Question 1 (Jun 2005, Q3)

Worked Solution

(i) Find $\int (2x+1)(x+3)\,\mathrm{d}x$.

Expand: $(2x+1)(x+3) = 2x^2 + 7x + 3$.

$$\int (2x^2 + 7x + 3)\,\mathrm{d}x = \frac{2x^3}{3} + \frac{7x^2}{2} + 3x + c$$

$\frac{2}{3}x^3 + \frac{7}{2}x^2 + 3x + c$

(ii) Evaluate $\int_0^9 \frac{1}{\sqrt{x}}\,\mathrm{d}x$.

$$\int_0^9 x^{-\frac{1}{2}}\,\mathrm{d}x = \left[2x^{\frac{1}{2}}\right]_0^9 = 2(3) - 0 = 6$$

6

Question 2 (Jun 2007, Q6)

Worked Solution

(a)(i) Find $\int x(x^2-4)\,\mathrm{d}x$.

Expand: $x(x^2-4) = x^3 - 4x$.

$$\int (x^3 - 4x)\,\mathrm{d}x = \frac{x^4}{4} - 2x^2 + c$$

$$\frac{x^4}{4} - 2x^2 + c$$

(a)(ii) Evaluate $\int_1^6 x(x^2-4)\,\mathrm{d}x$.

$$\left[\frac{x^4}{4} - 2x^2\right]_1^6 = \left(\frac{1296}{4} - 72\right) - \left(\frac{1}{4} - 2\right) = (324 - 72) - \left(-\frac{7}{4}\right) = 252 + \frac{7}{4} = \frac{1015}{4}$$

$253\frac{3}{4}$

(b) Find $\int \frac{6}{x^3}\,\mathrm{d}x = \int 6x^{-3}\,\mathrm{d}x$.

$$= \frac{6x^{-2}}{-2} + c = -3x^{-2} + c$$

$$-\frac{3}{x^2} + c$$

Question 3 (Jan 2009, Q1)

Worked Solution

(i) Find $\displaystyle\int (x^3 + 8x - 5)\,\mathrm{d}x$.

$$= \frac{x^4}{4} + 4x^2 - 5x + c$$

$\dfrac{x^4}{4} + 4x^2 - 5x + c$

(ii) Find $\displaystyle\int 12\sqrt{x}\,\mathrm{d}x = \int 12x^{\frac{1}{2}}\,\mathrm{d}x$.

$$= \frac{12x^{\frac{3}{2}}}{\frac{3}{2}} + c = 8x^{\frac{3}{2}} + c$$

$8x^{\frac{3}{2}} + c$

Question 4 (Jun 2009, Q4)

Worked Solution

(i) Expand $(x^2-5)^3$ using the binomial theorem:

$$(x^2)^3 + 3(x^2)^2(-5) + 3(x^2)(-5)^2 + (-5)^3 = x^6 - 15x^4 + 75x^2 - 125$$

$(x^2-5)^3 = x^6 - 15x^4 + 75x^2 - 125$

(ii) Hence find $\int (x^2-5)^3 \, dx$:

$$\int (x^6 - 15x^4 + 75x^2 - 125) \, dx = \frac{x^7}{7} - 3x^5 + 25x^3 - 125x + c$$

$\frac{x^7}{7} - 3x^5 + 25x^3 - 125x + c$

Question 5 (Jan 2011, Q6a,b(i))

Worked Solution

(a) Find $\displaystyle\int \frac{x^3 + 3x^{\frac{1}{2}}}{x}\,\mathrm{d}x$.

Divide each term by x:

$$\int \left(x^2 + 3x^{-\frac{1}{2}}\right)\mathrm{d}x = \frac{x^3}{3} + \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{1}{3}x^3 + 6x^{\frac{1}{2}} + c$$

$\frac{1}{3}x^3 + 6\sqrt{x} + c$

(b)(i) Find, in terms of a ($a > 2$), the value of $\displaystyle\int_2^a 6x^{-4}\,\mathrm{d}x$:

$$\left[\frac{6x^{-3}}{-3}\right]_2^a = \left[-2x^{-3}\right]_2^a = -2a^{-3} - (-2\cdot 2^{-3}) = \frac{1}{4} - \frac{2}{a^3}$$

$\frac{1}{4} - \frac{2}{a^3}$

Question 6 (Jan 2013, Q9)

Worked Solution

$\int_a^{2a} \frac{2x^3 - 5x^2 + 4}{x^2}\,dx = 0,\ a > 0.$

(i) Divide each term by x^2: $\frac{2x^3}{x^2} - \frac{5x^2}{x^2} + \frac{4}{x^2} = 2x - 5 + 4x^{-2}$.

$$\int (2x - 5 + 4x^{-2})\,dx = x^2 - 5x - 4x^{-1}$$

Apply limits a to $2a$:

$$\left[x^2 - 5x - \frac{4}{x}\right]_a^{2a} = \left(4a^2 - 10a - \frac{2}{a}\right) - \left(a^2 - 5a - \frac{4}{a}\right) = 3a^2 - 5a + \frac{2}{a}$$

Set equal to 0, multiply by a:

$$3a^3 - 5a^2 + 2 = 0 \qquad \checkmark$$

$3a^3 - 5a^2 + 2 = 0$ (shown)

(ii) Show $a = 1$ is a root and find the other positive root.

