Definite and Indefinite Integrals (From OCR 4722)

Q1, (Jun 2005, Q3)

(i)	$\int (2x^2 + 7x + 3)\,\mathrm{d}x$ $= \frac{2}{3}x^3 + \frac{7}{2}x^2 + 3x + c$	M1 A1 A1 B1	 4	For expanding and integration attempt For at least one term correct For all three terms correct For addition of arbitrary constant, and no $\int$ or dx
(ii)	$\left[2x^{\frac{1}{2}}\right]_0^9$ $= 6$	M1 M1 A1	 3 **7**	For integral of the form $kx^{\frac{1}{2}}$ For evaluating at least F(9), following attempt at integration For final answer of 6 only

Q2, (Jun 2007, Q6)

(a) **(i)**	$\int x^3 - 4x = \frac{1}{4}x^4 - 2x^2 + c$	M1 A1 B1	 **3**	Expand and attempt integration Obtain ¼ $x^4 - 2x^2$ (A0 if $\int$ or dx still present) + c (mark can be given in (b) if not gained here)
(ii)	$\left[\frac{1}{4}x^4 - 2x^2\right]_1^6$ =(324 – 72) – (¼ – 2) = 253¾	M1 A1	 **2**	Use limits correctly in integration attempt (ie F(6) – F(1)) Obtain 253¾ (answer only is M0A0)
(b)	$\int 6x^{-3}\mathrm{d}x = -3x^{-2} + c$	B1 M1 A1	 **3** **8**	Use of $\frac{1}{x^3} = x^{-3}$ Obtain integral of the form kx^{-2} Obtain correct $-3x^{-2}$ (+ c) (A0 if $\int$ or dx still present, but only penalise once in question)

Q3, (Jan 2009, Q1)

(i)	$\int (x^3 + 8x - 5)\mathrm{d}x = \frac{1}{4}x^4 + 4x^2 - 5x + c$	M1 A1 A1	 **3**	Attempt integration – increase in power for at least 2 terms Obtain at least 2 correct terms Obtain $\frac{1}{4}x^4 + 4x^2 - 5x + c$ (and no integral sign or dx)
(ii)	$\int 12x^{\frac{1}{2}}\mathrm{d}x = 8x^{\frac{3}{2}} + c$	B1 M1 A1	 **3** **6**	State or imply $\sqrt{x} = x^{\frac{1}{2}}$ Obtain $kx^{\frac{3}{2}}$ Obtain $8x^{\frac{3}{2}} + c$ (and no integral sign or dx) (only penalise lack of + c, or integral sign or dx once)

Q4, (Jun 2009, Q4)

(i) $(x^2-5)^3 = (x^2)^3 + 3(x^2)^2(-5) + 3(x^2)(-5)^2 + (-5)^3$	M1*		Attempt expansion, with product of powers of x^2 and ± 5, at least 3 terms
$= x^6 - 15x^4 + 75x^2 - 125$	M1*		Use at least 3 of binomial coeffs of 1, 3, 3, 1
	A1dep*		Obtain at least two correct terms, coeffs simplified
	A1	**4**	Obtain fully correct expansion, coeffs simplified
OR			
$(x^2-5)^3 = (x^2-5)(x^4-10x^2+25)$	M2		Attempt full expansion of all 3 brackets
$= x^6 - 15x^4 + 75x^2 - 125$	A1		Obtain at least two correct terms
	A1		Obtain full correct expansion
(ii) $\int (x^2-5)^3\,dx = \frac{1}{7}x^7 - 3x^5 + 25x^3 - 125x + c$	M1		Attempt integration of terms of form kx^n
	A1√		Obtain at least two correct terms, allow unsimplified coeffs
	A1		Obtain $\frac{1}{7}x^7 - 3x^5 + 25x^3 - 125x$
	B1	**4**	$+ c$, and no dx or $\int$ sign

8

Q5, (Jan 2011, Q6a,bi)

