Binomial Expansion Exam Questions Sheet 2 MS

Q1.

Question Number	Scheme	Marks
	$\left(3-\frac{1}{3}x\right)^5$ - $3^5+{}^5C_1 3^4(-\frac{1}{3}x)+{}^5C_2 3^3(-\frac{1}{3}x)^2+{}^5C_3 3^2(-\frac{1}{3}x)^3...$ First term of 243 $\left({}^5C_1\times...\times x\right)+\left({}^5C_2\times...\times x^2\right)+\left({}^5C_3\times...\times x^3\right)...$ $=(243..)\ -\frac{405}{3}x+\frac{270}{9}x^2-\frac{90}{27}x^3...$ $=(243..)-135x+30x^2-\frac{10}{3}x^3..$	 B1 M1 A1 A1 (4) **[4]**
Alternative method	$\left(3-\frac{1}{3}x\right)^5=3^5(1-\frac{x}{9})^5$ $3^5(1+{}^5C_1(-\frac{1}{9}x)+{}^5C_2(-\frac{1}{9}x)^2+{}^5C_3(-\frac{1}{9}x)^3\ \ ...)$ Scheme is applied exactly as before	
	Notes B1: The constant term should be 243 in their expansion M1: Two of the three binomial coefficients must be correct and must be with the correct power of x. Accept ${}^5C_1$ or $\binom{5}{1}$ or 5 **as a coefficient, and** ${}^5C_2$ or $\binom{5}{2}$ or 10 **as another and** ${}^5C_3$ or $\binom{5}{3}$ or 10 as **another……..** Pascal's triangle may be used to establish coefficients. NB: If they only include the first two of these terms then the M1 may be awarded. A1: Two of the final three terms correct – may be unsimplified i.e. two of $-135x+30x^2-\frac{10}{3}x^3$ correct, or two of $-\frac{405}{3}x+\frac{270}{9}x^2-\frac{90}{27}x^3$ (may be just two terms) A1: All three final terms correct and simplified. (Can be listed with commas or appear on separate lines. Accept in reverse order.) Accept correct alternatives to $-\frac{10}{3}$ e.g. $-3\frac{1}{3}$ or $-3.\dot{3}$ the recurring must be clear. 3.3 is not acceptable. Allow e.g. $+-135x$	
	e.g. The common error $3^5+{}^5C_1 3^4(-\frac{1}{3})x+{}^5C_2 3^3(-\frac{1}{3})x^2+{}^5C_3 3^2(-\frac{1}{3})x^3=(243)-135x-90x^2-30x^3$ would earn B1, M1, A0, A0, so 2/4 If extra terms are given then isw No negative signs in answer also earns B1, M1, A0, A0 If the series is divided through by 3 at the final stage after an error or omission resulting in all multiple of three coefficients then apply scheme to series before this division and ignore subsequent work (isw) **Special Case**: Only gives first three terms $=(243..)-135x+30x^2$ ... or $243-\frac{405}{3}x+\frac{270}{9}x^2$ Follow the scheme to give B1 M1 A1 A0 special case. (Do not treat as misread.) Answers such as $243\ +405-\frac{1}{3}x+270-\frac{1}{9}x^2+90-\frac{1}{27}x^3$.. gain no credit as the binomial coefficients are not linked to the x terms.	

Q2.

Question Number	Scheme		Marks
Way 1	$\left(2-\frac{x}{4}\right)^{10}$		
	$2^{10}+\binom{10}{1}2^9\left(-\frac{1}{4}x\right)+\binom{10}{2}2^8\left(-\frac{1}{4}x\right)^2+\ldots$	For **either** the x term **or** the x^2 term including a correct binomial coefficient with a correct power of x	M1
		First term of 1024	B1
	$=1024-1280x+720x^2$	**Either** $-1280x$ **or** $720x^2$ (Allow +-1280x here)	A1
		Both $-1280x$ and $720x^2$ (Do not allow +-1280x here)	A1 [4]
Way 2	$\left(2-\frac{x}{4}\right)^{10}=2^k\left(1-10\times\frac{x}{8}+\frac{10\times 9}{2}\left(-\frac{x}{8}\right)^2\right)$		M1
	$1024(1\pm\ldots\ldots.)$		
	$=1024-1280x+720x^2$		B1A1 A1 [4]

