Area Between a Curve and the x-Axis

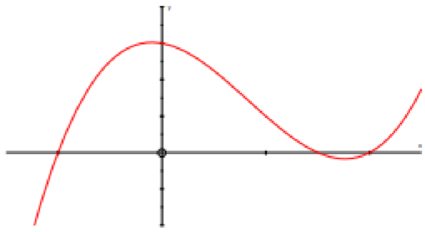
Q1, (Jun 2010, Q6a)

$\int_3^5 (x^2 + 4x)\,dx = \left[\frac{1}{3}x^3 + 2x^2\right]_3^5$	**M1**		Attempt integration
$= (^{125}/_3 + 50) - (9 + 18)$	**A1**		Obtain $^1/_3x^3 + 2x^2$
$= 64\ ^2/_3$	**M1**		Use limits $x = 3, 5$ – correct order & subtraction
	A1	**4**	Obtain $64\ ^2/_3$ or any exact equiv

Q2, (Jun 2006, Q4)

(i)	Intersect where $x^2 + x - 2 = 0 \Rightarrow x = -2, 1$	M1 A1	2	For finding *x* at both intersections For both values correct
(ii)	Area under curve is $\left[4x - \frac{1}{3}x^3\right]_{-2}^{1}$	M1 M1		For integration attempt with any one term correct For use of limits – subtraction and correct order
	i.e. $\left(4 - \frac{1}{3}\right) - \left(-8 + \frac{8}{3}\right) = 9$	A1		For correct area of 9
	Area of triangle is 4½	M1 A1		Attempt area of triangle (½*bh* or integration) Obtain area of triangle as 4 ½
	Hence shaded area is 9 – 4½ = 4½	A1	6	Obtain correct final area of 4 ½
	OR	M1		Attempt subtraction – either order
	Area under curve is $\int_{-2}^{1} (2 - x - x^2)\,dx$	M1		For integration attempt with any one term correct
	$= \left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x\right]_{-2}^{1}$	A1		Obtain $\pm\left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x\right]$
	$= \left(-\frac{1}{3} - \frac{1}{2} + 2\right) - \left(\frac{8}{3} - 2 - 4\right)$	M1		For use of limits – subtraction and correct order
	$= 4\frac{1}{2}$	A1		Obtain ± 4 ½ - consistent with their order of subtraction
		A1	<u>8</u>	Obtain 4 ½ only, following correct method only

Q3, (Jan 2006, Q8)

(i)	$-2+k+1+6=0 \Rightarrow k=-5$	M1		For attempting f(–1)
		A1		For equating f(-1) to 0 and deducing the correct value of k **AG**
	OR	M1		Match coefficients and attempt k
		A1		Show k = -5
	OR	B2		Following division, state remainder is 0, hence $(x+1)$ **is** a factor, hence k = -5
	EITHER: $(x+1)(2x^2-7x+6)$	B1		For correct leading term $2x^2$
		M1		For attempt at complete division by f(x) by $(x+1)$ or equiv.
		A1		For completely correct quadratic factor
	$=(x+1)(x-2)(2x-3)$	A1		For all three factors correct
	OR: f(2) = 16 – 20 – 2 + 6 = 0	M1		For further relevant use of the factor theorem
	Hence $(x-2)$ is a factor			
	Third factor is $(2x-3)$	A1		For correct identification of factor $(x-2)$
	Hence f(x) $=(x+1)(x-2)(2x-3)$	M1		For any method for the remaining factor
		A1	**6**	For all three factors correct
(ii)	$\int_{-1}^{2} f(x)dx = \left[\frac{1}{2}x^4 - \frac{5}{3}x^3 - \frac{1}{2}x^2 + 6x\right]_{-1}^{2}$	B1√		For any two terms integrated correctly
		B1√		For all four terms integrated correctly
	$=\left(8-\frac{40}{3}-2+12\right)-\left(\frac{1}{2}+\frac{5}{3}-\frac{1}{2}-6\right)$	M1		For evaluation of F(2) – F(–1)
	$= 9$	A1	**4**	For correct value 9
(iii)		B1		For sketch of positive cubic, with three distinct, non-zero, roots
		B1	**2**	For correct explanation that some of the area is below the axis
			12	

Q4, (Jan 2012, Q7b)

