part (iii): $y = \log_2(x-3)$ passes through $(b, 1.8)$. Find b to 3 s.f.

$\log_2(b-3) = 1.8 \implies b - 3 = 2^{1.8} \implies b = 3 + 2^{1.8} \approx 3 + 3.482 \approx 6.48.$

$b \approx 6.48$

Part (iv): P on $y = \log_2 x$ and Q on $y = \log_2(x-3)$, both with x-coordinate c. Given $PQ = 4$, find the exact value of c.

Both P and Q have the same x-coordinate, so:

$$PQ = \log_2 c - \log_2(c-3) = 4.$$

Step 1: Combine logarithms.

$$\log_2\left(\frac{c}{c-3}\right) = 4 \implies \frac{c}{c-3} = 2^4 = 16.$$

Step 2: Solve for c.

$$c = 16(c-3) = 16c - 48 \implies -15c = -48 \implies c = \frac{48}{15} = \frac{16}{5}.$$

$c = \dfrac{16}{5}$

Question 7 (OCR 4722, Jun 2013, Q8)

Worked Solution

The diagram shows $y = a^x$ and $y = 4b^x$.

Part (i)(a): Intersection of $y = a^x$ with y-axis.

When $x = 0$: $y = 1$.

$(0, 1)$

Part (i)(b): Intersection of $y = 4b^x$ with y-axis.

When $x = 0$: $y = 4$.

$(0, 4)$

Part (i)(c): Possible values of a and b.

From the diagram $y = a^x$ is increasing $(a > 1)$ and $y = 4b^x$ is decreasing $(0 < b < 1)$.

E.g. $a = 2$, $b = \frac{1}{2}$

Part (ii): Given $ab = 2$, show the x-coordinate of the intersection is $\dfrac{2}{2\log_2 a - 1}$.

Set $a^x = 4b^x$. Take $\log_2$:

$$x \log_2 a = \log_2 4 + x \log_2 b = 2 + x \log_2 b.$$

Since $b = \frac{2}{a}$: $\log_2 b = \log_2 2 - \log_2 a = 1 - \log_2 a$. Substitute:

$$x \log_2 a = 2 + x(1 - \log_2 a) = 2 + x - x \log_2 a.$$

$$2x \log_2 a - x = 2 \implies x(2\log_2 a - 1) = 2 \implies x = \frac{2}{2\log_2 a - 1}. \quad \square$$

$$x = \frac{2}{2\log_2 a - 1}$$

End of Worked Solutions