Area Between the Curve and the x-Axis Exam Questions Sheet 2 MS

Q1.

Question Number	Scheme	Marks
	$y=(1+x)(4-x)=4+3x-x^2$ M: Expand, giving 3 (or 4) terms	M1
	$\int(4+3x-x^2)\,dx=4x+\frac{3x^2}{2}-\frac{x^3}{3}$ M: Attempt to integrate	M1 A1
	$=[\ldots\ldots\ldots\ldots]_{-1}^{4}=\left(16+24-\frac{64}{3}\right)-\left(-4+\frac{3}{2}+\frac{1}{3}\right)=\frac{125}{6}$ $\left(=20\frac{5}{6}\right)$	M1 A1 (5) **[5]**
Notes	**M1** needs expansion, there may be a slip involving a sign or simple arithmetical error e.g. $1\times4=5$, but there needs to be a 'constant' an 'x term' and an 'x^2 term'. The x terms do not need to be collected. (Need not be seen if next line correct) Attempt to integrate means that $x^n\to x^{n+1}$ for at least one of the terms, then **M1** is awarded (even 4 becoming $4x$ is sufficient) – one correct power sufficient. **A1** is for correct answer only, not follow through. But allow $2x^2-\frac{1}{2}x^2$ or any correct equivalent. Allow + c, and even allow an evaluated extra constant term. **M1**: Substitute limit 4 and limit -1 into a changed function (must be -1) and indicate subtraction (either way round). **A1** must be exact, not 20.83 or similar. If recurring indicated can have the mark. Negative area, even if subsequently positive loses the A mark.	
Special cases	(i) Uses calculator method: **M1** for expansion (if seen) **M1** for limits if answer correct, so 0 , 1 or 2 marks out of 5 is possible (Most likely **M0 M0 A0 M1 A0**) (ii) Uses trapezium rule : not exact, no calculus – 0/5 unless expansion mark **M1** gained. (iii) Using original method, but then change all signs after expansion is likely to lead to: **M1 M1 A0, M1 A0 i.e. 3/5**	

Q2.

Question Number	Scheme		Marks
(a)	$\left\{\int\left(3x - x^{\frac{3}{2}}\right)\mathrm{d}x\right\} = \dfrac{3x^2}{2} - \dfrac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)}\ \{+c\}$	Either $3x \to \pm\lambda x^2$ or $x^{\frac{3}{2}} \to \pm\mu x^{\frac{5}{2}}$, $\lambda, \mu \neq 0$	M1
		At least one term correctly integrated	A1
		Both terms correctly integrated	A1
			[3]
(b)	$0 = 3x - x^{\frac{3}{2}} \Rightarrow 0 = 3 - x^{\frac{1}{2}}$ or $0 = x\left(3 - x^{\frac{1}{2}}\right) \Rightarrow x = ...$	Sets $y = 0$, in order to find the correct $x^{\frac{1}{2}} = 3$ or $x = 9$	M1
	$\left\{\text{Area}(S) = \left[\dfrac{3x^2}{2} - \dfrac{2}{5}x^{\frac{5}{2}}\right]_0^9\right\}$		
	$= \left(\dfrac{3(9)^2}{2} - \left(\dfrac{2}{5}\right)(9)^{\frac{5}{2}}\right) - \{0\}$	Applies the limit 9 on an integrated function with no wrong lower limit.	ddM1
	$\left\{= \left(\dfrac{243}{2} - \dfrac{486}{5}\right) - \{0\}\right\} = \dfrac{243}{10}$ or 24.3	$\dfrac{243}{10}$ or 24.3	A1 oe
			[3]
			6

Question Notes

(a)	M1	Either $3x \to \pm\lambda x^2$ or $x^{\frac{3}{2}} \to \pm\mu x^{\frac{5}{2}}$, $\lambda, \mu \neq 0$
	1st A1	At least one term correctly integrated. Can be simplified or un-simplified but power must be simplified. Then isw.
	2nd A1	Both terms correctly integrated. Can be un-simplified (as in the scheme) but the n+1 in each denominator and power should be a single number. (e.g. 2 – not 1+1) Ignore subsequent work if there are errors simplifying. Ignore the omission of "$+c$". Ignore integral signs in their answer.
(b)	1st M1	Sets $y = 0$, and reaches the correct $x^{\frac{1}{2}} = 3$ or $x = 9$ (isw if $x^{\frac{1}{2}} = 3$ is followed by $x = \sqrt{3}$) Just seeing $x = \sqrt{3}$ without the correct $x^{\frac{1}{2}} = 3$ gains M0. May just see $x = 9$. Use of trapezium rule to find area is M0A0 as hence implies integration needed.
	ddM1	This mark is dependent on the two previous method marks and needs both to have been awarded. Sees the limit 9 substituted in an integrated function. (Do not follow through their value of x) Do not need to see MINUS 0 but if another value is used as lower limit – this is M0. This mark may be implied by 9 in the limit and a correct answer.
	A1	$\dfrac{243}{10}$ or 24.3
	Common Error	Common Error $0 = 3x - x^{\frac{3}{2}} \Rightarrow x^{\frac{1}{2}} = 3$ so $x = \sqrt{3}$ Then uses limit $\sqrt{3}$ etc gains M1 M0 A0 so 1/3

Q3.