(a)	$\int(x^3 - 6x^2 + 4x - 24)\,dx$	**M1**	Expand and attempt int
	$= \frac{1}{4}x^4 - 2x^3 + 2x^2 - 24x + c$	**A1ft**	Obtain at least two correct (algebraic) terms
		A1	Obtain fully correct expression, inc + c
		[3]	
(b)	$\int 6x^{\frac{3}{2}}\,dx = \frac{12}{5}x^{\frac{5}{2}}$	**M1**	Obtain $kx^{\frac{5}{2}}$
		A1	Obtain $\frac{12}{5}x^{\frac{5}{2}}$, or any exact equiv
	$\int(8x^{-2} - 2)\,dx = -8x^{-1} - 2x$	**M1**	Obtain at least one of $-8x^{-1}$ and $-2x$
	$\left[\frac{12}{5}x^{\frac{5}{2}}\right]_0^1 = \frac{12}{5}$	**A1**	Obtain $-8x^{-1} - 2x$
	$\left[-8x^{-1} - 2x\right]_1^2 = (-8) - (-10) = 2$		
	hence total area $= \frac{22}{5}$	**B1**	State or imply that pt of intersection is (2, 0)
		M1	Use limits correctly at least once

Q5, (Jan 2008, Q7)

(i)	Some of the area is below the x-axis	B1	1	Refer to area / curve below x-axis or 'negative area'…
(ii)		M1		Attempt integration with any one term correct
		A1		Obtain $\frac{1}{3}x^3 - \frac{3}{2}x^2$
	$\left[\frac{1}{3}x^3 - \frac{3}{2}x^2\right]_0^3 = \left(9 - \frac{27}{2}\right) - (0 - 0)$	M1		Use limits 3 (and 0) – correct order / subtraction
	$= -4\frac{1}{2}$	A1		Obtain (-)4½
	$\left[\frac{1}{3}x^3 - \frac{3}{2}x^2\right]_3^5 = \left(\frac{125}{3} - \frac{75}{2}\right) - \left(9 - \frac{27}{2}\right)$	M1		Use limits 5 and 3 – correct order / subtraction
	$= 8\frac{2}{3}$	A1		Obtain $8\frac{2}{3}$ (allow 8.7 or better)
	Hence total area is $13\frac{1}{6}$	A1	7	Obtain total area as $13\frac{1}{6}$, or exact equiv
				SR: if no longer $\int f(x)dx$, then B1 for using [0, 3] and [3, 5]
			8	

Q6, (Jan 2007, Q10)

(i) $0 = 1 - \frac{3}{\sqrt{9}}$	B1	1	Verification of (9, 0), with at least one step shown
(ii) $\int_9^a 1 - 3x^{-\frac{1}{2}}\,dx = \left[x - 6\sqrt{x}\right]_9^a$	M1		Attempt integration – increase in power for at least 1 term
	A1		For second term of form $kx^{½}$
	A1		For correct integral
$= \left(a - 6\sqrt{a}\right) - \left(9 - 6\sqrt{9}\right)$	M1		Attempt F(a) – F(9)
$= a - 6\sqrt{a} + 9$	A1		Obtain $a - 6\sqrt{a} + 9$
$a - 6\sqrt{a} + 9 = 4$	M1		Equate expression for area to 4
$a - 6\sqrt{a} + 5 = 0$ $\left(\sqrt{a} - 1\right)\left(\sqrt{a} - 5\right) = 0$	M1		Attempt to solve 'disguised' quadratic
$\sqrt{a} = 1, \sqrt{a} = 5$	A1		Obtain at least $\sqrt{a} = 5$
$a = 1$, $a = 25$ but $a > 9$, so $a = 25$	A1	9	Obtain $a = 25$ only
		10	

Q7, (Jan 2011, Q9)