Notes

M1: For either the x term or the x^2 term having correct structure i.e. a correct binomial coefficient in any form with the correct power of x. Condone sign errors and condone missing brackets and allow alternative forms for binomial coefficients e.g. ${}^{10}C_1$ or $\binom{10}{1}$ or even $\left(\frac{10}{1}\right)$ or 10. The powers of 2 or of ¼ may be wrong or missing.

B1: Award this for 1024 when first seen as a distinct constant term (not $1024x^0$) and not 1 + 1024

A1: For one correct term in x with coefficient simplified. Either -1280x or $720x^2$ (allow +-1280x here)

Allow $720x^2$ to come from $\left(\frac{x}{4}\right)^2$ with no negative sign. So use of + sign throughout could give M1 B1 A1 A0

A1: For both correct simplified terms i.e. -1280x and $720x^2$ (**Do not** allow +-1280x here)

Allow terms to be listed for full marks e.g. 1024, $-1280x$, $+720x^2$

N.B. If they follow a correct answer by a factor such as $512-640x+360x^2$ then isw

Terms may be listed. Ignore any extra terms.

Notes for Way 2

M1: Correct structure for at least one of the underlined terms. i.e. a correct binomial coefficient in any form with the correct power of x. Condone sign errors and condone missing brackets and allow alternative forms for binomial coefficients e.g. ${}^{10}C_1$ or $\binom{10}{1}$ or even $\left(\frac{10}{1}\right)$ or 10. k may even be 0 or 2^k may not be seen. Just consider the bracket for this mark.

B1: Needs 1024(1…. To become 1024

A1, A1: as before

Q3.

Question Number	Scheme		Marks
	$\left(1+\frac{3x}{2}\right)^8$		
	$1+12x$	Both terms correct as printed (allow $12x^1$ but not 1^8)	B1
	$...+\frac{8(7)}{2!}\left(\frac{3x}{2}\right)^2+\frac{8(7)(6)}{3!}\left(\frac{3x}{2}\right)^3+...$ $...+{}^8C_2\left(\frac{3x}{2}\right)^2+{}^8C_3\left(\frac{3x}{2}\right)^3+...$	$\left(\frac{8(7)}{2!}\times...\times x^2\right)$ or $\left(\frac{8(7)(6)}{3!}\times...\times x^3\right)$ or $\left({}^8C_2\times...\times x^2\right)$ or $\left({}^8C_3\times...\times x^3\right)$ M1: For <u>either</u> the x^2 term <u>or</u> the x^3 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 2 and/or 3 or signs) may be wrong or missing.	M1
	<u>Special Case:</u> Allow this M1 <u>only</u> for an attempt at a descending expansion provided the equivalent conditions are met for any term <u>other than the first</u> e.g. $...+8\left(\frac{3x}{2}\right)^7(1)+\frac{8(7)}{2!}\left(\frac{3x}{2}\right)^6(1)^2+...$ $...+{}^8C_1\left(\frac{3x}{2}\right)^7+{}^8C_2\left(\frac{3x}{2}\right)^6+...$		
	$...+63x^2+189x^3+...$	A1: **Either** $63x^2$ **or** $189x^3$ A1: **Both** $63x^2$ **and** $189x^3$	A1A1
	Terms may be listed but must be positive		
			[4]
			Total 4
	Note it is common not to square the 2 in the denominator of $\left(\frac{3x}{2}\right)$ and this gives $1+12x+126x^2+756x^3$. This could score B1M1A0A0.		
	Note $...+{}^8C_2\left(1^4+\frac{3x}{2}\right)^2+{}^8C_3\left(1^3+\frac{3x}{2}\right)^3+...$ would score M0 unless a correct method was implied by later work		

Q4.