Question Number	Scheme	Marks
(a)	$x^2+2x+2=10 \Rightarrow x^2+2x-8=0$ (so $(x+4)(x-2)=0$) $\Rightarrow x=$........	M1
	$x=-4,\ 2$	A1 (2)
(b) Way 1	$\int(x^2+2x+2)\,dx=\frac{x^3}{3}+\frac{2x^2}{2}+2x(+C)$	M1A1A1
	$\left[\frac{x^3}{3}+\frac{2x^2}{2}+2x\right]_{"-4"}^{"2"}=\left(\frac{8}{3}+\frac{8}{2}+4\right)-\left(-\frac{64}{3}+\frac{32}{2}-8\right)\quad(=24)$	M1
	Rectangle: $10\times(2--4)=60$	B1 cao
	$R="60"-"24"$	M1
	$=36$	A1 (7)
		Total 9
(b) Way 2	$\int(8-x^2-2x)\,dx=8x-\frac{x^3}{3}-\frac{2x^2}{2}(+C)$	M1 A1ft A1
	$\left[8x-\frac{x^3}{3}-\frac{2x^2}{2}\right]_{"-4"}^{"2"}=\left(16-\frac{8}{3}-4\right)-\left(-32+\frac{64}{3}-16\right)$ =(9.3 – (- 26.7))	M1
	Implied by final answer of 36 after correct work	B1
	$10-(x^2+2x+2)=8-x^2-2x,\ =36$	M1, A1
	Notes for Question	
(a)	M1 Set the curve equation equal to 10 and collect terms. Solves quadratic to $x=$ ..... A1 cao : Both values correct – allow $A=-4$, $B=2$	
(b)	M1: One correct integration A1: Two correct integrations(ft slips subtracting in Way 2) A1: All 3 terms correct **(penalise subtraction errors here in Way 2)** M1: Substitute their limits from (a) into the **integrated** function and subtract (either way round) B1: Way 1:Find area under the line by integration or area of rectangle – should be 60 here (no follow through) Way 2: (implied by final correct answer in second method) M1: Subtract one area from the other (implied by subtraction of functions in second method)- award even after differentiation A1: Must be 36 not –36.	
	Special case 1: Combines both methods. Uses Way 2 integration, but continues after reaching "36" to subtract "36" from rectangle giving answer as "24" This loses final M1 A1 *Special case* 2: Integrates (x^2+2x-8) between limits –4 and 2 to get -36 and then changes sign and obtains 36. Do not award final A mark – so M1A1A1M1B1M1A0 If the answer is left as -36, then M1A1A1M1B0M1A0 N.B. Allow full marks for modulus used earlier in working e.g. $\left\|\int_{-4}^{2}x^2+2x-2dx-\int_{-4}^{2}10dx\right\|$	

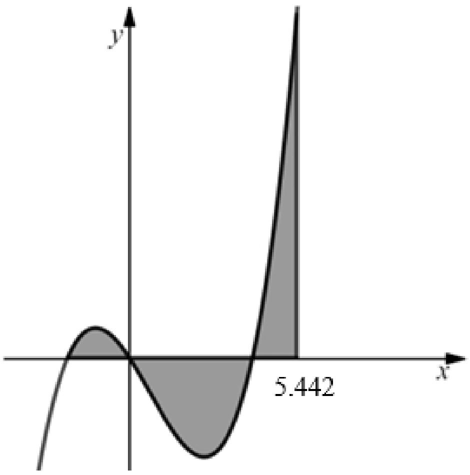
Q4.

