

Differentiation Exam Questions MS (From OCR MEI 4752 unless otherwise stated)

Q1, (Jan 2007, Q1)

$\frac{5}{2} \times 6x^{\frac{3}{2}}$	1+1	- 1 if extra term	2
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Q2, (Jan 2009, Q7)

$8x - x^{-2}$ o.e.	2	B1 each term	5
their $\frac{dy}{dx} = 0$	M1	s.o.i.	
correct step	DM1	s.o.i.	
$x = \frac{1}{2}$ c.a.o.	A1		

Q3 (Jun 2007, Q9)

i	$y' = 6x^2 - 18x + 12$ $= 12$ $y = 7$ when $x = 3$ tgt is $y - 7 = 12(x - 3)$ verifying $(-1, -41)$ on tgt	M1 M1 B1 M1 A1	condone one error subst of $x = 3$ in <u>their</u> y' f.t. $they$ y' or B2 for showing line joining $(3, 7)$ and $(-1, -41)$ has gradient 12	5
ii	$y' = 0$ soi quadratic with 3 terms $x = 1$ or 2 $y = 3$ or 2	M1 M1 A1 A1	Their y' Any valid attempt at solution or A1 for $(1, 3)$ and A1 for $(2, 2)$ marking to benefit of candidate	4
iii	cubic curve correct orientation touching x- axis only at $(0.2, 0)$ max and min correct curve crossing y axis only at -2	G1 G1 G1	f.t.	3

Q4, (Jun 2009, Q6)

$y = 3x^2 - 12x - 15$ use of $y' = 0$, s.o.i. ft $x = 5, -1$ c.a.o. $x < -1$ or $x > 5$ f.t.	M1 M1 A1 A1 A1	for two terms correct	5
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Q5, (Jan 2013, Q6)

$6x^2 + 18x - 24$	B1	
their $6x^2 + 18x - 24 = 0$ or > 0 or ≥ 0	M1	
-4 and $+1$ identified oe	A1	
$x < -4$ and $x > 1$ cao	A1	or $x \leq -4$ and $x \geq 1$
	[4]	

Q6, (Jun 2009, Q12)

i	6.1	2	M1 for $\frac{(3.1^2 - 7) - (3^2 - 7)}{3.1 - 3}$ o.e.	2
ii	$\frac{((3+h)^2 - 7) - (3^2 - 7)}{h}$ numerator = $6h + h^2$ $6 + h$	M1 M1 A1	s.o.i.	3
iii	as h tends to 0, grad. tends to 6 o.e. f.t. from " 6 " + h	M1 A1		2
iv	$y - 2 = "6" (x - 3)$ o.e. $y = 6x - 16$	M1 A1	6 may be obtained from $\frac{dy}{dx}$	2
v	At P, $x = 16/6$ o.e. or ft At Q, $x = \sqrt{7}$ 0.021 cao	M1 M1 A1		3

Q7, (Jan 2006, Q11)

i	$3x^2 - 6$	2	1 if one error	2
ii	$-\sqrt{2} < x < \sqrt{2}$	3	M1 for using their $y' = 0$ B1 f.t. for both roots found	3
iii	subst $x = -1$ in their $y' [= -3]$ $y = 7$ when $x = -1$ $y + 3x = 4$ $x^3 - 6x + 2 = -3x + 4$ $(2, -2)$ c.a.o.	B1 M1 A1 M1 A1, A1	f.t. f.t. 3 terms f.t.	6

Q8, (Jan 2007, Q5)

(i)	$-0.93, -0.930, -0.9297\dots$	2	M1 for grad = $(1 - \text{their } y_B)/(2 - 2.1)$ if M0, SC1 for 0.93 don't allow 1.9 recurring	
(ii)	answer strictly between 1.91 and 2 or 2 and 2.1	B1		
(iii)	$y = -8/x^3$, gradient = -1	M1A1		5

Q9, (Jun 2014, Q11)

(i)	$y' = 1 + 8x^{-3}$ $y'' = -24x^{-4}$ oe	M2 A1 [3]	M1 for just $8x^{-3}$ or $1 - 8x^{-3}$
(ii)	their $y' = 0$ soi $x = -2$ $y = -3$ substitution of $x = -2$: $\frac{-24}{(-2)^4}$ < 0 or $= -1.5$ oe correctly obtained isw	M1 A1 A1 M1 A1 [5]	A0 if more than one x -value A0 if more than one y -value or considering signs of gradient either side of -2 with negative x -values signs for gradients identified to verify maximum
(iii)	$y = -5$ soi substitution of $x = -1$ in their y' grad normal = $^{-1}/_{\text{their} - 7}$ $y - \text{their}(-5) = \text{their} \frac{1}{7}(x - -1)$ $-x + 7y + 34 = 0$ oe	B1 M1 M1* M1dep* A1 [5]	may be implied by -7 may be implied by eg $\frac{1}{7}$ or their $(-5) = \text{their} \frac{1}{7} \times (-1) + c$ allow eg $y - \frac{1}{7}x + \frac{34}{7} = 0$