Question Number	Scheme		Marks
	(a) $(2-9x)^4 = 2^4 + {}^4C_1 2^3(-9x) + {}^4C_2 2^2(-9x)^2$, (b) $f(x) = (1+kx)(2-9x)^4 = A - 232x + Bx^2$		
(a) **Way 1**	First term of 16 in their final series		B1
	At least one of $({}^4C_1 \times \ldots \times x)$ or $({}^4C_2 \times \ldots \times x^2)$		M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ **and** $+1944x^2$	A1
			[4]
(a) **Way 2**	$(2-9x)^4 = (4-36x+81x^2)(4-36x+81x^2)$		
		First term of 16 in their final series	B1
	$= 16 - 144x + 324x^2 - 144x + 1296x^2 + 324x^2$	Attempts to multiply a 3 term quadratic by the same 3 term quadratic to achieve either 2 terms in x or at least 2 terms in x^2.	M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ **and** $+1944x^2$	A1
			[4]
(a) **Way 3**	$\{(2-9x)^4 =\}\ 2^4\left(1-\frac{9}{2}x\right)^4$	First term of 16 in final series	B1
	$= 2^4\left(1 + 4\left(-\frac{9}{2}x\right) + \frac{4(3)}{2}\left(-\frac{9}{2}x\right)^2 + \ldots\right)$	At least one of $(4\times\ldots\times x)$ or $\left(\frac{4(3)}{2}\times\ldots\times x^2\right)$	M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ **and** $+1944x^2$	A1
			[4]
	Parts (b), (c) and (d) may be marked together		
(b)	$A = \text{"16"}$	Follow through their value from (a)	B1ft
			[1]
(c)	$\{(1+kx)(2-9x)^4\} = (1+kx)(16 - 288x + \{1944x^2 + \ldots\})$	*May be seen in part (b) or (d) and can be implied by work in parts (c) or (d).*	M1
	x terms: $-288x + 16kx = -232x$		
	giving, $16k = 56 \Rightarrow k = \frac{7}{2}$	$k = \frac{7}{2}$	A1
			[2]
(d)	x^2 terms: $1944x^2 - 288kx^2$		
	So, $B = 1944 - 288\left(\frac{7}{2}\right)$; $= 1944 - 1008 = 936$	See notes	M1
		936	A1
			[2]
			9

Q5.

Question Number	Scheme		Marks
(a)	$\{(3+bx)^5\} = (3)^5 + {}^5C_1(3)^4(bx) + {}^5C_2(3)^3(bx)^2 + \dots$	243 as a constant term seen.	B1
	$= 243 + 405bx + 270b^2x^2 + \dots$	$405bx$	B1
		$({}^5C_1 \times \dots \times x)$ or $({}^5C_2 \times \dots \times x^2)$	M1
		$270b^2x^2$ or $270(bx)^2$	A1 [4]
(b)	$\{2(\text{coeff } x) = \text{coeff } x^2\} \Rightarrow 2(405b) = 270b^2$	Establishes an equation from their coefficients. Condone 2 on the wrong side of the equation.	M1
	So, $\left\{b = \frac{810}{270} \Rightarrow\right\} b = 3$	$b = 3$ (Ignore $b = 0$, if seen.)	A1
			[2]
			6