	Scheme	Marks	AOs
	The overall method of finding the x coordinate of A.	M1	3.1a
	$y = 2x^3 - 17x^2 + 40x \Rightarrow \frac{dy}{dx} = 6x^2 - 34x + 40$	B1	1.1b
	$\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 34x + 40 = 0 \Rightarrow 2(3x-5)(x-4) = 0 \Rightarrow x = ...$	M1	1.1b
	Chooses $x = 4$ ~~$x = \frac{5}{3}$~~	A1	3.2a
	$\int 2x^3 - 17x^2 + 40x \, dx = \left[\frac{1}{2}x^4 - \frac{17}{3}x^3 + 20x^2\right]$	B1	1.1b
	Area $= \frac{1}{2}(4)^4 - \frac{17}{3}(4)^3 + 20(4)^2$	M1	1.1b
	$= \frac{256}{3}$ *	A1*	2.1
		(7)	
		(7 marks)	

Notes
M1: An overall problem -solving method mark to find the minimum point. To score this you need to see • an attempt to differentiate with at least **two correct terms** • an attempt to set their $\frac{dy}{dx} = 0$ and then solve to find x. Don't be overly concerned by the mechanics of this solution
B1: $\left(\frac{dy}{dx} =\right) 6x^2 - 34x + 40$ which may be unsimplified
M1: Sets their $\frac{dy}{dx} = 0$, which must be a 3TQ in x, and attempts to solve via factorisation, formula or calculator. If a calculator is used to find the roots, they must be correct for their quadratic. If $\frac{dy}{dx}$ is correct allow them to just choose the root 4 for M1 A1. Condone $(x-4)\left(x-\frac{5}{3}\right)$
A1: Chooses $x = 4$ This may be awarded from the upper limit in their integral
B1: $\int 2x^3 - 17x^2 + 40x \, dx = \left[\frac{1}{2}x^4 - \frac{17}{3}x^3 + 20x^2\right]$ which may be unsimplified
M1: Correct attempt at area. There may be slips on the integration but expect **two correct terms** The upper limit used must be their larger solution of $\frac{dy}{dx} = 0$ and the lower limit used must be 0. So if their roots are 6 and 10, then they must use 10 and 0. If only one value is found then the limits must be 0 to that value. Expect to see embedded or calculated values. Don't accept $\int_0^4 2x^3 - 17x^2 + 40x \, dx = \frac{256}{3}$ without seeing the integration and the embedded or calculated values
A1*: Area $= \frac{256}{3}$ **with** correct notation and no errors. Note that this is a given answer. For correct notation expect to see • $\frac{dy}{dx}$ or $\frac{d}{dx}$ used correctly at least once. If $f(x)$ is used accept $f'(x)$. Condone y' • $\int 2x^3 - 17x^2 + 40x \, dx$ used correctly at least once with or without the limits.

Q5.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$y = x(x+2)(x-4) = x^3 - 2x^2 - 8x$	B1	This mark is given for expanding brackets as a first step to a solution
	$\int_{-2}^{0} x^3 - 2x^2 - 8x \ \mathrm{d}x$	M1	This mark is given for a method to find the exact are of R_1
	$= \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0}$	M1	This mark is given for a method to evaluate the integral
	$= 0 - (4 - \frac{-16}{3} - 16) = \frac{20}{3}$	A1	This mark is given for a full method to show the exact value of R_1
(b)	$\frac{1}{4}b^4 - \frac{2}{3}b^3 - 4b^2 = -\frac{20}{3}$	M1	This mark is given for deducing the area of $R_2 = -\frac{20}{3}$
	$3b^4 - 8b^3 - 48b^2 + 80 = 0$	A1	This mark is given for rearranging the equation to a quartic
	$(b+2)^2(3b^2 - 20b + 20)$ $= (b^2 + 4b + 4)(3b^2 - 20b + 20)$	M1	This mark is given for expanding the equation given
	$= 3b^4 - 8b^3 - 48b^2 + 80 = 0$ The two equations are the same, so verified	A1	This mark is for showing, and stating, that the equations are the same

(c)		B1	This mark is given for a sketch of the curve with $b = 5.442$ shown
	Between $x = -2$ and $b = 5.442$, the area above the x-axis is the same as the area below the x-axis	B1	This mark is given for a valid explanation of the significance of the root 5.442

Q6.