6(a) $\int \frac{x^3 + 3x^{\frac{1}{2}}}{x} \mathrm{d}x = \int \left(x^2 + 3x^{-\frac{1}{2}} \right) \mathrm{d}x$	**M1**		Simplify and attempt integration	Need to attempt to divide both terms by x, or multiply entire numerator by x^{-1} – allow if intention is clear even if errors when simplifying, or one term doesn't actually change. Need to simplify each of the two terms as far as x^n before integrating. For integration attempt, need to increase power by 1 for at least one term.
$= \frac{1}{3}x^3 + 6x^{\frac{1}{2}} + c$	**A1**		Obtain at least one correct term	Allow unsimplified terms.
	A1		Obtain $\frac{1}{3}x^3 + 6x^{\frac{1}{2}}$	Coefficients must now be simplified. Could be $6\sqrt{x}$ for second term.
	B1	**4**	Obtain $+ c$	Not dependent on previous marks as long as no longer original function. B0 if integral sign or dx still present in answer. Ignore anything that appears on LHS of an equation eg $y = \ldots$, d$x = \ldots$ or even $\int = \ldots$
(b)(i) $\int_2^a 6x^{-4} \mathrm{d}x = \left[-2x^{-3} \right]_2^a$	**M1**		Obtain integral of the form kx^{-3}	Any k, as long as numerical, including unsimplified. Allow $+ c$. Condone integral sign or dx still present.
$= \frac{1}{4} - 2a^{-3}$	**M1**		Attempt F(a) – F(2)	Must be correct order and subtraction. $- 2a^{-3} - {}^{2}/_{8}$ is M0 unless clear evidence suggesting that there was an intention to subtract and that this is a sign error. Not dependent on first M mark, so substituting into their integration attempt (eg kx^{-5}) can still get M1, but using kx^{-4} is M0.
	A1	**3**	Obtain $\frac{1}{4} - 2a^{-3}$	Allow ${}^{2}/_{8}$ for ¼ , but not – $(-{}^{2}/_{8})$, but want 2 not ${}^{6}/_{3}$. A0 if $+ c$, integral sign or dx still present. isw any subsequent work, usually equating to 0 or writing as inequality.

Q6, (Jan 2013, Q9)

(i)		$\int(2x-5+4x^{-2})\,dx = x^2-5x-4x^{-1}$	M1	Attempt to rewrite integrand in a suitable form	Attempt to divide all 3 terms by x^2, or attempt to multiply all 3 terms by x^{-2} soi
		$(4a^2-10a-{}^2/_a)-(a^2-5a-{}^4/_a) = 0$	A1	Obtain $2x-5+4x^{-2}$	Allow if third term is written in fractional form
		$3a^2-5a+{}^2/_a=0$ $3a^3-5a^2+2=0$ **AG**	M1	Attempt integration of their integrand	Their integrand must be written as a polynomial ie with all terms of the form kx^n, and no brackets At least two terms must increase in power by 1 Allow if the – 5 disappears
			A1	Obtain $x^2-5x-4x^{-1}$	Allow unsimplified (eg ${}^4/_{-1}\,x^{-1}$)
			M1	Attempt use of limits	Must be F(2a) - F(a) ie subtraction with limits in the correct order Allow if no brackets ie $4a^2-10a-{}^2/_a - a^2-5a-{}^4/_a$ Must be in integration attempt, but allow M1 for limits following M0 for integration eg if fraction not dealt with before integrating
			A1	Equate to 0 and rearrange to obtain $3a^3-5a^2+2=0$	Must be equated to 0 before multiplying through by a At least one extra line of working required between $(4a^2-10a-{}^2/_a)-(a^2-5a-{}^4/_a)=0$ and the final answer **AG** so look carefully at working
			[6]		

(ii)		$f(1) = 3 - 5 + 2 = 0$ **AG**			**Allow working in *x* not *a* throughout**
		$f(a) = (a-1)(3a^2 - 2a - 2)$	B1	Confirm $f(1) = 0$ – detail required	$3(1)^3 - 5(1)^2 + 2 = 0$ is enough B0 for just $f(1) = 0$ If using division must show '0' on last line If using coefficient matching must show 'R = 0' If using inspection then there must be some indication of no remainder eg expand to show correct cubic
		$a = \frac{2 \pm \sqrt{4+24}}{6} = \frac{2 \pm 2\sqrt{7}}{6} = \frac{1 \pm \sqrt{7}}{3}$	M1	Attempt full division by $(a-1)$, or equiv method	Must be complete method - ie all 3 terms attempted Long division - must subtract lower line (allow one slip) Inspection - expansion must give at least three correct terms of the cubic Coefficient matching - must be valid attempt at all coeffs of quadratic, considering all relevant terms each time
		hence $a = \frac{1}{3}(1 + \sqrt{7})$	A1	Obtain $3a^2$ and one other correct term	Could be middle or final term depending on method Must be correctly obtained Coeff matching - allow for $A = 3$ etc
			A1	Obtain fully correct quotient	Could appear as quotient in long division, or as part of a product if using inspection. For coeff matching it must now be explicit not just $A = 3$, $B = -2$, $C = -2$
			M1	Attempt to solve quadratic	Using the quadratic formula, or completing the square (see guidance sheet) though negative root may be lost at any point M0 if factorising attempt as expected root is a surd Quadratic must come from division attempt, even if this was not good enough for first M1
			A1 **[6]**	Obtain $\frac{1}{3}(1 + \sqrt{7})$ only	Must give the positive root only, so A0 if negative root still present (but condone $a = 1$ also given) Allow aef but must be a simplified surd as per request on question paper (ie simplify $\sqrt{28}$)

