Mark Scheme

Q1.

Question Number	Scheme	Marks
	$\int (8x^3 + 4)\,dx = \frac{8x^4}{4} + 4x$	M1, A1
	$= 2x^4 + 4x + c$	A1
		(3 marks)

Notes

M1 $x^n \to x^{n+1}$ so $x^3 \to x^4$ or $4 \to 4x$ or $4x^1$

A1 This is for either term with coefficient unsimplified (power must be simplified) – so $\frac{8}{4}x^4$ or $4x$ (accept $4x^1$)

A1 Fully correct simplified solution with c i.e. $2x^4 + 4x + c$ [allow $2x^4 + 4x + cx^0$]

If the answer is given as $\int 2x^4 + 4x + c$, with an integral sign – having never been seen as the fully correct simplified answer without an integral sign − then give M1A1A0 but allow anything before the = sign e.g. $y = 2x^4 + 4x + c$, $f(x) = 2x^4 + 4x + c$, $\int = 2x^4 + 4x + c$, etc....

If this answer is followed by (for example) $x^4 + 2x + k$ then treat this as **isw** (ignore subsequent work) If they follow it by finding a value for c, also **isw**, provided correct answer with c has been seen and credited

Q2.

Question Number	Scheme	Notes	Marks
	$\int (2x^4 - \frac{4}{\sqrt{x}} + 3)\,dx$		
	$\frac{2}{5}x^5 - \frac{4}{\frac{1}{2}}x^{\frac{1}{2}} + 3x$	M1: $x^n \to x^{n+1}$. One power increased by 1 but not for just + c. This could be for 3 → 3x or for $x^n \to x^{n+1}$ on what they think $\frac{1}{\sqrt{x}}$ is as a power of x.	M1A1A1
		A1: One of these 3 terms correct. Allow un-simplified e.g. $\frac{2x^{4+1}}{4+1}$, $-\frac{4x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$, $3x^1$	
		A1: Two of these 3 terms correct. Allow un-simplified e.g. $\frac{2x^{4+1}}{4+1}$, $-\frac{4x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$, $3x^1$	
	$= \frac{2}{5}x^5 - 8x^{\frac{1}{2}} + 3x + c$	<u>**Complete fully correct simplified expression appearing all on one line with constant.**</u> Allow 0.4 for $\frac{2}{5}$. Do not allow $3x^1$ for $3x$. Allow $\sqrt{x}$ or $x^{0.5}$ for $x^{\frac{1}{2}}$	A1
	Ignore any spurious integral signs and ignore subsequent working following a fully correct answer.		
			[4]
			4 marks

Q3.

Question Number	Scheme		Marks
	$\int\left(2x^5-\frac{1}{4}x^{-3}-5\right)\mathrm{d}x$		
	Ignore any spurious integral signs throughout		
	$x^n \to x^{n+1}$	Raises any of their powers by 1. E.g. $x^5 \to x^6$ or $x^{-3} \to x^{-2}$ or $k \to kx$ or $x^{\text{their } n} \to x^{\text{their } n+1}$. Allow the powers to be un-simplified e.g. $x^5 \to x^{5+1}$ or $x^{-3} \to x^{-3+1}$ or $kx^0 \to kx^{0+1}$.	M1
	$2\times\frac{x^{5+1}}{6}$ or $-\frac{1}{4}\times\frac{x^{-3+1}}{-2}$	Any one of the first two terms correct <u>**simplified or un-simplified**</u>.	A1
	Two of: $\frac{1}{3}x^6,\ \frac{1}{8}x^{-2},\ -5x$	Any two correct <u>**simplified**</u> terms. Accept $+\frac{1}{8x^2}$ for $+\frac{1}{8}x^{-2}$ but not x^1 for x. Accept 0.125 for $\frac{1}{8}$ but $\frac{1}{3}$ would clearly need to be identified as 0.3 recurring.	A1
	$\frac{1}{3}x^6+\frac{1}{8}x^{-2}-5x+c$	All correct and simplified and including + c all on one line. Accept $+\frac{1}{8x^2}$ for $+\frac{1}{8}x^{-2}$ but not x^1 for x. Apply isw here.	A1
			(4 marks)

Q4.