Question Number	Scheme	Marks
(a)	Seeing -4 and 2.	B1 (1)
(b)	$x(x+4)(x-2) = \underline{x^3 + 2x^2 - 8x}$ or $\underline{x^3 - 2x^2 + 4x^2 - 8x}$ (without simplifying)	B1
	$\int(x^3 + 2x^2 - 8x)\,dx = \frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}\{+c\}$ or $\frac{x^4}{4} - \frac{2x^3}{3} + \frac{4x^3}{3} - \frac{8x^2}{2}\{+c\}$	M1A1ft
	$\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}\right]_{-4}^{0} = (0) - \left(64 - \frac{128}{3} - 64\right)$ or $\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}\right]_{0}^{2} = \left(4 + \frac{16}{3} - 16\right) - (0)$	dM1
	One integral $= \pm 42\frac{2}{3}$ (42.6 or awrt 42.7) **or** other integral $= \pm 6\frac{2}{3}$ (6.6 or awrt 6.7)	A1
	Hence Area $=$ "*their* $42\frac{2}{3}$" $+$ "*their* $6\frac{2}{3}$" or Area $=$ "*their* $42\frac{2}{3}$" $-$ "$-$ *their* $6\frac{2}{3}$"	dM1
	$= 49\frac{1}{3}$ or 49.3 or $\frac{148}{3}$ (NOT $-\frac{148}{3}$)	A1
	(An answer of $= 49\frac{1}{3}$ may not get the final two marks – check solution carefully)	(7) [8]
	Notes for Question	
(a)	B1: Need both –4 and 2. May see (-4,0) and (2,0) (correct) but allow (0,-4) and (0, 2) or $A = -4$, $B = 2$ or indeed any indication of -4 and 2 – check graph also	
(b)	B1: Multiplies out cubic correctly (terms may not be collected, but if they are, mark collected terms here) M1: Tries to integrate their expansion with $x^n \to x^{n+1}$ for at least one of the terms A1ft: completely correct integral **following through** from their CUBIC expansion (if only quadratic or quartic this is A0) dM1: (dependent on previous M) substituting EITHER $-a$ and 0 and subtracting either way round OR similarly for 0 and b. **If their limits $-a$ and b are used in ONE integral, apply the Special Case below.** A1: Obtain **either** $\pm 42\frac{2}{3}$ (or 42.6 or awrt 42.7)*from the integral from -4 to 0* **or** $\pm 6\frac{2}{3}$ (6.6 or awrt 6.7) *from the integral from 0 to 2;* NO follow through on their cubic (allow decimal or improper equivalents $\frac{128}{3}$ or $\frac{20}{3}$) isw such as subtracting from rectangles. This will be penalized in the next two marks, which will be M0A0. dM1 (depends on first method mark) **Correct method to obtain shaded area** so adds two positive numbers (areas) together or uses their **positive** value minus **their negative** value, **obtained from** two separate **definite integrals**. A1: Allow 49.3, 49.33, 49.333 etc. Must follow correct logical work with **no errors** seen. For full marks on this question there must be **two** definite integrals, from -4 to 0 and from 0 to 2, though the evaluations for 0 may not be seen. (Trapezium rule **gets no marks after first two B marks**)	
(b)	**Special Case: one integral only from $-a$ to b:** B1M1A1 available as before, then $\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}\right]_{-4}^{2} = (4 + \frac{16}{3} - 16) - \left(64 - \frac{128}{3} - 64\right) = -6\frac{2}{3} + 42\frac{2}{3} = \ldots$ **dM1** for correct use of their limits $-a$ and b and **subtracting** either way round. A1 for 36: NO follow through. Final M and A marks not available. Max 5/7 for part (b)	

Q7.

Question Number	Scheme	Marks
(a)	$\left(\frac{dy}{dx}=\right)8+2x-3x^2$	M1 A1
	$3x^2-2x-8=0 \quad (3x+4)(x-2)=0 \quad x=2$	A1 cso (3)
(b)	Area of triangle = $\frac{1}{2}\times 2\times 22$	M1 A1
	$\int 10+8x+x^2-x^3\,dx = 10x+\frac{8x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}$	M1 A1 A1
	$\left[10x+\frac{8x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\right]_0^2 = ...\left(=20+16+\frac{8}{3}-4\right)$	M1
	Area of $R = 34\frac{2}{3}-22=\frac{38}{3}\left(=12\frac{2}{3}\right)$ (Or $12.\dot{6}$)	M1 A1 (8)
		(11 marks)

Q8.

