

Exponential and Logarithmic Equations Exam Questions MS

Q1, (OCR 4722, Jun 2009, Q3)

$$\log 7^x = \log 2^{x+1}$$

$$x \log 7 = (x+1) \log 2$$

$$x(\log 7 - \log 2) = \log 2$$

$$x = 0.553$$

- M1 Introduce logarithms throughout, or equiv with base 7 or 2
- M1 Drop power on at least one side
- A1 Obtain correct linear equation (allow with no brackets)
- M1 **Either** expand bracket and attempt to gather x terms,
or deal correctly with algebraic fraction
- A1 **5** Obtain $x = 0.55$, or rounding to this, with no errors seen

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Q2, (OCR 4722, Jun 2010, Q8)

a $\log 5^{3w-1} = \log 4^{250}$

$$(3w-1) \log 5 = 250 \log 4$$

$$3w-1 = \frac{250 \log 4}{\log 5}$$

$$w = 72.1$$

- M1* Introduce logarithms throughout
- M1* Use $\log a^b = b \log a$ at least once
- A1 Obtain $(3w-1) \log 5 = 250 \log 4$ or equiv
- M1d* Attempt solution of linear equation
- A1 **5** Obtain 72.1, or better

b $\log_x \frac{5y+1}{3} = 4$

$$\frac{5y+1}{3} = x^4$$

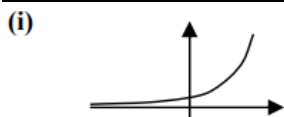
$$5y+1 = 3x^4$$

$$y = \frac{3x^4 - 1}{5}$$

- M1 Use $\log a - \log b = \log \frac{a}{b}$ or equiv
- M1 Use $f(y) = x^4$ as inverse of $\log_x f(y) = 4$
- M1 Attempt to make y the subject of $f(y) = x^4$
- A1 **4** Obtain $y = \frac{3x^4 - 1}{5}$, or equiv

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Q3, (OCR 4722, Jun 2008, Q8)



- M1** Attempt sketch of exponential graph (1st quad)
- if seen in 2nd quad must be approx correct
- A1** Correct graph in both quadrants
- B1** State or imply (0, 2) only

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(ii) $8^x = 2 \times 3^x$
 $\log_2 8^x = \log_2 (2 \times 3^x)$
 $x \log_2 8 = \log_2 2 + x \log_2 3$
 $3x = 1 + x \log_2 3$
 $x(3 - \log_2 3) = 1$, hence $x = \frac{1}{3 - \log_2 3}$ **A.G.**

- M1** Form equation in x and take logs (to any consistent base, or no base) – could use \log_8
- M1** Use $\log a^b = b \log a$
- M1** Use $\log ab = \log a + \log b$, or equiv with $\log a^b$
- M1** Use $\log_2 8 = 3$
- A1** Show given answer correctly

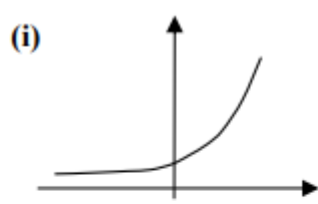
OR $8^x = 2 \times 3^x$

$2^{3x} = 2 \times 3^x$
 $2^{(3x-1)} = 3^x$
 $\log_2 2^{(3x-1)} = \log_2 3^x$
 $(3x - 1) \log_2 2 = x \log_2 3$
 $x(3 - \log_2 3) = 1$, hence $x = \frac{1}{3 - \log_2 3}$ **A.G.**

- M1** Use $8^x = 2^{3x}$
- M1** Attempt to rearrange equation to $2^k = 3^x$
- M1** Take logs (to any base)
- M1** Use $\log a^b = b \log a$
- A1** Show given answer correctly

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Q4, (OCR 4722, Jan 2010, Q9)



- M1** Reasonable graph in both quadrants
 - A1** Correct graph in both quadrants
- B1** 3 State or imply (0, 6)

(ii) $9^x = 150$
 $x \log 9 = \log 150$
 $x = 2.28$

- M1** Introduce logarithms throughout, or equiv with \log_9
- M1** Use $\log a^b = b \log a$ and attempt correct method to find x
- A1** 3 Obtain $x = 2.28$

