Area Between the Curve and the x-Axis Exam Questions Sheet 2

Q1.

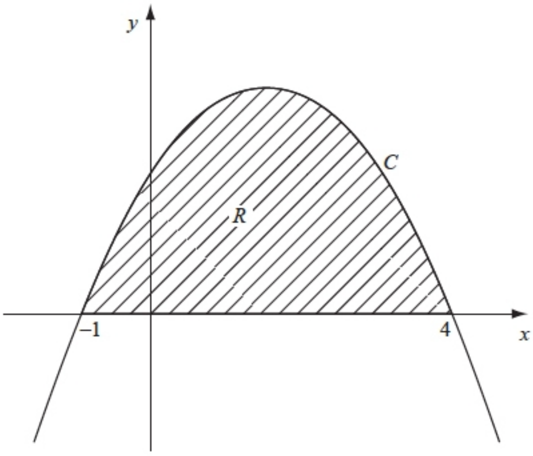


Figure 1

Figure 1 shows part of the curve C with equation $y = (1 + x)(4 - x)$.
The curve intersects the x-axis at $x = -1$ and $x = 4$. The region R, shown shaded in Figure 1, is bounded by C and the x-axis. Use calculus to find the exact area of R.

(5)

(Total 5 marks)

Q2.

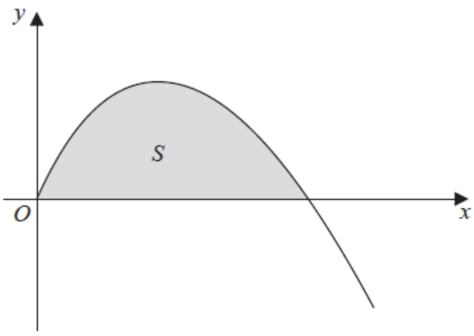


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 3x - x^{\frac{3}{2}}, \qquad x \geqslant 0$$

The finite region S, bounded by the x-axis and the curve, is shown shaded in Figure 3.

(a) Find

$$\int \left(3x - x^{\frac{3}{2}}\right) dx$$

(3)

(b) Hence find the area of S.

(3)

(Total for question = 6 marks)

Q3.

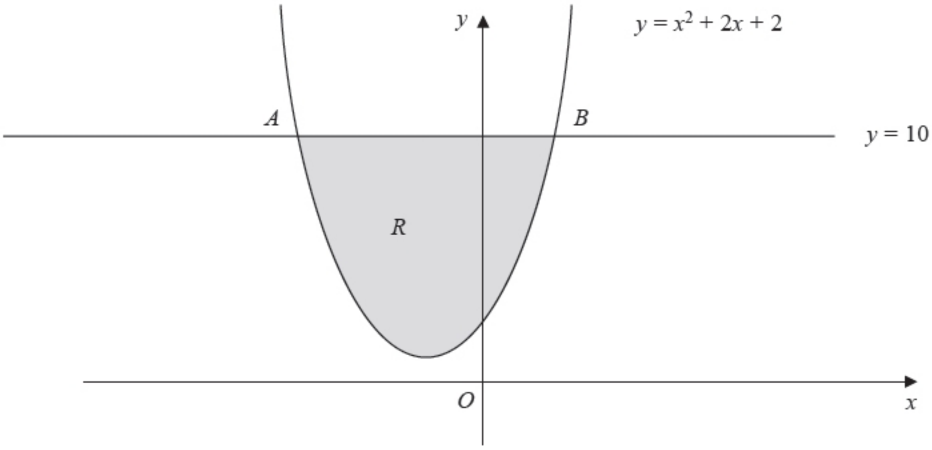


Figure 1

The line with equation $y = 10$ cuts the curve with equation $y = x^2 + 2x + 2$ at the points A and B as shown in Figure 1. The figure is not drawn to scale.

(a) Find by calculation the x-coordinate of A and the x-coordinate of B.

(2)

The shaded region R is bounded by the line with equation $y = 10$ and the curve as shown in Figure 1.

(b) Use calculus to find the exact area of R.

(7)

(Total 9 marks)

Q4.

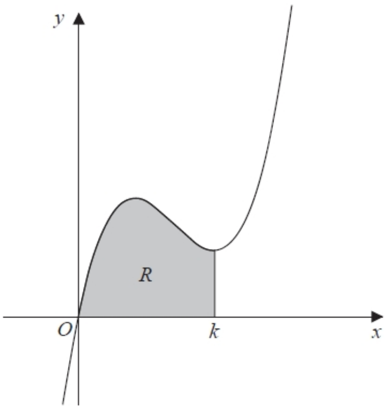


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2x^3 - 17x^2 + 40x$$

The curve has a minimum turning point at $x = k$.

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = k$.

Show that the area of R is $\frac{256}{3}$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(7)

(Total for question = 7 marks)

Q5.