(a)	The terms can be "listed" rather than added. Ignore any extra terms. 1st B1: A constant term of 243 seen. Just writing $(3)^5$ is B0. 2nd B1: Term must be simplified to $405bx$ for B1. The x is required for this mark. Note $405 + bx$ is B0. M1: For <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 3 and/or b) may be wrong or missing. <u>Allow</u> binomial coefficients such as $\binom{5}{2}, \left(\frac{5}{2}\right), \binom{5}{1}, \left(\frac{5}{1}\right), {}^5C_2, {}^5C_1$. A1: For either $270b^2x^2$ or $270(bx)^2$. (If $270bx^2$ follows $270(bx)^2$, isw and allow A1.) **<u>Alternative:</u>** Note that a factor of 3^5 can be taken out first: $3^5\left(1+\frac{bx}{3}\right)^5$, but the mark scheme still applies. **<u>Ignore subsequent working (isw)</u>**: Isw if necessary after correct working: e.g. $243 + 405bx + 270b^2x^2 + \dots$ leading to $9 + 15bx + 10b^2x^2 + \dots$ scores B1B1M1A1 isw. Also note that full marks could also be available in part (b), here. **<u>Special Case</u>**: Candidate writing down the first three terms in ***descending*** powers of x usually get $(bx)^5 + {}^5C_4(3)^1(bx)^4 + {}^5C_3(3)^2(bx)^3 + \dots = b^5x^5 + 15b^4x^4 + 90b^3x^3 + \dots$ So award SC: B0B0M1A0 for either $({}^5C_4 \times \dots \times x^4)$ or $({}^5C_3 \times \dots \times x^3)$
(b)	M1 for equating 2 times their coefficient of x to the coefficient of x^2 to get an equation in b, <u>or</u> equating their coefficient of x to 2 times that of x^2, to get an equation in b. Allow this M mark even if the equation is trivial, providing their coefficients from part (a) have been used, eg: $2(405b) = 270b$, but beware $b = 3$ from this, which is A0. <u>An equation in b alone</u> is required: e.g. $2(405b)x = 270b^2x^2 \Rightarrow b = 3$ or similar will be **Special Case** SC: M1A0 (as equation in coefficients only is not seen here). e.g. $2(405b)x = 270b^2x^2 \Rightarrow 2(405b) = 270b^2 \Rightarrow b = 3$ will get M1A1 (as coefficients rather than terms have now been considered). **Note:** Answer of 3 from no working scores M1A0. **Note:** The mistake $k\left(1+\frac{bx}{3}\right)^5$, $k \neq 243$ would give a maximum of 3 marks: B0B0M1A0, M1A1 **Note:** For $270bx^2$ in part (a), followed by $2(405b) = 270b^2 \Rightarrow b = 3$, in part (b), allow recovery M1A1.

Q6.

Question number	Scheme	Marks
(a).	$(1+\frac{x}{4})^8 = 1 + 2x + ..,$	B1
	$+\frac{8\times 7}{2}(\frac{x}{4})^2 + \frac{8\times 7\times 6}{2\times 3}(\frac{x}{4})^3,$	M1 A1
	$= \quad +\frac{7}{4}x^2 + \frac{7}{8}x^3$ or $= \quad +1.75x^2 + 0.875x^3$	A1 (4)
(b)	States or implies that $x = 0.1$	B1
	Substitutes their value of x (provided it is <1) into series obtained in (a)	M1
	i.e. 1 + 0.2 + 0.0175 + 0.000875, = 1.2184	A1 cao (3) 7
Alternative for (b) Special case	Starts again and expands $(1+0.025)^8$ to $1+ 8\times 0.025 + \frac{8\times 7}{2}(0.025)^2 + \frac{8\times 7\times 6}{2\times 3}(0.025)^3, = 1.2184$ (Or 1 + 1/5 + 7/400 + 7/8000 = 1.2184)	B1,M1,A1
Notes	(a) **B1** must be simplified The **method** mark **(M1)** is awarded for an attempt at Binomial to get the third and/or fourth term – need correct binomial coefficient combined with correct power of x. Ignore bracket errors or errors in powers of 4. Accept any notation for ${}^8C_2$ and ${}^8C_3$, e.g. $\binom{8}{2}$ and $\binom{8}{3}$ (unsimplified) or 28 and 56 from Pascal's triangle. (The terms may be listed without + signs) First **A1** is for two completely correct unsimplified terms **A1** needs the fully simplified $\frac{7}{4}x^2$ *and* $\frac{7}{8}x^3$. (b) **B1** – states or uses x =0.1 or $\frac{x}{4} = \frac{1}{40}$ **M1** for substituting their value of x ($0 < x < 1$) into expansion (e.g. 0.1 (correct) or 0.01, 0.00625 or even 0.025 but **not** 1 nor 1.025 which would earn M0) **A1** Should be answer printed cao (not answers which round to) and should follow correct work. **Answer with no working at all** is B0, M0, A0 States 0.1 then just writes down answer is B1 M0A0	