Question Number	Scheme		Marks
	$\int\left(\frac{1}{8}x^3+\frac{3}{4}x^2\right)dx = \frac{x^4}{32}+\frac{x^3}{4}\{+c\}$	M1: $x^n \to x^{n+1}$ on either term A1: $\frac{x^4}{32}+\frac{x^3}{4}$. Any correct simplified or un-simplified form. (+ c not required)	M1A1
	$\left[\frac{x^4}{32}+\frac{x^3}{4}\right]_{-4}^{2} = \left(\frac{16}{32}+\frac{8}{4}\right)-\left(\frac{256}{32}+\frac{(-64)}{4}\right)$ or $\left[\frac{x^4}{32}+\frac{x^3}{4}\right]_{-4}^{0} = (0)-\left(\frac{(-4)^4}{32}+\frac{(-4)^3}{4}\right)$ added to $\left[\frac{x^4}{32}+\frac{x^3}{4}\right]_{0}^{2} = \left(\frac{(2)^4}{32}+\frac{(2)^3}{4}\right)-(0)$		dM1
	Substitutes limits of 2 and −4 into an "integrated function" and subtracts either way round. Or substitutes limits of 0 and −4 **and** 2 and 0 into an "integrated function" and subtracts either way round and adds the two results.		
	$=\frac{21}{2}$	$\frac{21}{2}$ or 10.5	A1
	$\{$At $x=-4,\ y=-8+12=4$ or at $x=2,\ y=1+3=4\}$		
	Area of Rectangle $=6\times 4=24$ or Area of Rectangles $=4\times 4=16$ and $2\times 4=8$		M1
	Evidence of $(4--2)\times$ their y_{-4} **or** $(4--2)\times$ their y_2 or Evidence of $4\times$ their y_{-4} **and** $2\times$ their y_2		
	So, area(R) $=24-\frac{21}{2}=\frac{27}{2}$	dddM1: Area rectangle – integrated answer. Dependent on all previous method marks **and requires:** **Rectangle > integration > 0** A1: $\frac{27}{2}$ or 13.5	dddM1A1
			[7]
			Total 7

	Alternative:		
	$\pm\int \text{"their4"} - \left(\frac{1}{8}x^3 + \frac{3}{4}x^2\right) \mathrm{d}x$	Line – curve. **Condone missing brackets and allow either way round.**	4^{th} M1
	$= 4x - \frac{x^4}{32} - \frac{x^3}{4} \{+c\}$	M1: $x^n \to x^{n+1}$ on either curve term A1ft: "$-\frac{x^4}{32} - \frac{x^3}{4}$." Any correct simplified or un-simplified form of their curve terms, **follow through sign errors.** (+ c not required)	1^{st} M1, 1^{st} A1ft
	$[\ ]_{-4}^{2} = \underline{\underline{\left(8 - \frac{16}{32} - \frac{8}{4}\right) - \left(-16 - \frac{256}{32} - \frac{(-64)}{4}\right)}}$	2^{nd} M1 Substitutes limits of 2 and -4 into an "integrated curve" and subtracts either way round. 3^{rd} M1 for $\pm(\text{"8"} - \text{"}-16\text{"})$ **Substitutes limits into the 'line part' and subtracts either way round.** 2^{nd} A1 for correct $\pm$ (underlined expression). **Now needs to be correct but allow $\pm$ the correct expression.**	2^{nd} M1, 3^{rd} M1 2^{nd} A1
	$= \frac{27}{2}$	A1: $\frac{27}{2}$ or 13.5	3^{rd} A1
	If the final answer is -13.5 you can withhold the final A1 **If -13.5 then "becomes" +13.5 allow the A1**		

Q9.

Question number	Scheme		Marks
Method 1 (a)	Puts $10-x=10x-x^2-8$ and rearranges to give three term quadratic	Or puts $y=10(10-y)-(10-y)^2-8$ and rearranges to give three term quadratic	M1
	Solves their "$x^2-11x+18=0$" using acceptable method as in general principles to give $x=$	Solves their "$y^2-9y+8=0$" using acceptable method as in general principles to give $y=$	M1
	Obtains $x=2$, $x=9$ (may be on diagram or in part (b) in limits)	Obtains $y=8$, $y=1$ (may be on diagram)	A1
	Substitutes their x into a given equation to give $y=$ (may be on diagram)	Substitutes their y into a given equation to give $x=$ (may be on diagram or in part (b))	M1
	$y=8$, $y=1$	$x=2$, $x=9$	A1 (5)
(b)	$\int(10x-x^2-8)\,dx=\frac{10x^2}{2}-\frac{x^3}{3}-8x\{+c\}$		M1 A1 A1
	$\left[\frac{10x^2}{2}-\frac{x^3}{3}-8x\right]_2^9=(\ldots\ldots)-(\ldots\ldots)$		dM1
	$=90-\frac{4}{3}=88\frac{2}{3}$ or $\frac{266}{3}$		
	Area of trapezium $=\frac{1}{2}(8+1)(9-2)=31.5$		B1
	So area of R is $88\frac{2}{3}-31.5=57\frac{1}{6}$ or $\frac{343}{6}$		M1A1 cao (7)
			12 marks
Notes (a)	First **M1**: See scheme Second **M1**: See notes relating to solving quadratics Third **M1**: This may be awarded if one substitution is made Two correct Answers following tables of values, or from Graphical calculator are 5/5 **Just one pair of correct coordinates – no working or from table is M0M0A0M1A0**		
(b)	**M1**: $x^n \to x^{n+1}$ for any one term. **1st A1**: at least two out of three terms correct **2nd A1**: All three correct **dM1**: Substitutes 9 and 2 (or limits from part(a)) into an "integrated function" and subtracts, either way round (NB: If candidate **changes all signs** to get $\int(-10x+x^2+8)\,dx=-\frac{10x^2}{2}+\frac{x^3}{3}+8x\{+c\}$ This is M1 A1 A1 Then uses limits dM1 and trapezium is B1 Needs to *change sign of value obtained* from integration for final M1A1 so $-88\frac{2}{3}-31.5$ is M0A0) **B1**: Obtains 31.5 for area under line using any correct method (could be integration) or triangle minus triangle $\frac{1}{2}\times8\times8-\frac{1}{2}$ or rectangle plus triangle [may be implied by correct 57 1/6] **M1**: Their Area under curve – Their Area under line (if integrate both need same limits) **A1**: Accept 57.16recurring but not 57.16 **PTO for Alternative method**		