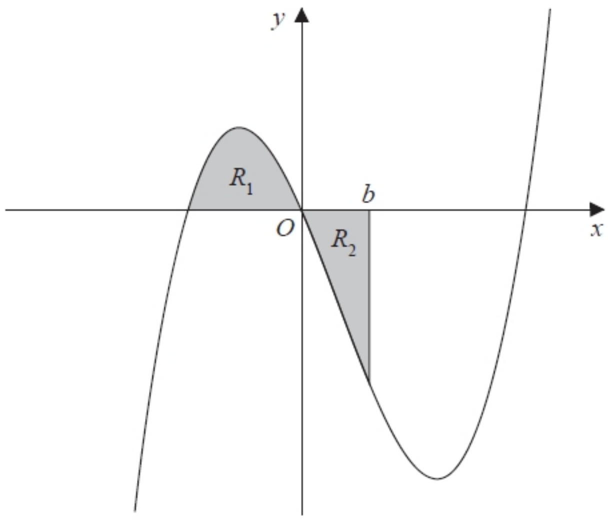


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x-axis.

(a) Show that the exact area of R_1 is $\frac{20}{3}$

(4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x-axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

(b) verify that b satisfies the equation

$$(b + 2)^2 (3b^2 - 20b + 20) = 0$$

(4)

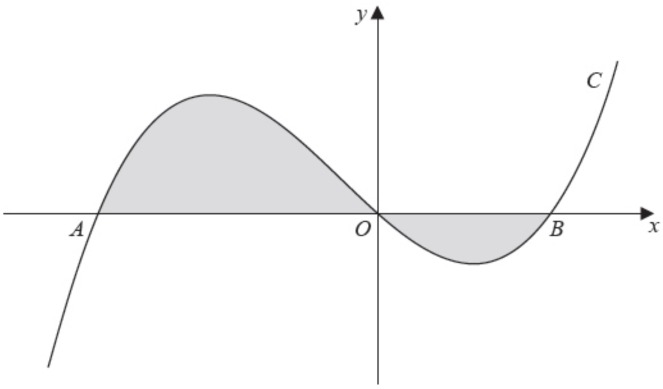
The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places. The value of b is therefore 1.225 to 3 decimal places.

(c) Explain, with the aid of a diagram, the significance of the root 5.442

(2)

(Total for question = 10 marks)

Q6.



The diagram shows a sketch of part of the curve C with equation

$$y = x(x + 4)(x - 2)$$

The curve C crosses the x-axis at the origin O and at the points A and B.

(a) Write down the x-coordinates of the points A and B.

(1)

The finite region, shown shaded in the diagram, is bounded by the curve C and the x-axis.

(b) Use integration to find the total area of the finite region shown shaded in the diagram.

(7)

(Total 8 marks)

Q7.

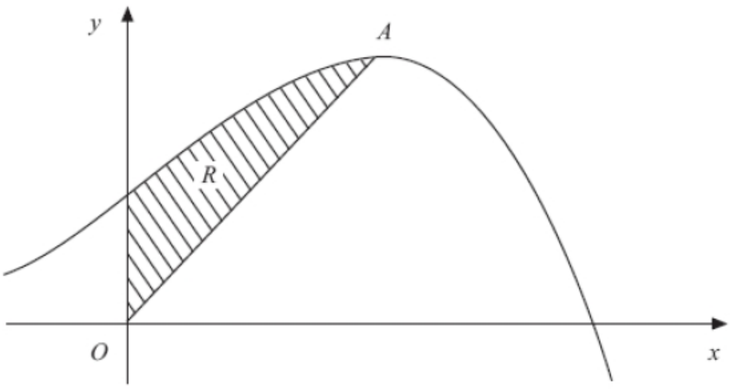


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = 10 + 8x + x^2 - x^3$.

The curve has a maximum turning point A.

(a) Using calculus, show that the x-coordinate of A is 2.

(3)

The region R, shown shaded in Figure 2, is bounded by the curve, the y-axis and the line from O to A, where O is the origin.

(b) Using calculus, find the exact area of R.

(8)

(Total for question = 11 marks)

Q8.

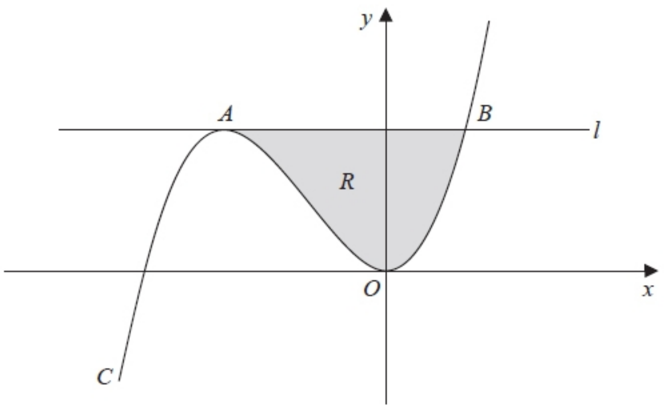


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{1}{8}x^3 + \frac{3}{4}x^2, \quad x \in \mathbb{R}$$

The curve C has a maximum turning point at the point A and a minimum turning point at the origin O.

The line l touches the curve C at the point A and cuts the curve C at the point B.

The x coordinate of A is -4 and the x coordinate of B is 2.

The finite region R, shown shaded in Figure 3, is bounded by the curve C and the line l.