Question Number	Scheme	Notes	Marks
	$\int 3x^2-\frac{4}{x^2}\mathrm{d}x=3\frac{x^3}{3}-4\frac{x^{-1}}{-1}$	M1: $x^n \to x^{n+1}$ for either term. If they write $\frac{4}{x^2}$ *as* $4x^2$ allow $x^2 \to x^3$ here.	M1,A1,A1
		A1: $3\frac{x^3}{3}$ **or** $-4\frac{x^{-1}}{-1}$ (one correct term which may be un-simplified)	
		A1: $3\frac{x^3}{3}$ **and** $-4\frac{x^{-1}}{-1}$ (both terms correct which may be un-simplified)	
	Note that M1A0A1 is not possible		
	$=x^3+\frac{4}{x}+c$ or $x^3+4x^{-1}+c$	Fully correct simplified answer with + c all appearing on the same line.	A1
			[4]

Q5.

Question Number	Scheme	Marks
(a)	$y = 2x^5 + \frac{6}{\sqrt{x}}$	
	$x^n \to x^{n-1}$	M1
	$\frac{dy}{dx} = 10x^4 - 3x^{-\frac{3}{2}}$ oe	A1A1
		(3)
(b)	$\int 2x^5 + \frac{6}{\sqrt{x}} dx$	
	$x^n \to x^{n+1}$	M1
	$= \frac{x^6}{3} + 12x^{\frac{1}{2}} + c$	A1 A1
		(3) (6 marks)

(a) M1 For $x^n \to x^{n-1}$. ie. x^4 or $x^{-\frac{3}{2}}$ or $\left(\frac{1}{x^{\frac{3}{2}}}\right)$ seen

A1 For $2 \times 5x^4$ **or** $6 \times -\frac{1}{2}x^{-\frac{3}{2}}$ (oe). (Ignore $+c$ for this mark)

A1 For simplified expression $10x^4 - 3x^{-\frac{3}{2}}$ or $10x^4 - \frac{3}{x^{\frac{3}{2}}}$ o.e. and no $+c$

Apply ISW here and award marks when first seen.

(b) M1 For $x^n \to x^{n+1}$. ie. x^6 or $x^{\frac{1}{2}}$ or $\left(\sqrt{x}\right)$ seen

Do not award for integrating their answer to part (a)

A1 For either $2\frac{x^6}{6}$ **or** $6 \times \frac{x^{\frac{1}{2}}}{\frac{1}{2}}$ or simplified or unsimplified equivalents

A1 For fully correct and simplified answer with $+c$.

Q6.

Question Number	Scheme	Marks
(a)	$\frac{dy}{dx} = 10x^4 - 3x^{-4}$ or $10x^4 - \frac{3}{x^4}$	M1 A1 A1 (3)
(b)	$\left(\int =\right) \frac{2x^6}{6} + 7x + \frac{x^{-2}}{-2} = \frac{x^6}{3} + 7x - \frac{x^{-2}}{2}$ $+ C$	M1 A1 A1 B1 (4) 7
	Notes (a) M1: Attempt to differentiate $x^n \to x^{n-1}$ (for any of the 3 terms) i.e. ax^4 or ax^{-4}, where a is any non-zero constant or the 7 differentiated to give 0 is sufficient evidence for M1 1st A1: One correct (non-zero) term, possibly unsimplified. 2nd A1: Fully correct **simplified** answer. (b) M1: Attempt to integrate $x^n \to x^{n+1}$ (i.e. ax^6 or ax or ax^{-2}, where a is any non-zero constant). 1st A1: Two correct terms, possibly unsimplified. 2nd A1: All three terms correct and **simplified**. Allow correct equivalents to printed answer, e.g. $\frac{x^6}{3} + 7x - \frac{1}{2x^2}$ or $\frac{1}{3}x^6 + 7x - \frac{1}{2}x^{-2}$ Allow $\frac{1x^6}{3}$ or $7x^1$ B1: $+ C$ appearing at any stage in part (b) (independent of previous work)	

Q7.

