Completing the Square Exam Questions Sheet 2

Q1.

$$f(x) = x^2 - 8x + 19$$

(a) Express $f(x)$ in the form $(x + a)^2 + b$, where a and b are constants.

(2)

The curve C with equation $y = f(x)$ crosses the y-axis at the point P and has a minimum point at the point Q.

(b) Sketch the graph of C showing the coordinates of point P and the coordinates of point Q.

(3)

(c) Find the distance PQ, writing your answer as a simplified surd.

(3)

(Total for question = 8 marks)

Q2.

$$f(x) = x^2 - 8x + 19$$

(a) Express $f(x)$ in the form $(x + a)^2 + b$, where a and b are constants.

(2)

The curve C with equation $y = f(x)$ crosses the y-axis at the point P and has a minimum point at the point Q.

(b) Sketch the graph of C showing the coordinates of point P and the coordinates of point Q.

(3)

(c) Find the distance PQ, writing your answer as a simplified surd.

(3)

(Total for question = 8 marks)

Q3.

$$4x - 5 - x^2 = q - (x + p)^2$$

where p and q are integers.

(a) Find the value of p and the value of q.

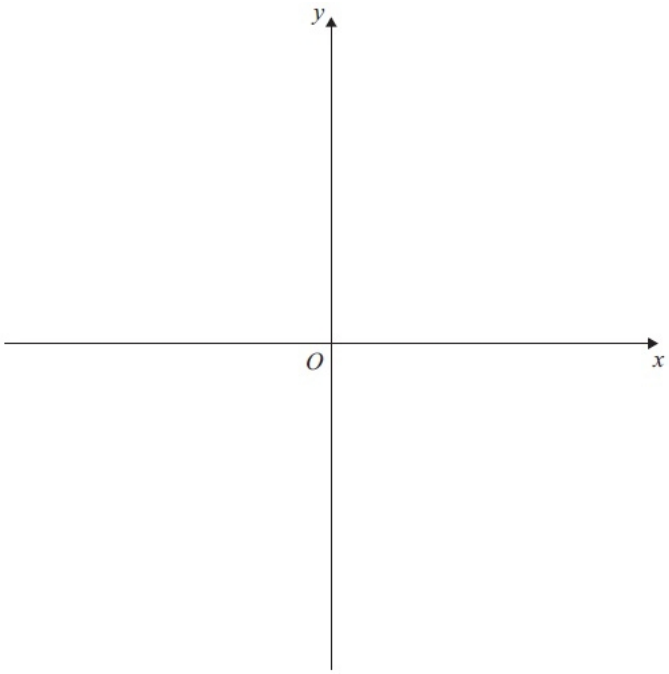
(3)

(b) Calculate the discriminant of $4x - 5 - x^2$

(2)

(c) On the axes on the next page, sketch the curve with equation $y = 4x - 5 - x^2$ showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(3)



(Total 8 marks)

Q4.

$f(x) = x^2 + (k+3)x + k$

where k is a real constant.

(a) Find the discriminant of f(x) in terms of k.

(2)

(b) Show that the discriminant of f(x) can be expressed in the form $(k + a)^2 + b$, where a and b are integers to be found.

(2)

(c) Show that, for all values of k, the equation f(x) = 0 has real roots.

(2)

(Total 6 marks)

Q5.

(a) Show that $x^2 + 6x + 11$ can be written as

$(x + p)^2 + q$

where p and q are integers to be found.

(2)

(b) In the space at the top of page 7, sketch the curve with equation $y = x^2 + 6x + 11$, showing clearly any intersections with the coordinate axes.

(2)

(c) Find the value of the discriminant of $x^2 + 6x + 11$

(2)

(Total 6 marks)

Q6.

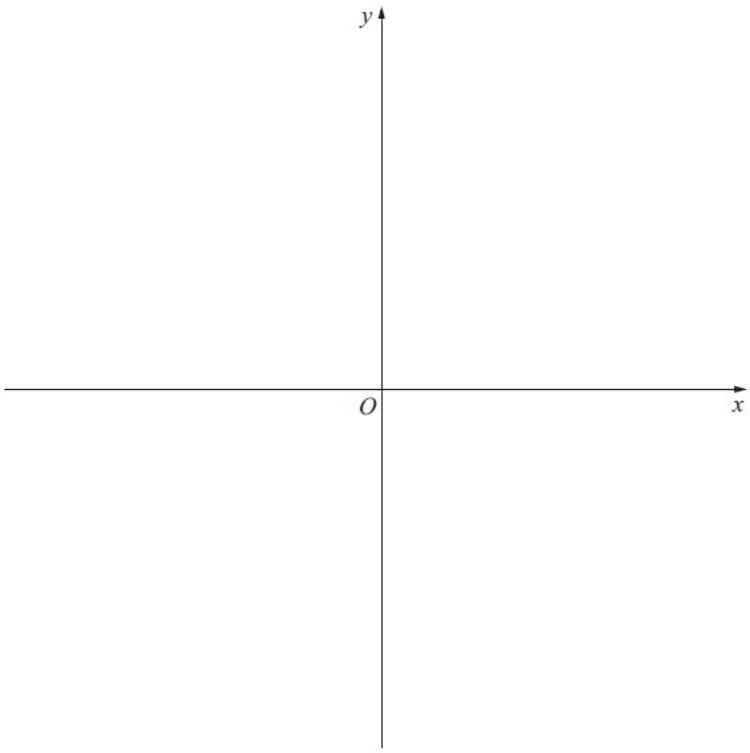
$$4x^2 + 8x + 3 = a(x + b)^2 + c$$

(a) Find the values of the constants a, b and c.

(3)

(b) On the axes below, sketch the curve with equation $y = 4x^2 + 8x + 3$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(4)



(Total 7 marks)

Q7.

Given that

$$f(x) = x^2 - 4x + 5 \qquad x \in \mathbb{R}$$

(a) express $f(x)$ in the form $(x + a)^2 + b$ where a and b are integers to be found.

(2)

The curve with equation $y = f(x)$

- meets the y-axis at the point P
- has a minimum turning point at the point Q

(b) Write down

(i) the coordinates of P
(ii) the coordinates of Q

(2)

(Total for question = 4 marks)