Exponential and Logarithmic Equations Exam Questions MS

Q1, (OCR 4722, Jun 2009, Q3)

$\log 7^x = \log 2^{x+1}$	M1		Introduce logarithms throughout, or equiv with base 7 or 2
$x \log 7 = (x+1)\log 2$	M1		Drop power on at least one side
	A1		Obtain correct linear equation (allow with no brackets)
$x(\log 7 - \log 2) = \log 2$	M1		**Either** expand bracket and attempt to gather x terms, **or** deal correctly with algebraic fraction
$x = 0.553$	A1	**5**	Obtain $x = 0.55$, or rounding to this, with no errors seen
		5	

Q2, (OCR 4722, Jun 2010, Q8)

a	$\log 5^{3w-1} = \log 4^{250}$	**M1***		Introduce logarithms throughout
	$(3w-1)\log 5 = 250 \log 4$	**M1***		Use $\log a^b = b \log a$ at least once
	$3w - 1 = \frac{250 \log 4}{\log 5}$	**A1**		Obtain $(3w-1)\log 5 = 250 \log 4$ or equiv
	$w = 72.1$	**M1d***		Attempt solution of linear equation
		A1	**5**	Obtain 72.1, or better
b	$\log_x \frac{5y+1}{3} = 4$	**M1**		Use $\log a - \log b = \log {}^a/_b$ or equiv
	$\frac{5y+1}{3} = x^4$	**M1**		Use $f(y) = x^4$ as inverse of $\log_x f(y) = 4$
	$5y + 1 = 3x^4$	**M1**		Attempt to make y the subject of $f(y) = x^4$
	$y = \frac{3x^4 - 1}{5}$	**A1**	**4**	Obtain $y = \frac{3x^4 - 1}{5}$, or equiv
			9	

Q3, (OCR 4722, Jun 2008, Q8)

(i)

M1		Attempt sketch of exponential graph (1st quad) - if seen in 2nd quad must be approx correct
A1		Correct graph in both quadrants
B1		State or imply (0, 2) only
3		

(ii) $8^x = 2 \times 3^x$

$\log_2 8^x = \log_2 (2 \times 3^x)$ — **M1** Form equation in x and take logs (to any consistent base, or no base) – could use $\log_8$

$x\log_2 8 = \log_2 2 + x\log_2 3$ — **M1** Use $\log a^b = b \log a$

M1 Use $\log ab = \log a + \log b$,or equiv with $\log a/b$

$3x = 1 + x\log_2 3$ — **M1** Use $\log_2 8 = 3$

$x(3 - \log_2 3) = 1$, hence $x = \dfrac{1}{3 - \log_2 3}$ **A.G.** — **A1** Show given answer correctly

OR $8^x = 2 \times 3^x$

$2^{3x} = 2 \times 3^x$ — **M1** Use $8^x = 2^{3x}$

$2^{(3x-1)} = 3^x$ — **M1** Attempt to rearrange equation to $2^k = 3^x$

$\log_2 2^{(3x-1)} = \log_2 3^x$ — **M1** Take logs (to any base)

$(3x - 1)\log_2 2 = x \log_2 3$ — **M1** Use $\log a^b = b \log a$

$x(3 - \log_2 3) = 1$, hence $x = \dfrac{1}{3 - \log_2 3}$ **A.G.** — **A1** Show given answer correctly

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Q4, (OCR 4722, Jan 2010, Q9)

(i)

M1		Reasonable graph in both quadrants
A1		Correct graph in both quadrants
B1	3	State or imply (0, 6)

(ii) $9^x = 150$ — M1 Introduce logarithms throughout, or equiv with $\log_9$

$x \log 9 = \log 150$ — M1 Use $\log a^b = b \log a$ and attempt correct method to find x

$x = 2.28$ — A1 3 Obtain $x = 2.28$

(iii) $6 \times 5^x = 9^x$ — M1 Form eqn in x and take logs throughout (any base)

$\log_3 (6 \times 5^x) = \log_3 9^x$ — M1 Use $\log a^b = b \log a$ correctly on $\log 5^x$ or $\log 9^x$ or legitimate combination of these two