Question number	Scheme	Marks
	(a) $\left(\frac{dy}{dx}\right) = 6x^1 + \frac{4}{2}x^{-\frac{1}{2}}$ or $\left(6x + 2x^{-\frac{1}{2}}\right)$	M1 A1 (2)
	(b) $6 + -x^{-\frac{3}{2}}$ or $6 + -1 \times x^{-\frac{3}{2}}$	M1 A1ft (2)
	(c) $x^3 + \frac{8}{3}x^{\frac{3}{2}} + C$ A1: $\frac{3}{3}x^3$ or $\frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}$ A1: both, simplified and $+C$	M1 A1 A1 (3)
		7
(a)	M1 for <u>some</u> attempt to differentiate: $x^n \to x^{n-1}$ Condone missing $\frac{dy}{dx}$ or $y = \ldots$ A1 for both terms correct, as written or better. No $+C$ here. Of course $\frac{2}{\sqrt{x}}$ is acceptable.	
(b)	M1 for some attempt to differentiate again. Follow through their $\frac{dy}{dx}$, at least one term correct or correct follow through. A1f.t. as written or better, follow through must have 2 <u>distinct</u> terms and simplified e.g. $\frac{4}{4} = 1$.	
(c)	M1 for some attempt to integrate: $x^n \to x^{n+1}$. Condone misreading $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ for y. ($+C$ alone is not sufficient) 1st A1 for either $\frac{3}{3}x^3$ or $\frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}$ (or better) $\frac{2}{3} \times 4x^{\frac{3}{2}}$ is OK here too but not for 2nd A1. 2nd A1 for <u>both</u> x^3 and $\frac{8}{3}x^{\frac{3}{2}}$ or $\frac{8}{3}x\sqrt{x}$ i.e. simplified terms <u>and</u> $+C$ all on one line. $2\frac{2}{3}$ instead of $\frac{8}{3}$ is OK	

Q8.

Question number	Scheme	Marks
	$\int x^{-\frac{1}{2}}\,dx = \frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}$ (Or equivalent, such as $2x^{\frac{1}{2}}$, or $2\sqrt{x}$) $\left[\frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right]_1^8 = 2\sqrt{8}-2=-2+4\sqrt{2}$ [or $4\sqrt{2}-2$, or $2(2\sqrt{2}-1)$, or $2(-1+2\sqrt{2})$]	M1 A1 M1 A1 (4) **4**
	1st M: $x^{-\frac{1}{2}} \rightarrow kx^{\frac{1}{2}},\ k \neq 0$. 2nd M: Substituting limits 8 and 1 into a 'changed' function (i.e. not $\frac{1}{\sqrt{x}}$ or $x^{-\frac{1}{2}}$), and subtracting, either way round. 2nd A: This final mark is still scored if $-2+4\sqrt{2}$ is reached via a decimal. N.B. Integration constant $+C$ may appear, e.g. $\left[\frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}+C\right]_1^8 = (2\sqrt{8}+C)-(2+C)=-2+4\sqrt{2}$ (Still full marks) <u>But</u>… a final answer such as $-2+4\sqrt{2}+C$ is A0. N.B. It will sometimes be necessary to 'ignore subsequent working' (isw) after a correct form is seen, e.g. $\int x^{-\frac{1}{2}}\,dx = \frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}$ (M1 A1), followed by incorrect simplification $\int x^{-\frac{1}{2}}\,dx = \frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = \frac{1}{2}x^{\frac{1}{2}}$ (still M1 A1)…. The second M mark is still available for substituting 8 and 1 into $\frac{1}{2}x^{\frac{1}{2}}$ and subtracting.	

Q9.