$a = 1$: $3 - 5 + 2 = 0$ ✓

Factor out $(a - 1)$: $3a^3 - 5a^2 + 2 = (a - 1)(3a^2 - 2a - 2)$.

Other roots from $3a^2 - 2a - 2 = 0$:

$$a = \frac{2 \pm \sqrt{4 + 24}}{6} = \frac{2 \pm \sqrt{28}}{6} = \frac{1 \pm \sqrt{7}}{3}$$

Taking the positive root: $a = \frac{1 + \sqrt{7}}{3}$.

$a = \frac{1 + \sqrt{7}}{3}$

Question 7 (Jun 2014, Q6)

Worked Solution

(i) Expand $\left(x^3 + \frac{2}{x^2}\right)^4$ using the binomial theorem:

$$= (x^3)^4 + 4(x^3)^3\left(\frac{2}{x^2}\right) + 6(x^3)^2\left(\frac{2}{x^2}\right)^2 + 4(x^3)\left(\frac{2}{x^2}\right)^3 + \left(\frac{2}{x^2}\right)^4$$
$$= x^{12} + 8x^7 + 24x^2 + 32x^{-3} + 16x^{-8}$$

$x^{12} + 8x^7 + 24x^2 + 32x^{-3} + 16x^{-8}$

(ii) Hence find $\int \left(x^3 + \frac{2}{x^2}\right)^4 \mathrm{d}x$:

$$= \frac{x^{13}}{13} + x^8 + 8x^3 - \frac{16}{x^2} - \frac{16}{7x^7} + c$$

$\frac{x^{13}}{13} + x^8 + 8x^3 - \frac{16}{x^2} - \frac{16}{7x^7} + c$

Question 8 (Jun 2015, Q6(i),(ii))

Worked Solution

$f(x) = x^3 - 19x + 30$.

(i) Express $f(x)$ as a product of three linear factors (given $x = 2$ is a root).

$f(2) = 8 - 38 + 30 = 0$ ✓, so $(x-2)$ is a factor.

Divide: $x^3 - 19x + 30 = (x-2)(x^2+2x-15) = (x-2)(x+5)(x-3)$.

$f(x) = (x-2)(x+5)(x-3)$

(ii) Evaluate $\int_{-5}^{3} f(x)\,dx$.

$$\int (x^3 - 19x + 30)\,dx = \frac{x^4}{4} - \frac{19x^2}{2} + 30x$$

$$\left[\frac{x^4}{4} - \frac{19x^2}{2} + 30x\right]_{-5}^{3} = \left(\frac{81}{4} - \frac{171}{2} + 90\right) - \left(\frac{625}{4} - \frac{475}{2} - 150\right)$$

$$= \left(\frac{81}{4} - \frac{342}{4} + \frac{360}{4}\right) - \left(\frac{625}{4} - \frac{950}{4} - \frac{600}{4}\right)$$

$$= \frac{99}{4} - \left(\frac{-925}{4}\right) = \frac{99+925}{4} = \frac{1024}{4} = 256$$

256

Question 9 (Jun 2016, Q5a,b(i))

Worked Solution

(a) Find $\int (x^2+2)(2x-3)\,\mathrm{d}x$.

Expand: $(x^2+2)(2x-3) = 2x^3 - 3x^2 + 4x - 6$.

$$\int (2x^3 - 3x^2 + 4x - 6)\,\mathrm{d}x = \frac{x^4}{2} - x^3 + 2x^2 - 6x + c$$

$\frac{x^4}{2} - x^3 + 2x^2 - 6x + c$

(b)(i) Find, in terms of a ($a > 1$), the value of $\int_1^a (6x^{-2} - 4x^{-3})\,\mathrm{d}x$.

$$\left[-6x^{-1} + 2x^{-2}\right]_1^a = \left(-\frac{6}{a} + \frac{2}{a^2}\right) - (-6+2) = -\frac{6}{a} + \frac{2}{a^2} + 4$$

$4 - \frac{6}{a} + \frac{2}{a^2}$

Question 10 (Jun 2018, Q4)

Worked Solution

(a) Find $\int_1^4 (3\sqrt{x} + 5)\,\mathrm{d}x$.

$3\sqrt{x} = 3x^{\frac{1}{2}}$:

$$\int (3x^{\frac{1}{2}} + 5)\,\mathrm{d}x = 2x^{\frac{3}{2}} + 5x$$

$$\left[2x^{\frac{3}{2}} + 5x\right]_1^4 = (2(8) + 20) - (2(1) + 5) = 36 - 7 = 29$$

29

(b) Find $\int \frac{6x^4 + 4}{x^2}\,\mathrm{d}x$.

Rewrite: $\frac{6x^4 + 4}{x^2} = 6x^2 + 4x^{-2}$.

$$\int (6x^2 + 4x^{-2})\,\mathrm{d}x = 2x^3 + \frac{4x^{-1}}{-1} + c = 2x^3 - \frac{4}{x} + c$$

$2x^3 - \frac{4}{x} + c$

End of Worked Solutions