Q10, (Jun 2016, Q10)

(i)	$\frac{(5.1^2 - 10.2) - (5^2 - 10)}{5.1 - 5}$ oe 8.1	M1 A1 [2]	condone omission of brackets
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(ii)	$\frac{(5+h)^2 - 2(5+h) - \text{their } 15}{h} \text{ oe}$ <p>25 + 10h + h² - 10 - 2h oe seen</p> <p>numerator is 8h + h²</p> <p>8 + h isw</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[4]</p>	<p>condone omission of brackets</p> <p>allow one sign error</p>
(iii)	<p>$h \rightarrow 0$</p> <p>their 8</p>	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>may be embedded; allow eg “tends to 0”</p> <p>FT their $k + h$ from part (ii)</p>
(iv)	<p>$y = 8x - 25$ isw</p> <p>non-zero numerical value for x-intercept on their straight line found</p> <p>[$x =$] 3.125 oe</p> <p>$\frac{1}{2} \times$ their non-zero y-intercept \times their $\frac{25}{8}$</p> <p>$\frac{625}{16}$ or $39\frac{1}{16}$ or 39.0625 isw</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>or $y - 15 = 8(x - 5)$ isw or $y = 8x + c$ and $c = -25$ stated isw</p> <p>may be embedded in calculation for area</p> <p>condone arithmetic slips in finding values of intercepts</p> <p>accept rounded to 1 dp or better for A1; but A0 if final answer negative</p>

Q11, (OCR 4721, Jun 2015, Q9)

(i)	$\frac{dy}{dx} = 6x^2 - 2ax + 8$ <p>When $x = 4$, $\frac{dy}{dx} = 104 - 8a$</p> $\frac{dy}{dx} = 0 \text{ gives } a = 13$	<p>M1 A1 M1 M1 A1 [5]</p>	<p>Attempt to differentiate, at least two non-zero terms correct Fully correct Substitutes $x = 4$ into their $\frac{dy}{dx}$ Sets their $\frac{dy}{dx}$ to 0. Must be seen</p>
(ii)	$\frac{d^2y}{dx^2} = 12x - 26$ <p>When $x = 4$, $\frac{d^2y}{dx^2} > 0$ so minimum</p>	<p>M1 A1 [2]</p>	<p>Correct method to find nature of stationary point e.g. substituting $x = 4$ into second derivative (at least one term correct from their first derivative in (i)) and consider the sign www</p>
(iii)	$6x^2 - 26x + 8 = 0$ $(3x - 1)(x - 4) = 0$ $x = \frac{1}{3}$	<p>M1 M1 A1 [3]</p>	<p>Sets their derivative to zero Correct method to solve quadratic (appx 1) oe</p>

Q12, (OCR 4721, Jun 2016, Q8)

(i)	$y_1 = 50, y_2 = 2(5 + h)^2$ $\frac{(50 + 20h + 2h^2) - 50}{(5 + h) - 5}$ $20 + 2h$	<p>B1 M1 A1 [3]</p>	<p>Finds y coordinates at 5 and $5 + h$ Correct method to find gradient of a line segment; at least 3/4 values correct Fully correct working to give answer AG</p>
(ii)	<p>e.g. "As h tends to zero, the gradient will be 20"</p>	<p>B1 [1]</p>	<p>Indicates understanding of limit See Appendix 2 for examples</p>
(iii)	<p>Gradient of normal = $-\frac{1}{20}$</p> $y - 50 = -\frac{1}{20}(x - 5), x = 0$ <p>50¼</p>	<p>B1 M1 A1 [3]</p>	<p>Gradient of line must be numerical negative reciprocal of their gradient at A through their A Correct coordinate in any form e.g. $\frac{201}{4}, \frac{1005}{20}$</p>