(i)	$f(3) = -108 + 81 + 30 - 3 = 0$ hence $(x - 3)$ is a factor	B1		Show that $f(3) = 0$, detail required	Substitute $x = 3$ and confirm $f(3) = 0$ – must show detail of substitution rather than just state $f(3) = 0$. Allow $f(3) = -4 \times 3^3 + 9 \times 3^2 + 10 \times 3 - 3 = 0$ for B1.
		B1	2	State $(x - 3)$ as factor (allow $(3 - x)$ as the factor)	Not dependent on first B1. Must be seen in (i) so no back credit from (ii). Allow if not explicitly stated as factor (and allow $f(x) = x - 3$). Ignore other factors if also given at this stage.
ii)	$f(x) = (x - 3)(-4x^2 - 3x + 1)$ or $f(x) = (3 - x)(4x^2 + 3x - 1)$ or $f(x) = (x + 1)(-4x^2 + 13x - 3)$ or $f(x) = (-x - 1)(4x^2 - 13x + 3)$ or $f(x) = (1 - 4x)(x^2 - 2x - 3)$ or $f(x) = (4x - 1)(-x^2 + 2x + 3)$	M1		Attempt complete division by $(x - 3)$, or equiv (allow division by $(3 - x)$)	Must be a full attempt to find three term quadratic. Can use inspection, but must be a reasonable attempt at middle term, with first and last correct. Can use coefficient matching, but must be full method with reasonable attempts at all 3 coefficients. Allow M1 if actually factorising $-f(x)$.
		A1		Obtain $-4x^2 - 3x + c$ or $-4x^2 + bx + 1$ (or the negative of these if dividing by $(3 - x)$)	c, b non-zero constants. First option is likely to come from division, second option from inspection. Coefficient matching could lead to either. Allow A1 for negative of either of these from factorising $-f(x)$.
		A1	3	Obtain $(x - 3)(-4x^2 - 3x + 1)$ (or $(3 - x)(4x^2 + 3x - 1)$)	Needs to be written as a product as per request in question paper. Allow $-(x - 3)(4x^2 + 3x - 1)$, but $(x - 3)(4x^2 + 3x - 1)$ is A0. A0 if now 3 linear factors and product of linear and quadratic never seen. If using one of the other two correct factors then all three marks are available, and apply mark scheme as above ie M1 for full attempt at division or equiv, A1 for lead term plus one other correct and A1 for product of linear and quadratic. **SR:** If candidates initially state three linear factors and then expand to get the product of a linear and quadratic as requested award **B3** if fully correct and simplified otherwise **B0**.
iii)	$-4x^2 - 3x + 1 = 0$ $(1 - 4x)(x + 1) = 0$ $x = \frac{1}{4}$, $x = -1$	M1		Attempt to solve quadratic	If factorising, needs to give two correct terms when brackets expanded. If using formula allow sign slips only – need to substitute and attempt one further step. If completing the square must get to $(x + p) = \pm\sqrt{q}$, with reasonable attempts at p and q.
		A1	2	Obtain $(\frac{1}{4}, 0)$, $(-1, 0)$	Condone only x values given rather than coordinates. Allow if $x = 3$ is still present as well.

(iv)	$\int f(x)dx = -x^4 + 3x^3 + 5x^2 - 3x$	B1		Obtain $-x^4 + 3x^3 + 5x^2 - 3x$	Allow unsimplified coefficients. Condone + c.
	$F(3) - F(\frac{1}{4}) = (36) - (^{-101}/_{256}) = 36^{101}/_{256}$ $F(\frac{1}{4}) - F(-1) = (^{-101}/_{256}) - (4) = -4^{101}/_{256}$	M1*		Attempt F(3) – F(¼) or F(¼) – F(-1)	Allow use of incorrect limits from their (iii). Limits need to be in correct order, and subtraction. Allow slips when evaluating but clear subtraction attempt must be seen or implied at least once. If minimal method shown then it must appear to be a plausible attempt eg F(3) = 198 or even F(3) – F(¼) = 198.4.
		A1		Obtain at least one correct area, including decimal equivs	Obtain $36^{101}/_{256}$ or $^{9317}/_{256}$ or 36.4 or $-4^{101}/_{256}$ or $-{}^{1125}/_{256}$ or – 4.4 Can get A1 if both areas attempted and one is correct but the other isn't.
		M1d*		Attempt full method to find total area including dealing correctly with negative area	Need to see modulus of negative integral from attempt at F(¼) – F(-1) (just changing sign from –ve to +ve is sufficient). If values incorrect in (iii) then can only get this mark if their integral gives negative value. Need to have positive integral from F(3) – F(¼).
	Hence area = $36^{101}/_{256} + 4^{101}/_{256} = 40^{101}/_{128}$	A1	5 **12**	Obtain $40^{101}/_{128}$ or $^{5221}/_{128}$ or 40.8	Allow exact fraction (including unsimplified ie $^{10442}/_{256}$), or decimal answer to 3dp or better (rounding to 40.8 with no errors seen) **SR:** If candidate attempts F(3) – F(¼) **and** F(-1) – F(¼) as an alternative method for dealing with negative area then mark as **B1** correct integral **M2** complete method **A1** obtain one correct area **A1** obtain correct total area Any attempts using this method must be fully supported by evidence of intention, especially -1 as top limit and ¼ as bottom limit used consistently throughout integration attempt. It should not be awarded if candidate appears to have simply confused their order of subtraction.