$\log_3 6 + x \log_3 5 = x \log_3 9$ — M1 Use $\log ab = \log a + \log b$ correctly on $\log (6 \times 5^x)$ or $\log 6$

$\log_3 3 + \log_3 2 + x \log_3 5 = 2x$ — M1 Use $\log_3 9 = 2$ or equiv (need base 3 throughout that line)

$x(2 - \log_3 5) = 1 + \log_3 2$

$x = \dfrac{1 + \log_3 2}{2 - \log_3 5}$ **A.G.** — A1 5 Obtain $x = \dfrac{1 + \log_3 2}{2 - \log_3 5}$ convincingly (inc base 3 throughout)

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Q5, (OCR 4722 Jan 2009, Q8)

(a)(i) $\log_a xy = p+q$	B1	**1**	State $p+q$ cwo
(ii) $\log_a\left(\frac{a^2x^3}{y}\right) = 2+3p-q$	M1		Use $\log a^b = b\log a$ correctly at least once
	M1		Use $\log\frac{a}{b} = \log a - \log b$ correctly
	A1	**3**	Obtain $2+3p-q$
(b)(i) $\log_{10}\frac{x^2-10}{x}$	B1	**1**	State $\log_{10}\frac{x^2-10}{x}$ (with or without base 10)
(ii) $\log_{10}\frac{x^2-10}{x} = \log_{10} 9$	B1		State or imply that $2\log_{10} 3 = \log_{10} 3^2$
$\frac{x^2-10}{x} = 9$	M1		Attempt correct method to remove logs
$x^2-9x-10=0$	A1		Obtain correct $x^2-9x-10=0$ aef, no fractions
$(x-10)(x+1)=0$	M1		Attempt to solve three term quadratic
$x=10$	A1	**5**	Obtain $x=10$ only
		10	

Q6, (OCR 4722, Jan 2013, Q8)

(i)		Translation of 3 units in positive x-direction	B1 B1	State translation State or imply 3 units in positive x-direction
(ii)		$a = 8$	B1 **[1]**	State 8
(iii)		$b - 3 = 2^{1.8}$ $b = 6.48$	B1 B1 **[2]**	State or imply $b - 3 = 2^{1.8}$ Obtain 6.48, or better
(iv)		$\log_2 c - \log_2(c-3) = 4$ $\log_2 {}^{c}/_{c-3} = 4$ ${}^{c}/_{c-3} = 2^4$ $c = 16c - 48$ $c = {}^{48}/_{15} = {}^{16}/_{5}$	M1 M1 A1 A1 **[4]**	Equate difference in y-coordinates to ± 4 Use $\log a - \log b = \log {}^{a}/_{b}$ Obtain ${}^{c}/_{c-3} = 2^4$ Obtain ${}^{16}/_{5}$ oe

Q7, (OCR 4722, Jun 2013, Q8)

(i)	**(a)**	(0, 1)	B1 **[1]**	State (0, 1)
	(b)	(0, 4)	B1 **[1]**	State (0, 4)
	(c)	State a possible value for a	B1	Must satisfy $a > 1$
		State a possible value for b	B1 **[2]**	Must satisfy $0 < b < 1$

(ii)		$\log_2 a^x = \log_2(4b^x)$	M1	Equate a^x and $4b^x$ and introduce logarithms at some stage
		$\log_2 a^x = \log_2 4 + \log_2 b^x$	M1	Use $\log ab = \log a + \log b$ correctly
		$x\log_2 a = \log_2 4 + x\log_2 b$	M1	Use $\log a^b = b \log a$ correctly at least once
		$x\log_2 a = \log_2 4 + x\log_2 ({}^2/_a)$	B1	Use $b = {}^2/_a$ to produce a correct equation in a and x only
		$x\log_2 a = 2 + x\log_2 2 - x\log_2 a$ $x(2\log_2 a - 1) = 2$ $x = \dfrac{2}{2\log_2 a - 1}$ **AG**	A1	Obtain given relationship with no wrong working
			[5]	