Q7.

Question number	Scheme	Marks
	(a) $1+6kx$ [Allow unsimplified versions, e.g. $1^6+6(1^5)kx$, ${}^6C_0+{}^6C_1kx$]	B1
	$+\frac{6\times5}{2}(kx)^2+\frac{6\times5\times4}{3\times2}(kx)^3$ [See below for acceptable versions]	M1 A1 (3)
	N.B. THIS NEED NOT BE SIMPLIFIED FOR THE A1 (isw is applied)	
	(b) $6k=15k^2$ $k=\frac{2}{5}$ (or equiv. fraction, or 0.4) (Ignore $k=0$, if seen)	M1 A1cso (2)
	(c) $c=\frac{6\times5\times4}{3\times2}\left(\frac{2}{5}\right)^3=\frac{32}{25}$ (or equiv. fraction, or 1.28)	A1cso (1)
	(Ignore x^3, so $\frac{32}{25}x^3$ is fine)	
		6
	(a) The terms can be 'listed' rather than added. M1: Requires correct structure: 'binomial coefficients' (perhaps from Pascal's triangle), increasing powers of x. Allow a 'slip' or 'slips' such as: $+\frac{6\times5}{2}kx^2+\frac{6\times5\times4}{3\times2}kx^3$, $+\frac{6\times5}{2}(kx)^2+\frac{6\times5}{3\times2}(kx)^3$ $+\frac{5\times4}{2}kx^2+\frac{5\times4\times3}{3\times2}kx^3$, $+\frac{6\times5}{2}x^2+\frac{6\times5\times4}{3\times2}x^3$ <u>But</u>: $15+k^2x^2+20+k^3x^3$ or similar is M0. Both x^2 and x^3 terms must be seen. $\binom{6}{2}$ and $\binom{6}{3}$ or equivalent such as ${}^6C_2$ and ${}^6C_3$ are acceptable, and even $\left(\frac{6}{2}\right)$ and $\left(\frac{6}{3}\right)$ are acceptable for the method mark. A1: Any correct (possibly unsimplified) version of these 2 terms. $\binom{6}{2}$ and $\binom{6}{3}$ or equivalent such as ${}^6C_2$ and ${}^6C_3$ are acceptable. <u>Descending powers of x:</u> Can score the M mark if the required first 4 terms are not seen. <u>Multiplying out</u> $(1+kx)(1+kx)(1+kx)(1+kx)(1+kx)(1+kx)$: M1: A full attempt to multiply out (power 6) B1 and A1 as on the main scheme. (b) M: Equating the coefficients of x and x^2 (even if trivial, e.g. $6k=15k$). Allow this mark also for the 'misread': equating the coefficients of x^2 and x^3. An equation in k alone is required for this M mark, although… …condone $6kx=15k^2x^2\Rightarrow\left(6k=15k^2\Rightarrow\right)k=\frac{2}{5}$.	

Q8.