Method 2 for (b)	Area of R $=\int_2^9(10x-x^2-8)-(10-x)\,dx$	3rd M1 (in (b)): Uses difference between two functions in integral.	
	$\int_2^9 -x^2+11x-18\,dx$	M: $x^n \to x^{n+1}$ for any one term.	M1
	$=-\frac{x^3}{3}+\frac{11x^2}{2}-18x\{+c\}$	A1 at least two out of these three simplified terms Correct integration. (Ignore $+c$).	A1 A1
	$\left[-\frac{x^3}{3}+\frac{11x^2}{2}-18x\right]_2^9=(\ldots\ldots)-(\ldots\ldots)$	Substitutes 9 and 2 (or limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	This mark is implied by final answer which rounds to 57.2		B1
	See above working(allow bracketing errors) to decide to award 3rd M1 mark for (b) here:		M1
	$40.5-(-16\frac{2}{3})=57\frac{1}{6}$ cao		A1 (7)
Special case of above method	$\int_2^9 x^2-11x+18\,dx=\frac{x^3}{3}-\frac{11x^2}{2}+18x\{+c\}$		M1A1A1
	$\left[\frac{x^3}{3}-\frac{11x^2}{2}+18x\right]_2^9=(\ldots\ldots)-(\ldots\ldots)$		DM1
	This mark is implied by final answer which rounds to 57.2 (not -57.2)		B1
	Difference of functions implied (see above expression)		M1
	$40.5-(-16\frac{2}{3})=57\frac{1}{6}$ cao		A1 (7)
Special Case 2	Integrates expression in y e.g. "$y^2-9y+8=0$": This can have first M1 in part (b) and no other marks. (It is not a method for finding this area)		
Notes	Take away trapezium again having used Method 2 loses last two marks Common Error: Integrates $-x^2+9x-18$ is likely to be M1A1A0dM1B0M1A0 Integrates $2-11x-x^2$ is likely to e M1A0A0dM1B0M1A0 Writing $\int_2^9(10x-x^2-8)-(10-x)\,dx$ only earns final M mark		

Q10.

Question Number	Scheme		Marks
(a)	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$		
	{Curve = Line} $\Rightarrow -x^2 + 2x + 24 = x + 4$	Eliminating y correctly.	B1
	$x^2 - x - 20 \{= 0\} \Rightarrow (x-5)(x+4) \{= 0\} \Rightarrow x = \ldots$	Attempt to solve a *resulting* quadratic to give $x =$ their values.	M1
	So, $x = 5, -4$	Both $x = 5$ and $x = -4$.	A1
	So corresponding y-values are $y = 9$ and $y = 0$.	See notes below.	B1ft [4]
(b)	$\left\{\int(-x^2 + 2x + 24)\,dx\right\} = -\frac{x^3}{3} + \frac{2x^2}{2} + 24x \{+c$	M1: $x^n \to x^{n+1}$ for any one term. 1st A1 at least two out of three terms. 2nd A1 for correct answer.	M1A1A1
	$\left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{-4}^{5} = (\ldots\ldots) - (\ldots\ldots)$	Substitutes 5 and -4 (or their limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{\left(-\frac{125}{3} + 25 + 120\right) - \left(\frac{64}{3} + 16 - 96\right) = \left(103\frac{1}{3}\right) - \left(-58\frac{2}{3}\right) = 162\right\}$		
	Area of $\Delta = \frac{1}{2}(9)(9) = 40.5$	Uses correct method for finding area of triangle.	M1
	So area of R is $162 - 40.5 = 121.5$	Area under curve – Area of triangle.	M1
		121.5	A1 oe cao
			[7]
			11