Use integration to find the area of the finite region R.

(7)

(Total 7 marks)

Q9.

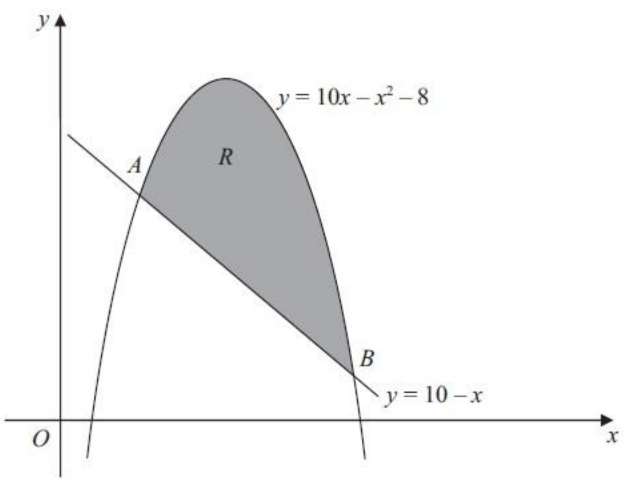


Figure 2

Figure 2 shows the line with equation $y = 10 - x$ and the curve with equation $y = 10x - x^2 - 8$

The line and the curve intersect at the points A and B, and O is the origin.

(a) Calculate the coordinates of A and the coordinates of B.

(5)

The shaded area R is bounded by the line and the curve, as shown in Figure 2.

(b) Calculate the exact area of R.

(7)

(Total 12 marks)

Q10.

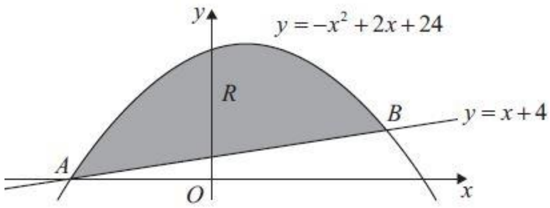


Figure 3

The straight line with equation $y = x + 4$ cuts the curve with equation $y = -x^2 + 2x + 24$ at the points A and B, as shown in Figure 3.

(a) Use algebra to find the coordinates of the points A and B.

(4)

The finite region R is bounded by the straight line and the curve and is shown shaded in Figure 3.

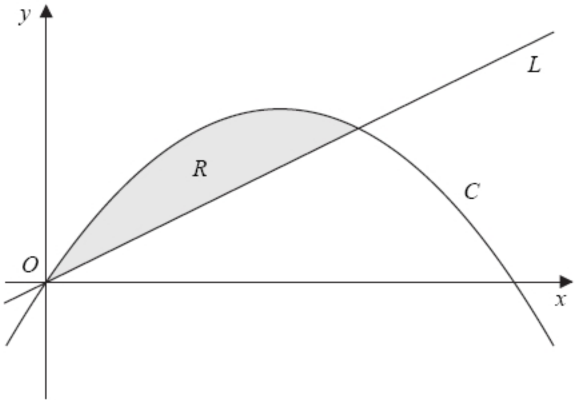
(b) Use calculus to find the exact area of R.

(7)

(Total 11 marks)

Q11.

Figure 2



In Figure 2 the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects the x-axis at $x = 0$ and $x = 6$.

(1)

(b) Show that the line L intersects the curve C at the points (0, 0) and (4, 8).

(3)

The region R, bounded by the curve C and the line L, is shown shaded in Figure 2.

(c) Use calculus to find the area of R.

(6)

(Total 10 marks)

Q12.

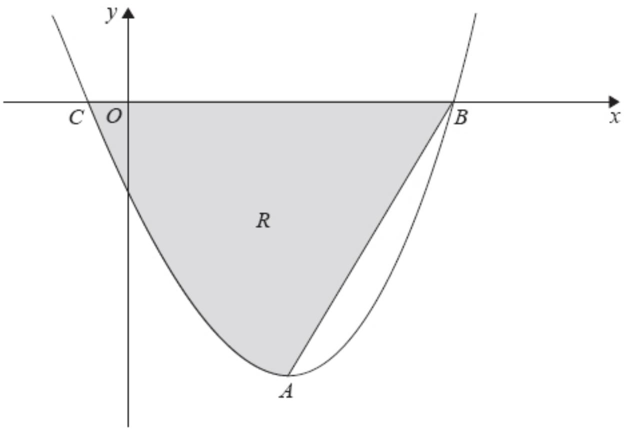


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 4x^3 + 9x^2 - 30x - 8, \quad -0.5 \leq x \leq 2.2$$

The curve has a turning point at the point *A*.

(a) Using calculus, show that the *x* coordinate of *A* is 1

(3)

The curve crosses the *x*-axis at the points *B* (2, 0) and $C\left(-\frac{1}{4}, 0\right)$

The finite region *R*, shown shaded in Figure 2, is bounded by the curve, the line *AB*, and the *x*-axis.

(b) Use integration to find the area of the finite region *R*, giving your answer to 2 decimal places.

(7)

(Total for question = 10 marks)