Q7, (Jun 2014, Q6)

(i)		$(x^3)^4 + 4(x^3)^3(2x^{-2}) + 6(x^3)^2(2x^{-2})^2 + 4(x^3)(2x^{-2})^3 + (2x^{-2})^4$ $= x^{12} + 8x^7 + 24x^2 + 32x^{-3} + 16x^{-8}$	M1*	Attempt expansion – products of powers of x^3 and $2x^{-2}$	Must attempt at least 4 terms Each term must be an attempt at a product, including binomial coeffs if used Allow M1 if no longer $2x^{-2}$ due to index errors Allow M1 for no, or incorrect, binomial coeffs Powers of x^3 and $2x^{-2}$ must be intended to sum to 4 within each term (allow slips if intention correct) Allow M1 even if powers used incorrectly with $2x^{-2}$ ie only applied to x^{-2} and not to 2 as well Allow M1 for expansion of bracket in $x^k(1 + 2x^{-5})^4$ with $k = 3$ or 12 only, or $x^k(x^5 + 2)^4$ with $k = -2$ or -8 only, oe
			M1d*	Attempt to use correct binomial coeffs	At least 4 correct from 1, 4, 6, 4, 1 - allow missing or incorrect (but not if raised to a power) May be implied rather than explicit Must be numerical eg ${}^4C_1$ is not enough They must be part of a product within each term The coefficient must be used in an attempt at the relevant term ie $6(x^3)^3(2x^{-2})$ is M0 Allow M1 for correct coefficients when expanding the bracket in $x^k(1 + 2x^{-5})^4$ or $x^k(x^5 + 2)^4$ $x^{12} + 8x^7 + 12x^2 + 8x^{-3} + 2x^{-8}$ gets M1 M1 implied (even if no method seen) – will also get the first A1 as well
			A1	Obtain two correct simplified terms	Either linked by '+' or as part of a list Powers and coefficients must be simplified
			A1	Obtain a further two correct terms	Either linked by '+' or as part of a list Powers and coefficients must be simplified
			A1 **[5]**	Obtain a fully correct expansion	Terms must be linked by '+' and not just commas Powers and coefficients must be simplified A0 if subsequent attempt to simplify indices (eg x by x^8)

					SR for reasonable expansion attempt: M2 for attempt involving all 4 brackets resulting in a quartic with at most one term missing A1 for two correct, simplified, terms A1 for a further two correct, simplified, terms A1 for fully correct, simplified, expansion
(ii)		${}^{1}/_{13}\, x^{13} + x^{8} + 8x^{3} - 16x^{-2} - {}^{16}/_{7}\, x^{-7} + c$	M1*	Attempt integration	Increase in power by 1 for at least three terms (other terms could be incorrect) Can still gain M1 if their expansion does not have 5 terms Allow if the three terms include x^{-1} becoming $k\ln x$ (but not x^{0})
			A1FT	Obtain at least 3 correct terms, following their **(i)**	Allow unsimplified coefficients
			A1	Obtain fully correct expression	Coefficients must be fully simplified, inc x^{8} not $1x^{8}$ isw subsequent errors eg $16x^{-2}$ then being written with 16 as well as x^{2} in the denominator of a fraction
			B1d*	$+\ c$, and no dx or integral sign in answer	Ignore notation on LHS such as $\int = \dots$, $y = \dots$, ${}^{dy}/_{dx} = \dots$
			[4]		

Q8, (Jun 2015, Q6i,ii)

(i)		$f(x) = (x-2)(x^2 + 2x - 15)$	B1	State or imply that $(x-2)$ is a factor	Could be stated explicitly, or implied by using it in an attempt at the quotient or a factorisation attempt Could also give $(2-x)$ as the factor
			M1	Attempt complete division, or equiv	Must be dividing by $(x-2)$, or by one of the two other correct factors (or the negative of any of these factors) No need to show zero remainder as told that $x = 2$ is a root Must be complete method - ie all 3 terms attempted Long division - must subtract lower line (allow one slip) Inspection - expansion must give at least three correct terms of the cubic Coefficient matching - must be valid attempt at all coeffs of quadratic, considering all relevant terms each time Synthetic division - must be using 2 (not -2) and adding within each column (allow one slip); expect to see $\begin{array}{c\|cccc} 2 & 1 & 0 & -19 & 30 \\ & & 2 & 4 & \\ \hline & 1 & 2 & -15 & \end{array}$
			A1	Obtain correct quotient of $x^2 + 2x - 15$ CWO	Or correct quotient for their factor Could be stated explicitly, seen in division attempt or implied by $A = 1$, $B = 2$, $C = -15$
		$= (x-2)(x+5)(x-3)$	A1	Obtain $(x-2)(x+5)(x-3)$	Must be written as a product of the three linear factors Allow any equiv eg $(2-x)(x+5)(3-x)$ Full credit for repeated use of factor theorem, or just writing down correct product Ignore any subsequent reference to roots **SR** A fully correct factorisation resulting from division by $(x+5)$ or $(x-3)$ can still get full credit, even though the root of $x = 2$ was not used
			[4]		