Question Number	Scheme		Marks
Aliter (b) Way 2	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$		
	Area of $R = \int_{-4}^{5} (-x^2 + 2x + 24) - (x + 4)\,dx$	3rd M1: Uses integral of $(x + 4)$ with correct ft limits. 4th M1: Uses "curve" – "line" function with correct ft limits.	
	$= -\frac{x^3}{3} + \frac{x^2}{2} + 20x \{+c\}$	M: $x^n \to x^{n+1}$ for any one term.	M1
		A1 at least two out of three terms	A1ft
		Correct answer (Ignore $+c$).	A1
	$\left[-\frac{x^3}{3} + \frac{x^2}{2} + 20x\right]_{-4}^{5} = (\ldots\ldots) - (\ldots\ldots)$	Substitutes 5 and -4 (or *their limits* from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{\left(-\frac{125}{3} + \frac{25}{2} + 100\right) - \left(\frac{64}{3} + 8 - 80\right) = \left(70\frac{5}{6}\right) - \left(-50\frac{2}{3}\right)\right\}$		
		See above working to decide to award 3rd M1 mark here:	M1
		See above working to decide to award 4th M1 mark here:	M1
	So area of R is $= 121.5$	121.5	A1 oe cao
			[7]
			11

Q11.

Question Number	Scheme	Marks
(a)	Either solving $0 = x(6 - x)$ and showing $x = 6$ (and $x = 0$) or showing (6,0) (and $x = 0$) satisfies $y = 6x - x^2$ [allow for showing x = 6]	B1 (1)
(b)	Solving $2x = 6x - x^2$ $(x^2 = 4x)$ to x = ….	M1
	$x = 4$ (and $x = 0$)	A1
	Conclusion: when $x = 4$, $y = 8$ and when $x = 0$, $y = 0$,	A1 (3)
(c)	(Area =) $\int_{(0)}^{(4)}(6x - x^2)\,dx$ Limits not required	M1
	Correct integration $3x^2 - \frac{x^3}{3}$ (+ c)	A1
	Correct use of correct limits on their result above (see notes on limits) $[\text{"}3x^2 - \frac{x^3}{3}\text{"}]^4 - [\text{"}3x^2 - \frac{x^3}{3}\text{"}]_0$ with limits substituted $[= 48 - 21\frac{1}{3} = 26\frac{2}{3}]$	M1
	Area of triangle = 2 × 8 =16 (Can be awarded even if no M scored, i.e. B1)	A1
	Shaded area = ± (area under curve − area of triangle) applied correctly	M1
	$(= 26\frac{2}{3} - 16)$ $= 10\frac{2}{3}$ (awrt 10.7)	A1 (6)**[10]**

Notes	(b) In scheme first A1: need only give $x = 4$ If *verifying approach* used: Verifying (4,8) satisfies both the line and the curve M1(attempt at both), Both shown successfully A1 For final A1, (0,0) needs to be mentioned ; accept " clear from diagram" (c) **Alternative** Using Area = $\pm\int_{(0)}^{(4)}\{(6x-x^2); \quad -2x\}\,dx$ approach (i) If candidate integrates separately can be marked as main scheme If combine to work with = $\pm\int_{(0)}^{(4)}(4x-x^2)\,dx$, **first** M mark and **third** M mark $= (\pm)\,[\,2x^2 - \frac{x^3}{3}\ (+c)\,]$ A1, Correct use of correct limits on their result **second** M1, Totally correct, unsimplified ± expression (may be implied by correct ans.) A1 $10\frac{2}{3}$ A1 [Allow this if, having given - $10\frac{2}{3}$, they correct it] M1 *for correct use of correct limits*: Must substitute correct limits for their strategy into a changed expression and subtract, either way round, e.g $\pm\{[\ ]^4 - [\ ]_0\}$ If a long method is used, e,g, finding three areas, this mark only gained for correct strategy and all limits need to be correct for this strategy. **Final M1: limits for area under curve and triangle must be the same.** S.C.(1) $\int_0^6(6x-x^2)dx - \int_0^6 2x\,dx = \left[3x^2 - \frac{x^3}{3}\right]_0^6 - \left[x^2\right]_0^6 = \ldots$ award M1A1 MO(limits)AO(triangle)M1(bod)A0 (2) If, having found ± correct answer, thinks this is not complete strategy and does more, do not award final 2 A marks Use of trapezium rule: M0A0MA0possibleA1for triangle M1(if correct application of trap. rule from $x = 0$ to $x = 4$) A0	

Q12.

