

Question 1

Worked Solution

Part (i) — Express $3x^2 + 12x + 7$ in the form $3(x + a)^2 + b$:

Factor out 3 from the first two terms:

$$3x^2 + 12x + 7 = 3(x^2 + 4x) + 7$$

Complete the square inside the bracket:

$$= 3[(x + 2)^2 - 4] + 7 = 3(x + 2)^2 - 12 + 7$$

$$3x^2 + 12x + 7 = 3(x + 2)^2 - 5 \quad (a = 2, b = -5)$$

Part (ii) — Line of symmetry of $y = 3x^2 + 12x + 7$:

The minimum occurs at $x = -2$, so the line of symmetry is:

$$x = -2$$

Question 2

Worked Solution

Part (i) — Express $2x^2 + 12x + 13$ in the form $a(x + b)^2 + c$:

Factor out 2 from the first two terms:

$$2x^2 + 12x + 13 = 2(x^2 + 6x) + 13$$

Complete the square:

$$= 2[(x + 3)^2 - 9] + 13 = 2(x + 3)^2 - 18 + 13$$

$$2x^2 + 12x + 13 = 2(x + 3)^2 - 5 \quad (a = 2, b = 3, c = -5)$$

Part (ii) — Solve $2x^2 + 12x + 13 = 0$ in simplified surd form:

Using the completed square form:

$$2(x + 3)^2 - 5 = 0 \implies (x + 3)^2 = \frac{5}{2}$$

$$x + 3 = \pm \sqrt{\frac{5}{2}}$$

$$x = -3 \pm \sqrt{\frac{5}{2}}$$

$$x = -3 \pm \sqrt{\frac{5}{2}}$$

Question 3

Worked Solution

Part (i) — Express $2x^2 - 24x + 80$ in the form $a(x - b)^2 + c$:

Factor out 2:

$$2x^2 - 24x + 80 = 2(x^2 - 12x) + 80$$

Complete the square:

$$= 2[(x - 6)^2 - 36] + 80 = 2(x - 6)^2 - 72 + 80$$

$$2x^2 - 24x + 80 = 2(x - 6)^2 + 8 \quad (a = 2, b = 6, c = 8)$$

Part (ii) — Line of symmetry of $y = 2x^2 - 24x + 80$:

$$x = 6$$

Part (iii) — Tangent at the minimum point:

The minimum occurs at $(6, 8)$. At a minimum the tangent is horizontal, so:

$$y = 8$$

Question 4

Worked Solution**Part (i) — Express $2x^2 - 6x + 11$ in the form $p(x + q)^2 + r$:**

Factor out 2:

$$2x^2 - 6x + 11 = 2(x^2 - 3x) + 11$$

Complete the square:

$$= 2 \left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} \right] + 11 = 2 \left(x - \frac{3}{2}\right)^2 - \frac{9}{2} + 11$$

$$= 2 \left(x - \frac{3}{2}\right)^2 + \frac{13}{2}$$

$$2x^2 - 6x + 11 = 2\left(x - \frac{3}{2}\right)^2 + \frac{13}{2} \quad \left(p = 2, q = -\frac{3}{2}, r = \frac{13}{2}\right)$$

Part (ii) — Coordinates of the vertex of $y = 2x^2 - 6x + 11$:The minimum is at $x = \frac{3}{2}$, $y = \frac{13}{2}$.

$$\text{Vertex} = \left(\frac{3}{2}, \frac{13}{2}\right)$$

Question 5

Worked Solution

Part (i) — Express $5x^2 + 20x - 8$ in the form $p(x + q)^2 + r$:

Factor out 5:

$$5x^2 + 20x - 8 = 5(x^2 + 4x) - 8$$

Complete the square:

$$= 5[(x + 2)^2 - 4] - 8 = 5(x + 2)^2 - 20 - 8$$

$$5x^2 + 20x - 8 = 5(x + 2)^2 - 28 \quad (p = 5, q = 2, r = -28)$$

Part (ii) — Line of symmetry of $y = 5x^2 + 20x - 8$:

The minimum occurs at $x = -2$.

$$x = -2$$

Question 6

Worked Solution

Part (i) — Express $2x^2 - 20x + 49$ in the form $p(x - q)^2 + r$:

Factor out 2:

$$2x^2 - 20x + 49 = 2(x^2 - 10x) + 49$$

Complete the square:

$$= 2[(x - 5)^2 - 25] + 49 = 2(x - 5)^2 - 50 + 49$$

$$2x^2 - 20x + 49 = 2(x - 5)^2 - 1 \quad (p = 2, q = 5, r = -1)$$

Part (ii) — Coordinates of the vertex of $y = 2x^2 - 20x + 49$:

The minimum is at $x = 5, y = -1$.

$$\text{Vertex} = (5, -1)$$

End of Worked Solutions — ALevelMathsRevision.com
