Definite and Indefinite Integrals Exam Questions Sheet 2

Q1.
Find

$$\int(8x^3 + 4)\mathrm{d}x$$

giving each term in its simplest form.

(3)

(Total 3 marks)

Q2.
Find

$$\int\left(2x^4 - \frac{4}{\sqrt{x}} + 3\right)\mathrm{d}x$$

giving each term in its simplest form.

(4)

(Total for question = 4 marks)

Q3.
Find

$$\int\left(2x^5 - \frac{1}{4x^3} - 5\right)\mathrm{d}x$$

giving each term in its simplest form.

(4)

(Total for question = 4 marks)

Q4.
Find

$$\int\left(3x^2 - \frac{4}{x^2}\right)\mathrm{d}x$$

giving each term in its simplest form.

(4)

(Total 4 marks)

Q5.

Given that $y = 2x^5 + \frac{6}{\sqrt{x}}$, $x > 0$, find in their simplest form

(a) $\frac{dy}{dx}$

(3)

(b) $\int y\,dx$

(3)

(Total 6 marks)

Q6.

Given that $y = 2x^5 + 7 + \frac{1}{x^3}, x \neq 0,$ find, in their simplest form,

(a) $\frac{dy}{dx}$,

(3)

(b) $\int y\,dx$.

(4)

(Total 7 marks)

Q7.

Given that $y = 3x^2 + 4\sqrt{x}$, $x > 0$, find

(a) $\frac{dy}{dx}$,

(2)

(b) $\frac{d^2y}{dx^2}$,

(2)

(c) $\int y\,dx$,

(3)

(Total 7 marks)

Q8.

Evaluate $\int_1^8 \frac{1}{\sqrt{x}}dx,$ giving your answer in the form $a + b\sqrt{2}$, where a and b are integers.

(4)

(Total 4 marks)

Q9.

Use integration to find

$$\int_1^{\sqrt{3}} \left(\frac{x^3}{6} + \frac{1}{3x^2} \right) dx$$

giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(5)

(Total 5 marks)

Q10.

Given that k is a positive constant and $\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) dx = 4$

(a) show that $3k + 5\sqrt{k} - 12 = 0$

(4)

(b) Hence, using algebra, find any values of k such that

$$\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) dx = 4$$

(4)

(Total for question = 8 marks)

Q11.

Given that

$$f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0$$

show that $\int_1^{2\sqrt{2}} f(x) dx = 16 + 3\sqrt{2}$

(5)

(Total for question = 5 marks)

Q12.

(a) Given that k is a constant, find

$$\int\left(\frac{4}{x^3}+kx\right)\mathrm{d}x$$

simplifying your answer.

(3)

(b) Hence find the value of k such that

$$\int_{0.5}^{2}\left(\frac{4}{x^3}+kx\right)\mathrm{d}x = 8$$

(3)

(Total for question = 6 marks)

Q13.

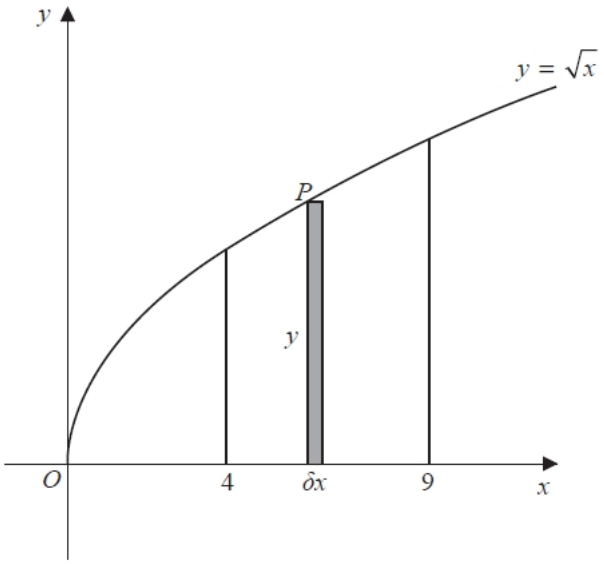

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \sqrt{x}$

The point P(x, y) lies on the curve.

The rectangle, shown shaded on Figure 3, has height y and width δx.

Calculate

$$\lim_{\delta x \to 0}\sum_{x=4}^{9}\sqrt{x}\,\delta x$$

(Total for question = 3 marks)