Question Number	Scheme		Marks
	$\left\{\int\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)dx\right\} = \frac{x^4}{6(4)}+\frac{x^{-1}}{(3)(-1)}$	M1: $x^n \to x^{n+1}$ A1: At least one of either $\frac{x^4}{6(4)}$ or $\frac{x^{-1}}{(3)(-1)}$. A1: $\frac{x^4}{6(4)}+\frac{x^{-1}}{(3)(-1)}$ or equivalent. e.g. $\frac{\frac{x^4}{6}}{4}+\frac{\frac{x^{-1}}{3}}{-1}$ (they will lose the final mark if they cannot deal with this correctly)	M1A1A1
	Note that some candidates may change the function prior to integrating e.g. $\int \frac{x^3}{6}+\frac{1}{3x^2}dx = \int 3x^5+6dx$ in which case allow the M1 if $x^n \to x^{n+1}$ for their changed function and allow the M1 for limits if scored		
	$\left\{\int_1^{\sqrt{3}}\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)dx\right\} = \left(\frac{(\sqrt{3})^4}{24}+\frac{(\sqrt{3})^{-1}}{-1(3)}\right)-\left(\frac{(1)^4}{24}+\frac{(1)^{-1}}{-1(3)}\right)$		dM1
	2[nd] dM1: For using limits of $\sqrt{3}$ and 1 on an integrated expression and subtracting the correct way round. **The 2[nd] M1 is dependent on the 1[st] M1 being awarded.**		
	$=\left(\frac{9}{24}-\frac{1}{3\sqrt{3}}\right)-\left(\frac{1}{24}-\frac{1}{3}\right)=\frac{2}{3}-\frac{1}{9}\sqrt{3}$	$\frac{2}{3}-\frac{1}{9}\sqrt{3}$ or $a=\frac{2}{3}$ and $b=-\frac{1}{9}$. Allow equivalent fractions for a and/or b and 0.6 recurring and/or 0.1 recurring but do **not** allow $\frac{6-\sqrt{3}}{9}$	A1cso
	This final mark is cao and cso – there must have been no previous errors		
			Total 5
	Common Errors (Usually 3 out of 5)		
	$\left\{\int\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)dx\right\} = \int\left(\frac{x^3}{6}+3x^{-2}\right)dx = \frac{x^4}{6(4)}+\frac{3x^{-1}}{(-1)}$ M1A1A0 $\left\{\int_1^{\sqrt{3}}\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)dx\right\} = \left(\frac{(\sqrt{3})^4}{24}+\frac{3(\sqrt{3})^{-1}}{-1}\right)-\left(\frac{(1)^4}{24}+\frac{3(1)^{-1}}{-1}\right)$ dM1 $=\left(\frac{9}{24}-\frac{3}{\sqrt{3}}\right)-\left(\frac{1}{24}+\frac{3}{-1}\right)=\frac{10}{3}-\sqrt{3}$ A0		
	$\left\{\int\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)dx\right\} = \int\left(\frac{x^3}{6}+(3x)^{-2}\right)dx = \frac{x^4}{6(4)}+\frac{(3x)^{-1}}{(-1)}$ M1A1A0 $\left\{\int_1^{\sqrt{3}}\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)dx\right\} = \left(\frac{(\sqrt{3})^4}{24}+\frac{(3\sqrt{3})^{-1}}{-1}\right)-\left(\frac{(1)^4}{24}+\frac{(3\times 1)^{-1}}{-1}\right)$ dM1 $=\left(\frac{9}{24}-\frac{1}{3\sqrt{3}}\right)-\left(\frac{1}{24}-\frac{1}{3}\right)=\frac{2}{3}-\frac{\sqrt{3}}{9}$ A0 **Note this is the correct answer but follows incorrect work.**		

Q10.

Question	Scheme	Marks	AOs
(a)	$x^n \to x^{n+1}$	M1	1.1b
	$\int \left(\frac{5}{2\sqrt{x}}+3\right)\mathrm{d}x = 5\sqrt{x}+3x$	A1	1.1b
	$\left[5\sqrt{x}+3x\right]_1^k = 4 \Rightarrow 5\sqrt{k}+3k-8=4$	dM1	1.1b
	$3k+5\sqrt{k}-12=0$ *	A1*	2.1
		(4)	
(b)	$3k+5\sqrt{k}-12=0 \Rightarrow \left(3\sqrt{k}-4\right)\left(\sqrt{k}+3\right)=0$	M1	3.1a
	$\sqrt{k}=\frac{4}{3},(-3)$	A1	1.1b
	$\sqrt{k}=\ldots \Rightarrow k=\ldots$ oe	dM1	1.1b
	$k=\frac{16}{9}$, ~~9~~	A1	2.3
		(4)	
			(8 marks)