Question Number	Scheme	Marks
(a)	$\frac{dy}{dx} = 12x^2 + 18x - 30$	M1
	Either: Substitute $x = 1$ to give $\frac{dy}{dx} = 12 + 18 - 30 = 0$ **Or:** Solve $\frac{dy}{dx} = 12x^2 + 18x - 30 = 0$ to give $x =$	A1
	Either: So turning point (all correct work so far) **Or:** Deduce $x = 1$ from correct work	A1cso (3)
(b) Way 1	When $x = 1$, $y = 4 + 9 - 30 - 8 = -25$	B1
	Area of triangle $ABP = \frac{1}{2} \times 1 \times 25 = 12.5$ (Where P is at (1, 0))	B1
	Way 1: $\int (4x^3 + 9x^2 - 30x - 8)\,dx = x^4 + \frac{9}{3}x^3 - \frac{30x^2}{2} - 8x \{+c\}$ *or* $x^4 + 3x^3 - 15x^2 - 8x \{+c\}$	M1A1
	$\left[x^4 + 3x^3 - 15x^2 - 8x\right]_{-\frac{1}{4}}^{1} = (1 + 3 - 15 - 8) - \left(\left(-\frac{1}{4}\right)^4 + 3\left(-\frac{1}{4}\right)^3 - 15\left(-\frac{1}{4}\right)^2 - 8\left(-\frac{1}{4}\right)\right)$ $= (-19) - \frac{261}{256}$ or $-19 - 1.02$	dM1
	So Area $=$ "*their* 12.5" + "*their* $20\frac{5}{256}$" or "12.5" + "20.02" or "12.5" + "*their* $\frac{5125}{256}$"	ddM1
	$= 32.52$ (NOT -32.52)	A1 (7) **[10]**
	Less efficient alternative methods for first two marks in part (b) with Way 1 or 2 For first mark: Finding equation of the line AB as $y = 25x - 50$ as this implies the -25	B1
	For second mark: Integrating to find triangle area $\int_1^2 (25x - 50)\,dx = \left[\frac{25}{2}x^2 - 50x\right]_1^2 = -50 + 37.5 = -12.5$ so area is 12.5 Then mark as before if they use Method in original scheme	B1

(b) Way 2	**Way 2:** Those who use area for **original curve between -1/4 and 2** and subtract area **between line and curve between 1 and 2** have a correct (long) method.	
	The first B1 (if y=−25 is not seen) is for equation of straight line $y = 25x - 50$	B1
	The second B1 may be implied by final answer correct, or 4.5 seen for area of "segment shaped" region between line and curve, or by area between line and axis/triangle found as 12.5	B1
	$\int (4x^3 + 9x^2 - 55x + 42)\,dx = x^4 + \frac{9}{3}x^3 - \frac{55x^2}{2} + 42x \{+c\}$ (or integration as in Way 1)	M1A1
	The dM1 is for correct use of the different correct limits for each of the two areas: i.e. $\left[x^4 + 3x^3 - 15x^2 - 8x\right]_{-\frac{1}{4}}^{2} = (16 + 24 - 60 - 16) - \left(\left(-\frac{1}{4}\right)^4 + 3\left(-\frac{1}{4}\right)^3 - 15\left(-\frac{1}{4}\right)^2 - 8\left(-\frac{1}{4}\right)\right)$ **And** $\left[x^4 + \frac{9}{3}x^3 - \frac{55x^2}{2} + 42x\right]_1^2 = 16 + 24 - 110 + 84 - (1 + 3 - 27.5 + 42)$	dM1
	So Area =their $\left[x^4 + 3x^3 - 15x^2 - 8x\right]_{-\frac{1}{4}}^{2}$ **minus their** $\left[x^4 + \frac{9}{3}x^3 - \frac{55x^2}{2} + 42x\right]_1^2$ i.e. "*their* 37.0195" − "*their* 4.5" (with both sets of limits correct for the integral)	ddM1
	Reaching $= 32.52$ (NOT -32.52) See over for special case with wrong limits	A1
	NB: Those who attempt curve − line wrongly with limits −1/4 to 2 may earn M1A1 for correct integration of their cubic. Usually e.g. $\int (4x^3 + 9x^2 - 55x + 42)\,dx = x^4 + \frac{9}{3}x^3 - \frac{55x^2}{2} + 42x \{+c\}$ (They will not earn any of the last 3 marks) They may also get first B1 mark for the correct equation of the straight line (usually seen but may be implied by correct line −curve equation) and second B1 if they also use limits 1 and 2 to obtain 4.5 (or find the triangle area 12.5).	M1A1