Question Number	Scheme	Marks
(a)	$\left(1+\frac{1}{2}x\right)^{10} = 1 + \underline{\binom{10}{1}\left(\frac{1}{2}x\right)+\binom{10}{2}\left(\frac{1}{2}x\right)^2+\binom{10}{3}\left(\frac{1}{2}x\right)^3}$	M1 A1
	$= 1 + 5x$; $+ \frac{45}{4}$ (or $11.25)x^2 + 15x^3$ (coeffs need to be these, i.e, simplified)	A1; A1 (4)
	[Allow A1A0, if totally correct with unsimplified, single fraction coefficients)	
(b)	$(1 + \frac{1}{2}\times 0.01)^{10} = 1 + 5(0.01) + (\frac{45}{4}\ or 11.25)(0.01)^2 + 15(0.01)^3$	M1 A1√
	$= 1 + 0.05 + 0.001125 + 0.000015$	
	$= 1.05114$ cao	A1 (3) **[7]**
Notes:	(a) For M1 first A1: Consider underlined expression only. M1 Requires correct structure for at least two of the three terms: (i) Must be attempt at binomial coefficients. [Be generous :allow all notations e.g. $^{10}C_2$, even $\left(\frac{10}{2}\right)$; allow "slips".] (ii) Must have increasing powers of x , (iii) May be listed, need not be added; *this applies for all marks.* First A1: Requires **all three** correct terms but need not be simplified, allow 1^{10} etc, $^{10}C_2$ etc, and condone omission of brackets around powers of ½x Second A1: Consider as B1: **1 + 5x can score A1 on Epen, even after M0** (b) For M1: Substituting **their** (0.01) into their (a) result [0.1, 0.001, 0.25, 0.025,0.0025 acceptable but **not** 0.005 or 1.005] First A1 (f.t.): Substitution of (**0.01**) into **their 4 termed** expression in (a) **Answer with no working** scores **no marks** (calculator gives this answer)	

Q9.

Question Number	Scheme	Marks
	(a) $(1+ax)^7 = 1+7ax....$ or $1+7(ax)....$ (Not unsimplified versions)	B1
	$+\frac{7\times6}{2}(ax)^2+\frac{7\times6\times5}{6}(ax)^3$ Evidence from one of these terms is enough	M1
	$+21a^2x^2$ or $+21(ax)^2$ or $+21(a^2x^2)$	A1
	$+35a^3x^3$ or $+35(ax)^3$ or $+35(a^3x^3)$	A1 (4)
	(b) $21a^2 = 525$	M1
	$a=\pm5$ (Both values are required) (The answer $a=5$ with no working scores M1 A0)	A1 (2)
		6
	(a) The terms can be 'listed' rather than added. M1: Requires correct structure: a correct binomial coefficient in any form (perhaps from Pascal's triangle) with the correct power of x. Allow missing a's and wrong powers of a, e.g. $\frac{7\times6}{2}ax^2$, $\frac{7\times6\times5}{3\times2}x^3$ However, $21+a^2x^2+35+a^3x^3$ or similar is M0. $1+7ax+21+a^2x^2+35+a^3x^3=57+.....$ scores the B1 (isw). $\binom{7}{2}$ and $\binom{7}{3}$ or equivalent such as ${}^7C_2$ and ${}^7C_3$ are acceptable, but not $\left(\frac{7}{2}\right)$ or $\left(\frac{7}{3}\right)$ (unless subsequently corrected). 1st A1: Correct x^2 term. 2nd A1: Correct x^3 term (The binomial coefficients must be simplified). Special case: If $(ax)^2$ and $(ax)^3$ are seen within the working, but then lost… … A1 A0 can be given if $21ax^2$ and $35ax^3$ are both achieved. a's omitted throughout: Note that only the M mark is available in this case. (b) M: Equating their coefficent of x^2 to 525. An equation in a or a^2 alone is required for this M mark, but allow 'recovery' that shows the required coefficient, e.g. $21a^2x^2=525 \Rightarrow 21a^2=525$ is acceptable, but $21a^2x^2=525 \Rightarrow a^2=25$ is not acceptable. After $21ax^2$ in the answer for (a), allow 'recovery' of a^2 in (b) so that full marks are available for (b) (but not retrospectively for (a)).	