(iii) $6 \times 5^x = 9^x$
 $\log_3 (6 \times 5^x) = \log_3 9^x$
 $\log_3 6 + x \log_3 5 = x \log_3 9$
 $\log_3 3 + \log_3 2 + x \log_3 5 = 2x$
 $x(2 - \log_3 5) = 1 + \log_3 2$
 $x = \frac{1 + \log_3 2}{2 - \log_3 5}$ **A.G.**

- M1** Form eqn in x and take logs throughout (any base)
- M1** Use $\log a^b = b \log a$ correctly on $\log 5^x$ or $\log 9^x$ or legitimate combination of these two
- M1** Use $\log ab = \log a + \log b$ correctly on $\log (6 \times 5^x)$ or $\log 6$
- M1** Use $\log_3 9 = 2$ or equiv (need base 3 throughout that line)
- A1** 5 Obtain $x = \frac{1 + \log_3 2}{2 - \log_3 5}$ convincingly
(inc base 3 throughout)

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(a)(i) $\log_a xy = p + q$	B1	1	State $p + q$ cwo
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(ii) $\log_a \left(\frac{a^2 x^3}{y}\right) = 2 + 3p - q$	M1		Use $\log a^b = b \log a$ correctly at least once
	M1		Use $\log \frac{a}{b} = \log a - \log b$ correctly
	A1	3	Obtain $2 + 3p - q$
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(b)(i) $\log_{10} \frac{x^2 - 10}{x}$	B1	1	State $\log_{10} \frac{x^2 - 10}{x}$ (with or without base 10)
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(ii) $\log_{10} \frac{x^2 - 10}{x} = \log_{10} 9$	B1		State or imply that $2 \log_{10} 3 = \log_{10} 3^2$
$\frac{x^2 - 10}{x} = 9$	M1		Attempt correct method to remove logs
$x^2 - 9x - 10 = 0$	A1		Obtain correct $x^2 - 9x - 10 = 0$ aef, no fractions
$(x - 10)(x + 1) = 0$	M1		Attempt to solve three term quadratic
$x = 10$	A1	5	Obtain $x = 10$ only

Q6, (OCR 4722, Jan 2013, Q8)

(i)		Translation of 3 units in positive x -direction	B1	State translation
			B1	State or imply 3 units in positive x -direction
(ii)		$a = 8$	B1 [1]	State 8
(iii)		$b - 3 = 2^{1.8}$ $b = 6.48$	B1	State or imply $b - 3 = 2^{1.8}$
			B1 [2]	Obtain 6.48, or better
(iv)		$\log_2 c - \log_2(c - 3) = 4$ $\log_2 {}^c/c-3 = 4$ ${}^c/c-3 = 2^4$ $c = 16c - 48$ $c = 48/15 = 16/5$	M1	Equate difference in y -coordinates to ± 4
			M1	Use $\log a - \log b = \log a/b$
			A1	Obtain ${}^c/c-3 = 2^4$
			A1	Obtain $16/5$ oe
			[4]	

Q7, (OCR 4722, Jun 2013, Q8)

(i)	(a)	(0, 1)	B1	State (0, 1)
			[1]	
	(b)	(0, 4)	B1	State (0, 4)
			[1]	
	(c)	State a possible value for a	B1	Must satisfy $a > 1$
		State a possible value for b	B1	Must satisfy $0 < b < 1$
			[2]	

(ii)

$$\log_2 a^x = \log_2(4b^x)$$

$$\log_2 a^x = \log_2 4 + \log_2 b^x$$

$$x \log_2 a = \log_2 4 + x \log_2 b$$

$$x \log_2 a = \log_2 4 + x \log_2 (2/a)$$

$$x \log_2 a = 2 + x \log_2 2 - x \log_2 a$$

$$x (2 \log_2 a - 1) = 2$$

$$x = \frac{2}{2 \log_2 a - 1} \quad \mathbf{AG}$$

M1

Equate a^x and $4b^x$ and introduce logarithms at some stage

M1

Use $\log ab = \log a + \log b$ correctly

M1

Use $\log a^b = b \log a$ correctly at least once

B1

Use $b = 2/a$ to produce a correct equation in a and x only

A1

Obtain given relationship with no wrong working

[5]