Question 1

Worked Solution

$f(x) = x^2 - 8x + 19$

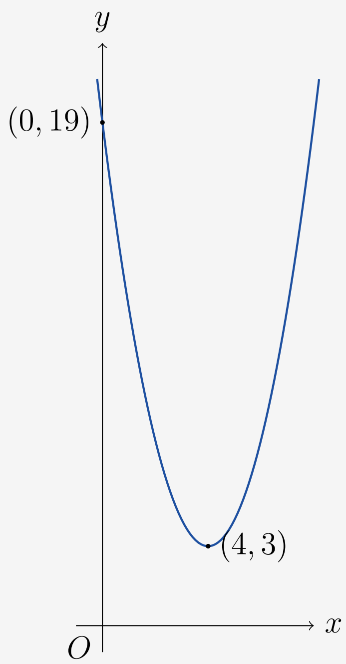
Part (a) — Express in the form $(x + a)^2 + b$:

$$x^2 - 8x + 19 = (x - 4)^2 - 16 + 19 = (x - 4)^2 + 3$$

$$f(x) = (x - 4)^2 + 3 \quad (a = -4,\ b = 3)$$

Part (b) — Sketch of C:

- y-intercept P: set $x = 0$: $f(0) = 19$, so $P = (0, 19)$
- Minimum point Q: from completed square, minimum at $(4, 3)$, so $Q = (4, 3)$
- U-shaped parabola with no x-intercepts (minimum value $3 > 0$)



Part (c) — Distance PQ:

$P = (0, 19)$ and $Q = (4, 3)$. Using Pythagoras:

$$PQ^2 = (4 - 0)^2 + (3 - 19)^2 = 16 + 256 = 272$$

$$PQ = \sqrt{272} = \sqrt{16 \times 17} = 4\sqrt{17}$$

$$PQ = 4\sqrt{17}$$

Question 2

Worked Solution

Note: Question 2 has the same function $f(x) = x^2 - 8x + 19$ and the same parts as Question 1. The worked solution is identical.

Part (a):

$$f(x) = (x-4)^2 + 3 \quad (a = -4,\ b = 3)$$

Part (b): U-shaped parabola with $P = (0, 19)$ and $Q = (4, 3)$ as labelled coordinates.

Part (c):

$$PQ = 4\sqrt{17}$$

Question 3

Worked Solution

$4x - 5 - x^2 = q - (x + p)^2$

Part (a) — Find p and q:

Rewrite the left-hand side by completing the square. First rearrange:

$$4x - 5 - x^2 = -(x^2 - 4x) - 5$$

Complete the square:

$$= -\left[(x-2)^2 - 4\right] - 5 = -(x-2)^2 + 4 - 5 = -(x-2)^2 - 1$$

Matching to $q - (x + p)^2$:

$$q - (x + p)^2 = -1 - (x - 2)^2$$

$$p = -2, \quad q = -1$$

Part (b) — Discriminant of $4x - 5 - x^2$:

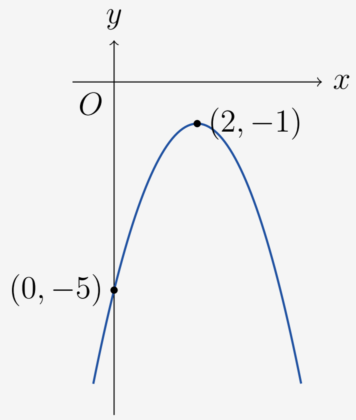
Write as $-x^2 + 4x - 5$, so $a = -1$, $b = 4$, $c = -5$:

$$\Delta = b^2 - 4ac = 4^2 - 4(-1)(-5) = 16 - 20 = -4$$

$$\Delta = -4$$

Part (c) — Sketch of $y = 4x - 5 - x^2$:

- Negative leading coefficient: $\cap$-shaped parabola
- Discriminant < 0: no real roots, curve does not cross the x-axis
- Maximum at $(2, -1)$ — this lies *below* the x-axis, so the entire curve is below the x-axis
- y-intercept: $x = 0 \Rightarrow y = -5$, so curve crosses y-axis at $(0, -5)$


y
x
O
(2, −1)
(0, −5)

Question 4

Worked Solution

$f(x) = x^2 + (k+3)x + k$, where k is a real constant.

Part (a) — Find the discriminant in terms of k:

With $a = 1$, $b = k+3$, $c = k$:

$$\Delta = b^2 - 4ac = (k+3)^2 - 4k$$

$$\Delta = (k+3)^2 - 4k = k^2 + 2k + 9$$

Part (b) — Show Δ can be written as $(k+a)^2 + b$:

Expand and complete the square:

$$(k+3)^2 - 4k = k^2 + 6k + 9 - 4k = k^2 + 2k + 9$$

$$= (k+1)^2 - 1 + 9 = (k+1)^2 + 8$$

$$\Delta = (k+1)^2 + 8 \quad (a = 1,\ b = 8)$$

Part (c) — Show $f(x) = 0$ has real roots for all k:

For real roots we need $\Delta \geq 0$.