(ii)		$\left[\frac{1}{4}x^4 - \frac{19}{2}x^2 + 30x\right]_{-5}^{3}$	M1*	Attempt integration	Increase in power by 1 for at least 2 terms
			A1	Obtain correct integral	Could also have + c present; condone dx or ∫ still present
		= 24.75 – (– 231.25)	M1d*	Attempt correct use of limits	Must be F(3) – F(-5) Must be attempting the value of the requested definite integral, so M0 if instead attempting area (ie using x = 2 as a limit)
		= 256	A1	Obtain 256	A0 for 256 + c Answer only is 0/4 - need to see evidence of integration, but use of limits does not need to be explicit
			[4]		

Q9, (Jun 2016, Q5a,bi)

(a)		$\int(2x^3-3x^2+4x-6)\,dx$ $=\frac{1}{2}x^4-x^3+2x^2-6x+c$	M1	Expand brackets and attempt integration	Must be reasonable attempt to expand brackets, resulting in at least 3 terms, but allow slip(s) Integration attempt must have an increase in power by 1 for at least 3 of their terms
			A1FT	Obtain at least three correct (algebraic) terms	Following their expansion Allow unsimplified coefficients
			A1	Obtain fully correct expression, including $+c$	Coefficients must now be fully simplified A0 if integral sign or dx still present in final answer, but allow $\int = \ldots$
			[3]		
(b)	(i)	$\left[-6x^{-1}+2x^{-2}\right]_1^a$	M1	Attempt integration	Integral must be of the form $k_1x^{-1}+k_2x^{-2}$, any k_1 and k_2 as long as numerical
		$=(-6a^{-1}+2a^{-2})-(-6+2)$ $=4-6a^{-1}+2a^{-2}$	A1	Obtain fully correct expression	Allow unsimplified coefficients Allow presence of $+c$
			M1	Attempt correct use of limits	Must be F(a) – F(1) ie correct order and subtraction Allow F(x) to be any function with indices changed from the original, even if differentiation appears to have been attempted
			A1	Obtain $4-6a^{-1}+2a^{-2}$ aef	Coefficients should now be simplified, and constant terms combined Could use negative indices, or write as fractions A0 if $+c$ present in final answer A0 if integral sign or dx still present in final answer, but condone presence for first 3 marks ISW any subsequent work, such as further attempts at simplification, multiplying by a^2, equating to a constant, or writing as an inequality
			[4]		

Q10, (Jun 2018, Q4)

(a)		$\int(3\sqrt{x}+5)\mathrm{d}x = 2x^{\frac{3}{2}}+5x$	M1*	Attempt integration	Obtain expression of the form $ax^{\frac{3}{2}}+bx$, any non-zero a and b
			A1	Obtain fully correct integral	Allow unsimplified coefficients May also include $+c$
		$\left[2x^{\frac{3}{2}}+5x\right]_1^4 = 36-7$	M1d*	Attempt correct use of limits	Must have integral of correct form Correct order and subtraction
		$=29$	A1	Obtain 29	Answer only gets full marks
			[4]		
(b)		$\int(6x^2+4x^{-2})\mathrm{d}x$	B1	Rewrite integrand as $6x^2+4x^{-2}$	Any two term equiv, such as $6x^2+\frac{4}{x^2}$
		$=2x^3-4x^{-1}+c$	M1	Attempt integration	Obtain expression of the form ax^3+bx^{-1}, any non-zero a and b
			A1	Obtain fully correct integral, including $+c$	Coefficients must be simplified Allow equivs such as $2x^3-\frac{4}{x}+c$ A0 if dx or integral sign still present in final answer Allow MR on coefficients, but not on indices
			[3]		**OR** M1 – attempt integration by parts (correct parts) A1 – obtain $-\frac{6x^4+4}{x}+\int 24x^2\mathrm{d}x$, or better A1 – obtain $2x^3-4x^{-1}+c$ (must be simplified)