Question 1

Worked Solution

Expand $\left(3 - \frac{1}{3}x\right)^5$, first 4 terms in ascending powers of x.

Use the binomial theorem with $a = 3$, $b = -\frac{1}{3}x$, $n = 5$:

$$(a+b)^5 = a^5 + \binom{5}{1}a^4b + \binom{5}{2}a^3b^2 + \binom{5}{3}a^2b^3 + \cdots$$

Constant: $3^5 = 243$

x **term:** $5 \cdot 3^4 \cdot \left(-\frac{1}{3}x\right) = 5 \cdot 81 \cdot \left(-\frac{1}{3}\right)x = -135x$

x^2 **term:** $10 \cdot 3^3 \cdot \left(-\frac{1}{3}x\right)^2 = 10 \cdot 27 \cdot \frac{x^2}{9} = 30x^2$

x^3 **term:** $10 \cdot 3^2 \cdot \left(-\frac{1}{3}x\right)^3 = 10 \cdot 9 \cdot \left(-\frac{x^3}{27}\right) = -\frac{10}{3}x^3$

$$\left(3 - \frac{1}{3}x\right)^5 = 243 - 135x + 30x^2 - \frac{10}{3}x^3 + \cdots$$

Question 2

Worked Solution

Expand $\left(2 - \frac{x}{4}\right)^{10}$, first 3 terms in ascending powers of x.

Use $a = 2$, $b = -\frac{x}{4}$, $n = 10$:

Constant: $2^{10} = 1024$

x **term:** $\binom{10}{1} \cdot 2^9 \cdot \left(-\frac{x}{4}\right) = 10 \cdot 512 \cdot \left(-\frac{1}{4}\right) x = -1280x$

x^2 **term:** $\binom{10}{2} \cdot 2^8 \cdot \left(-\frac{x}{4}\right)^2 = 45 \cdot 256 \cdot \frac{x^2}{16} = 720x^2$

$$\left(2 - \frac{x}{4}\right)^{10} = 1024 - 1280x + 720x^2 + \cdots$$

Question 3

Worked Solution

Expand $\left(1+\frac{3x}{2}\right)^8$, first 4 terms in ascending powers of x.

Use $a=1$, $b=\frac{3x}{2}$, $n=8$:

Constant and x: $1+8\cdot\frac{3x}{2}=1+12x$

x^2 term: $\binom{8}{2}\left(\frac{3x}{2}\right)^2=28\cdot\frac{9x^2}{4}=63x^2$

x^3 term: $\binom{8}{3}\left(\frac{3x}{2}\right)^3=56\cdot\frac{27x^3}{8}=189x^3$

$$\left(1+\frac{3x}{2}\right)^8=1+12x+63x^2+189x^3+\cdots$$

Question 4

Worked Solution

(a) Expand $(2-9x)^4$, first 3 terms.

Use $a=2$, $b=-9x$, $n=4$:

$$2^4 + \binom{4}{1}(2)^3(-9x) + \binom{4}{2}(2)^2(-9x)^2 + \cdots$$

$$= 16 + 4\cdot 8\cdot(-9)x + 6\cdot 4\cdot 81\,x^2 + \cdots$$

$(2-9x)^4 = 16 - 288x + 1944x^2 + \cdots$

(b) $\mathrm{f}(x) = (1+kx)(2-9x)^4$. The constant term equals the constant term from part (a):

$A = 16$

(c) Multiplying out up to the x term:

$$(1+kx)(16-288x+\cdots) \implies \text{coeff of } x = -288 + 16k$$

Set equal to -232:

$$-288 + 16k = -232 \implies 16k = 56$$

$k = \dfrac{7}{2}$

(d) Coeff of x^2:

$$B = 1944 + (-288)\cdot k = 1944 - 288 \times \tfrac{7}{2} = 1944 - 1008$$

$B = 936$

Question 5

Worked Solution

(a) Expand $(3 + bx)^5$, first 3 terms.

Use $a = 3$, $b_0 = bx$, $n = 5$:

$$3^5 + \binom{5}{1}3^4(bx) + \binom{5}{2}3^3(bx)^2 + \cdots = 243 + 405bx + 270b^2x^2 + \cdots$$

$(3 + bx)^5 = 243 + 405bx + 270b^2x^2 + \cdots$

(b) Coeff of x^2 equals twice coeff of x:

$$270b^2 = 2 \times 405b \implies 270b = 810 \quad (b \neq 0)$$

$b = 3$

Question 6

Worked Solution

(a) Expand $\left(1+\frac{x}{4}\right)^8$, first 4 terms.

$$= 1 + 8 \cdot \frac{x}{4} + \binom{8}{2} \cdot \frac{x^2}{16} + \binom{8}{3} \cdot \frac{x^3}{64} + \cdots = 1 + 2x + \frac{7}{4}x^2 + \frac{7}{8}x^3 + \cdots$$

$\left(1+\frac{x}{4}\right)^8 = 1 + 2x + \frac{7}{4}x^2 + \frac{7}{8}x^3 + \cdots$

(b) Set $\frac{x}{4} = 0.025$, so $x = 0.1$:

$$(1.025)^8 \approx 1 + 2(0.1) + \frac{7}{4}(0.01) + \frac{7}{8}(0.001) = 1 + 0.2 + 0.0175 + 0.000875$$

$(1.025)^8 \approx 1.2184$ (4 d.p.)

Question 7

Worked Solution

(a) First 4 terms of $(1+kx)^6$:

$$(1+kx)^6 = 1 + 6(kx) + 15(kx)^2 + 20(kx)^3 + \cdots$$

$(1+kx)^6 = 1 + 6kx + 15k^2x^2 + 20k^3x^3 + \cdots$

(b) Coefficients of x and x^2 equal:

$$6k = 15k^2 \implies 6 = 15k \quad (k \neq 0)$$

$k = \dfrac{2}{5}$

(c) Coefficient of x^3:

$$20k^3 = 20 \times \left(\tfrac{2}{5}\right)^3 = 20 \times \tfrac{8}{125} = \tfrac{160}{125}$$

Coefficient of $x^3 = \dfrac{32}{25}$

Question 8

Worked Solution

(a) First 4 terms of $\left(1+\frac{x}{2}\right)^{10}$:

$$= 1 + 10 \cdot \frac{x}{2} + 45 \cdot \frac{x^2}{4} + 120 \cdot \frac{x^3}{8} + \cdots$$

$\left(1+\frac{x}{2}\right)^{10} = 1 + 5x + \frac{45}{4}x^2 + 15x^3 + \cdots$

(b) Set $1 + \frac{x}{2} = 1.005$, so $x = 0.01$:

$(1.005)^{10} \approx 1 + 5(0.01) + \frac{45}{4}(0.0001) + 15(0.000001) = 1 + 0.05 + 0.001125 + 0.000015$

$(1.005)^{10} \approx 1.05114$ (5 d.p.)

Question 9

Worked Solution

(a) First 4 terms of $(1 + ax)^7$:

$$= 1 + 7(ax) + 21(ax)^2 + 35(ax)^3 + \cdots$$

$(1 + ax)^7 = 1 + 7ax + 21a^2x^2 + 35a^3x^3 + \cdots$

(b) Coeff of x^2 is 525:

$$21a^2 = 525 \implies a^2 = 25$$

$a = \pm 5$

End of Worked Solutions