Q8, (Jun 2016, Q7)

(i)	$Q = x^2 - 4x + 3$ $R = 0$	M1	Attempt complete division by $(x + 1)$, or equiv	Must be complete method to obtain at least the quotient (ie all 3 terms attempted) but can get M1A1 if remainder not considered Long division - must subtract lower line (allow one slip) Inspection - expansion must give at least three correct terms of the cubic Coefficient matching - must be valid attempt at all coeffs of the quadratic, considering all relevant terms each time Synthetic division - must be using -1 (not 1) and adding within each column (allow one slip); expect to see $\begin{array}{r\|rrrr} -1 & 1 & -3 & -1 & 3 \\ & & -1 & 4 & (-3) \\ \hline & 1 & -4 & 3 & (0) \end{array}$ The values in brackets come from attempting R and are not required for M1
		A1	Obtain fully correct quotient	Quotient could be stated explicitly, seen in division attempt or in a factorised expression for f(x). Do not ISW if their explicitly stated quotient contradicts earlier working (eg correct in division but then stated as 'quotient = 3') If using coefficient matching then $A = 1$, $B = -4$, $C = 3$ is not sufficient for A1.

			A1	Obtain remainder as 0, must be stated explicitly	Not sufficient to just see 0 at bottom of division attempt (algebraic or synthetic) Allow ‘no remainder’ for ‘remainder = 0’ f(–1) = 0 is not sufficient for A1 unless identified as remainder If coefficient matching then allow $R = 0$ **SR B1** for remainder of 0 with nothing wrong seen - it could just be stated, or from f(–1), and could follow either M0 or M1 for attempt to find quotient. However, if remainder is attempted both by division attempt and f(–1) then mark final attempt at remainder
			[3]		
(ii)		$x^2 - 4x + 3 = (x - 1)(x - 3)$ hence $x = -1, 1, 3$	M1	Attempt to solve their quadratic quotient	Allow for solving any three term quadratic from their attempt at quotient, even if M0 in **(i)** See additional guidance for acceptable methods Could now be a different quotient if there is another division attempt with the factor as $(x - 1)$ or $(x - 3)$
			A1	Obtain $x = 1, 3$	M1A1 if both roots just stated with no method shown (but no partial credit if only one root correct)
			B1	State $x = -1$	Independent of M mark B0 if $x = -1$ is clearly as result of solving their quadratic quotient only Must be seen in **(ii)** - no back credit if only seen in **(i)**
			[3]		
(iii)		$\frac{dy}{dx} = 4x^3 - 12x^2 - 4x + 12$ $4x^3 - 12x^2 - 4x + 12 = 0$ hence $x^3 - 3x^2 - x + 3 = 0$ **AG**	M1	Attempt differentiation	Decrease in power by 1 for at least 3 of the terms (could include $9 \rightarrow 0$) Not sufficient to substitute their roots to show $y = 0$
			A1	Equate to 0 and rearrange to given answer	Must equate to 0 before dividing by 4
			[2]		