Q13, (OCR 4721, Jun 2016, Q11)

$$y = 4x^2 + ax^{-1} + 5$$

$$\frac{dy}{dx} = 8x - ax^{-2}$$

At stationary point, $8x - ax^{-2} = 0$

$$a = 8x^3 \text{ oe}$$

When $a = 8x^3, y = 32$

$$32 = 4x^2 + 8x^2 + 5$$

$$x = \frac{3}{2} \text{ oe}$$

$$a = 27$$

OR

$$y = 4x^2 + ax^{-1} + 5$$

$$\frac{dy}{dx} = 8x - ax^{-2}$$

$$32 = 4x^2 + ax^{-1} + 5$$

$$a = 27x - 4x^3$$

At stationary point, $8x - ax^{-2} = 0$

$$8x - (27x - 4x^3)x^{-2} = 0$$

$$x = \frac{3}{2} \text{ oe}$$

$$a = 27$$

B1	ax^{-1} soi
M1	Attempt to differentiate – at least one non-zero term correct
A1	Fully correct
M1	Sets their derivative to 0
A1	Obtains expression for a in terms of x , or x in terms of a www
M1	Substitutes their expression and 32 into equation of the curve to form single variable equation
A1	Obtains correct value for x . Allow $x = \sqrt{\frac{27}{12}}$.
	Ignore $-\frac{3}{2}$ given as well.
A1	Obtains correct value for a . Ignore -27 given as well.
[8]	
B1	ax^{-1} soi
M1	Attempt to differentiate – at least one non-zero term correct
A1	Fully correct
M1	Substitutes 32 into equation of the curve to find expression for a
A1	Obtains expression for a in terms of x www
M1	Sets derivative to zero and forms single variable equation
A1	Obtains correct value for x . Allow $x = \sqrt{\frac{27}{12}}$.
	Ignore $-\frac{3}{2}$ given as well.
A1	Obtains correct value for a . Ignore -27 given as well.

Q14, (OCR 4721, Jun 2017, Q11)

(i)	<p>Gradient of given line = 6</p> <p>Perpendicular gradient = $-\frac{1}{6}$</p> $\frac{dy}{dx} = -2kx^{-3}$ $-\frac{1}{6} = -2k(-3)^{-3}$ $k = -\frac{9}{4}$	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[6]</p>	<p>soi as gradient of the line</p> <p>Uses product of perpendicular gradients is -1 at some point; may be implied by later working.</p> <p>Attempt to differentiate (ax^{-3} seen)</p> <p>Fully correct</p> <p>Equates their derivative at $x = -3$ with their perpendicular gradient</p> <p>Correct value of k. Allow $-\frac{27}{12}$ etc.</p>
(ii)	<p>When $x = -3, y = -\frac{9}{4(-3)^2} = -\frac{1}{4}$</p> $y + \frac{1}{4} = 6(x + 3)$ $24x - 4y + 71 = 0$	<p>B1</p> <p>M1</p> <p>A1ft</p> <p>A1</p> <p>[4]</p>	<p>Correct value of y www</p> <p>Attempts equation of straight line through $(-3, y)$, any non-zero gradient. y must be from their k but allow slips for M mark.</p> <p>Correct equation in any form – gradient 6 but ft their value of $\frac{k}{9}$. Allow $6(x - -3)$</p> <p>Correct equation in required form i.e. $a(24x - 4y + 71) = 0$ for integer a, terms in any order. cao</p>

Q15, (Jun 2010, Q10)

(i)	$\frac{dy}{dx} = 4x^3$ when $x = 2$, $\frac{dy}{dx} = 32$ s.o.i. when $x = 2$, $y = 16$ s.o.i. $y = 32x - 48$ c.a.o.	M1 A1 B1 A1	i.s.w.
(ii)	34.481	2	M1 for $\frac{2.1^4 - 2^4}{0.1}$
(iii) (A)	$16 + 32h + 24h^2 + 8h^3 + h^4$ c.a.o.	3	B2 for 4 terms correct B1 for 3 terms correct
(iii) (B)	$32 + 24h + 8h^2 + h^3$ or ft	2	B1 if one error
(iii) (C)	as $h \rightarrow 0$, result \rightarrow their 32 from (iii) (B) gradient of tangent is limit of gradient of chord	1 1	

Q16, (OCR H230/02, Sample Question Paper, Q7)

$f(x+h) = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$	M1	1.1	Attempt at expansion with product of powers of x and h summing to 4 and some attempt at coefficients, not necessarily correct
$\frac{f(x+h) - f(x)}{h} = \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$	M1	1.1	Attempt $\frac{f(x+h) - f(x)}{h}$
$= 4x^3 + 6x^2h + 4xh^2 + h^3$	A1	1.1	Allow at most two errors All terms correct
As $h \rightarrow 0$ all the terms in h tend to zero.	A1	2.4	Accept some indication that as h tends to 0, the terms involving h vanish and leave $4x^3$
Therefore $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = 4x^3$	E1	2.1	Award for good use of language, and of limit and function notation
	[5]		