Notes

(a)

M1: For $x^n \to x^{n+1}$ on correct indices. This can be implied by the sight of either $x^{\frac{1}{2}}$ or x

A1: $5\sqrt{x}+3x$ or $5x^{\frac{1}{2}}+3x$ but may be unsimplified. Also allow with $+c$ and condone any spurious notation.

dM1: Uses both limits, subtracts, and sets equal to 4. They cannot proceed to the given answer without a line of working showing this.

A1*: Fully correct proof with no errors (bracketing or otherwise) leading to given answer.

(b)

M1: For a correct method of solving. This could be as the scheme, treating as a quadratic in $\sqrt{k}$ and using allowable method to solve including factorisation, formula etc.

Allow values for $\sqrt{k}$ to be just written down, e.g. allow $\sqrt{k}=\pm\frac{4}{3},(\pm 3)$

Alternatively score for rearranging to $5\sqrt{k}=12-3k$ and then squaring to get $\ldots k=(12-3k)^2$

A1: $\sqrt{k}=\frac{4}{3},(-3)$

Or in the alt method it is for reaching a correct 3TQ equation $9k^2-97k+144=0$

dM1: For solving to find at least one value for k. It is dependent upon the first M mark.
In the main method it is scored for squaring their value(s) of $\sqrt{k}$
In the alternative scored for solving their 3TQ by an appropriate method

A1: Full and rigorous method leading to $k=\frac{16}{9}$ only. The 9 must be rejected.

Q11.

Question	Scheme	Marks	AOs
	$f(x) = 2x + 3 + 12x^{-2}$	B1	1.1b
	Attempts to integrate	M1	1.1a
	$\int \left(+2x + 3 + \frac{12}{x^2} \right) dx = x^2 + 3x - \frac{12}{x}$	A1	1.1b
	$\left((2\sqrt{2})^2 + 3(2\sqrt{2}) - \frac{12(\sqrt{2})}{2 \times 2} \right) - (-8)$	M1	1.1b
	$= 16 + 3\sqrt{2}$ *	A1*	1.1b
			(5 marks)

Notes

B1: Correct function with numerical powers

M1: Allow for raising power by one. $x^n \to x^{n+1}$

A1: Correct three terms

M1: Substitutes limits and rationalises denominator

A1*: Completely correct, no errors seen.

Q12.

Question	Scheme	Marks	AOs
(a)	$x^n \to x^{n+1}$	M1	1.1b
	$\int \left(\frac{4}{x^3} + kx \right) dx = -\frac{2}{x^2} + \frac{1}{2} kx^2 + c$	A1 A1	1.1b 1.1b
		(3)	
(b)	$\left[-\frac{2}{x^2} + \frac{1}{2} kx^2 \right]_{0.5}^{2} = \left(-\frac{2}{2^2} + \frac{1}{2} k \times 4 \right) - \left(-\frac{2}{(0.5)^2} + \frac{1}{2} k \times (0.5)^2 \right) = 8$	M1	1.1b
	$7.5 + \frac{15}{8} k = 8 \Rightarrow k = \ldots$	dM1	1.1b
	$k = \frac{4}{15}$ oe	A1	1.1b
		(3)	
			(6 marks)

Q13.

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\,\delta x = \int_4^9 \sqrt{x}\ \mathrm{d}x$	B1	This mark is given for writing the expression for a sum as an integral
	$\left[\frac{2}{3}x^{\frac{3}{2}}\right]_4^9 = \frac{2}{3} \times 9^{\frac{3}{2}} - \frac{2}{3} \times 4^{\frac{3}{2}}$	M1	This mark is given for a method to evaluate the integral
	$= \frac{38}{3}$	A1	This mark is given for a correct evaluation of the integral
			(Total 3 marks)