(iv)		$\left[\frac{1}{5}x^5 - x^4 - \frac{2}{3}x^3 + 6x^2 + 9x\right]_{-1}^{3}$ $= \left(\frac{153}{5}\right) - \left(-\frac{53}{15}\right)$ $= \frac{512}{15}$	M1*	Attempt integration	Increase in power by 1 for at least 3 of the terms Must be integrating equation of curve, not f(x)
			A1	Obtain fully correct expression	Allow unsimplified coefficients Allow presence of + c
			M1d*	Attempt correct use of correct limits	No follow-through from incorrect roots in **(ii)** Must be F(3) – F(–1) ie correct order and subtraction Could find area between 1 and 3, but must double this for M1 If final area is incorrect then must see evidence of use of limits to award M1; if all that is shown is the difference of two numerical values then both must be correct eg just $\left(\frac{153}{5}\right) - \left(-\frac{23}{15}\right) = \frac{482}{15}$ is M0 as no evidence for second term
			A1	Obtain $^{512}/_{15}$, or any exact equiv	Decimal equiv must be exact ie $34.1\dot{3}$, so A0 for 34.13, 34.133... etc Allow A1 if exact value seen, but followed by decimal equiv Answer only is 0/4 - need to see evidence of integration, but use of limits does not need to be explicit
			[4]		

Q9, (Jan 2015, Q6)

(i)		$f(x) = (x - 2)(x^2 + 2x - 15)$	B1	State or imply that $(x - 2)$ is a factor	Could be stated explicitly, or implied by using it in an attempt at the quotient or a factorisation attempt Could also give $(2 - x)$ as the factor
			M1	Attempt complete division, or equiv	Must be dividing by $(x - 2)$, or by one of the two other correct factors (or the negative of any of these factors) No need to show zero remainder as told that $x = 2$ is a root Must be complete method - ie all 3 terms attempted Long division - must subtract lower line (allow one slip) Inspection - expansion must give at least three correct terms of the cubic Coefficient matching - must be valid attempt at all coeffs of quadratic, considering all relevant terms each time Synthetic division - must be using 2 (not -2) and adding within each column (allow one slip); expect to see $\begin{array}{c\|cccc} 2 & 1 & 0 & -19 & 30 \\ & & 2 & 4 & \\ \hline & 1 & 2 & -15 & \end{array}$
			A1	Obtain correct quotient of $x^2 + 2x - 15$ CWO	Or correct quotient for their factor Could be stated explicitly, seen in division attempt or implied by $A = 1$, $B = 2$, $C = -15$
		$= (x - 2)(x + 5)(x - 3)$	A1	Obtain $(x - 2)(x + 5)(x - 3)$	Must be written as a product of the three linear factors Allow any equiv eg $(2 - x)(x + 5)(3 - x)$ Full credit for repeated use of factor theorem, or just writing down correct product Ignore any subsequent reference to roots **SR** A fully correct factorisation resulting from division by $(x + 5)$ or $(x - 3)$ can still get full credit, even though the root of $x = 2$ was not used
			[4]		

(ii)		$\left[\frac{1}{4}x^4 - \frac{19}{2}x^2 + 30x\right]_{-5}^{3}$	M1*	Attempt integration	Increase in power by 1 for at least 2 terms
			A1	Obtain correct integral	Could also have + c present; condone dx or $\int$ still present
		$= 24.75 - (-231.25)$	M1d*	Attempt correct use of limits	Must be F(3) – F(-5) Must be attempting the value of the requested definite integral, so M0 if instead attempting area (ie using $x = 2$ as a limit)
		$= 256$	A1 **[4]**	Obtain 256	A0 for 256 + c Answer only is 0/4 - need to see evidence of integration, but use of limits does not need to be explicit
(iii)		Sketch positive cubic with 3 distinct roots	B1	Sketch f(x) for $-5 \leq x \leq 3$	Must be a positive cubic Allow if maximum point is on y-axis No need for roots to be labelled, but need one negative and two positive roots (or ft from an incorrect factorisation in (i) - could have fewer than 3 roots shown if this is consistent with their roots in required range) Graph must be sketched for at least $-5 \leq x \leq 3$, but it is fine if more is shown – only penalise explicitly incorrect roots
		Some of the area is below the x-axis which will make negative contribution to the total	B1 **[2]**	Explanation referring to the area below the x-axis giving a negative value	B0B1 is possible (including following no sketch at all) Need to mention 'negative' and identify the relevant area in some way eg 'below x-axis' or $2 \leq x \leq 3$ or clear shading Just referring to some area below x-axis is insufficient, as is any reference just to negative area B0 for statements indicating that some area is ignored / cannot be calculated within an otherwise correct statement A reason is required as to why (ii) is incorrect - it is not sufficient to just state that the actual area is larger, or to just describe how to find the area