Since $(k+1)^2 \geq 0$ for all real k:

$$\Delta = (k+1)^2 + 8 \geq 0 + 8 = 8 > 0$$

So $\Delta > 0$ for all values of k, which means $f(x) = 0$ always has two distinct real roots. $\square$

Question 5

Worked Solution

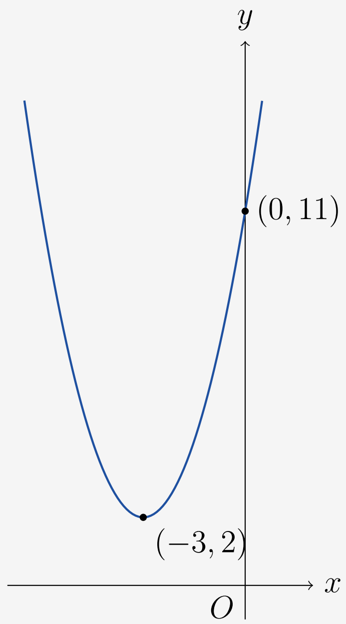
Part (a) — Show $x^2 + 6x + 11 = (x + p)^2 + q$:

$$x^2 + 6x + 11 = (x + 3)^2 - 9 + 11 = (x + 3)^2 + 2$$

$$x^2 + 6x + 11 = (x + 3)^2 + 2 \quad (p = 3,\ q = 2)\square$$

Part (b) — Sketch of $y = x^2 + 6x + 11$:

- U-shaped parabola
- Minimum at $(-3, 2)$, which is above the x-axis — no x-intercepts
- y-intercept: $x = 0 \Rightarrow y = 11$, so crosses y-axis at $(0, 11)$



Part (c) — Discriminant of $x^2 + 6x + 11$:

$a = 1$, $b = 6$, $c = 11$:

$$\Delta = 6^2 - 4(1)(11) = 36 - 44 = -8$$

$$\Delta = -8$$

Question 6

Worked Solution

$4x^2 + 8x + 3 = a(x+b)^2 + c$

Part (a) — Find a, b and c:

Factor out 4 from the first two terms:

$$4x^2 + 8x + 3 = 4(x^2 + 2x) + 3$$

Complete the square:

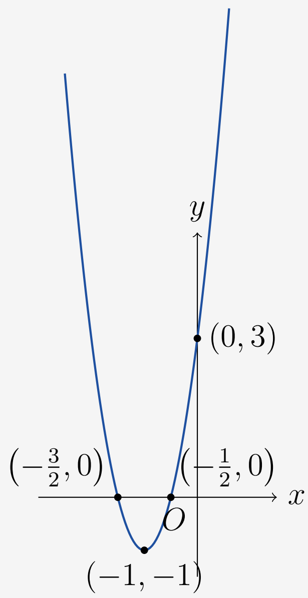
$$= 4\left[(x+1)^2 - 1\right] + 3 = 4(x+1)^2 - 4 + 3$$

$$4x^2 + 8x + 3 = 4(x+1)^2 - 1 \quad (a = 4,\ b = 1,\ c = -1)$$

Part (b) — Sketch of $y = 4x^2 + 8x + 3$:

- Minimum at $(-1, -1)$ — in the third quadrant
- y-intercept: $x = 0 \Rightarrow y = 3$, so $(0, 3)$
- x-intercepts: solve $4x^2 + 8x + 3 = 0 \Rightarrow (2x+1)(2x+3) = 0$:

$$x = -\tfrac{1}{2} \quad \text{and} \quad x = -\tfrac{3}{2}$$



Question 7

Worked Solution

$f(x) = x^2 - 4x + 5$

Part (a) — Express in the form $(x+a)^2 + b$:

$$x^2 - 4x + 5 = (x-2)^2 - 4 + 5 = (x-2)^2 + 1$$

$$f(x) = (x-2)^2 + 1 \quad (a = -2,\ b = 1)$$

Part (b) — Coordinates of P and Q:

(i) P is where the curve meets the y-axis: set $x = 0$:

$$f(0) = 0 - 0 + 5 = 5$$

$$P = (0,\ 5)$$

(ii) Q is the minimum turning point. From the completed square form, minimum is at $x = 2$, $y = 1$:

$$Q = (2,\ 1)$$

End of Worked Solutions — ALevelMathsRevision.com