Question 1

Worked Solution

Motorcyclist: $v = 7.2t - 0.45t^2$ $\mathrm{m\,s^{-1}}$ for $0 \le t \le T$, where acceleration $= 0$ at $t = T$. Constant speed 28.8 $\mathrm{m\,s^{-1}}$ for $t \ge T$.

Part (i): Value of T

$a = \frac{dv}{dt} = 7.2 - 0.9t$. Setting $a = 0$:

$$7.2 - 0.9T = 0 \implies T = 8 \text{ s}$$

$T = 8$ s

Part (ii): Show $v = 28.8$ when $t = T$

$v(8) = 7.2(8) - 0.45(64) = 57.6 - 28.8 = 28.8$ $\mathrm{m\,s^{-1}}$ ✓

Part (iii): Displacement from O when $t = 31$

Phase 1 ($0 \le t \le 8$): $s_1 = \int_0^8 (7.2t - 0.45t^2)\,dt = [3.6t^2 - 0.15t^3]_0^8 = 3.6(64) - 0.15(512) = 230.4 - 76.8 = 153.6$ m.

Phase 2 ($8 \le t \le 31$): constant speed 28.8 $\mathrm{m\,s^{-1}}$ for $(31 - 8) = 23$ s. $s_2 = 28.8 \times 23 = 662.4$ m.

Total displacement $= 153.6 + 662.4 = 816$ m.

Displacement $= 816$ m

Question 2

Worked Solution

Particle starts from rest at A at $t = 0$. For $0 \leq t \leq 4$: $a = 1.8t$ m s^{-2}. For $4 \leq t \leq 7$: $a = 7.2$ m s^{-2}.

Part (i): Velocity for $0 \leq t \leq 4$

$v = \int 1.8t\,dt = 0.9t^2 + C$. At $t = 0$, $v = 0$: $C = 0$.

$$v = 0.9t^2 \text{ m s}^{-1}$$

$v = 0.9t^2$ m s^{-1}

Part (ii): Show displacement from A is 19.2 m when $t = 4$

$s = \int_0^4 0.9t^2\,dt = [0.3t^3]_0^4 = 0.3(64) = 19.2$ m ✓

Part (iii): Displacement from A when $t = 7$

At $t = 4$: $v = 0.9(16) = 14.4$ m s^{-1}, $s = 19.2$ m.

For $4 \leq t \leq 7$: $a = 7.2$, $u = 14.4$, elapsed time = 3 s.

$$s_{4\to7} = 14.4(3) + \frac{1}{2}(7.2)(9) = 43.2 + 32.4 = 75.6 \text{ m}$$

Total: $19.2 + 75.6 = 94.8$ m.

Displacement = 94.8 m

Question 3

Worked Solution

$s = 0.001t^4 - 0.04t^3 + 0.6t^2$ for $0 \le t \le 10$.

Part (i): Show velocity is 4 $\mathbf{m\,s^{-1}}$ when $t = 10$

$v = \frac{ds}{dt} = 0.004t^3 - 0.12t^2 + 1.2t$

$v(10) = 0.004(1000) - 0.12(100) + 1.2(10) = 4 - 12 + 12 = 4\ \mathrm{m\,s^{-1}}$ ✓

Part (ii): Show velocity is zero when $t = 20$ (acceleration for $t \ge 10$: $a = (0.8 - 0.08t)$ $\mathbf{m\,s^{-2}}$)

At $t = 10$: $v = 4\ \mathrm{m\,s^{-1}}$.

For $t \ge 10$: $v = \int (0.8 - 0.08t)\,dt = 0.8t - 0.04t^2 + C$.

Using $v(10) = 4$: $4 = 8 - 4 + C \implies C = 0$.

So $v = 0.8t - 0.04t^2 = 0.04t(20 - t)$.

At $t = 20$: $v = 0.04(20)(0) = 0$ ✓

Part (iii): Displacement from A when $t = 20$

$s(10) = 0.001(10000) - 0.04(1000) + 0.6(100) = 10 - 40 + 60 = 30$ m.

For $10 \le t \le 20$: $s = \int v\,dt = 0.4t^2 - \frac{0.04t^3}{3} + C_1$.

At $t = 10$, $s = 30$: $C_1 = 30 - 40 + \frac{40}{3} = -10 + \frac{40}{3} = \frac{10}{3}$.

$s(20) = 0.4(400) - \frac{0.04(8000)}{3} + \frac{10}{3} = 160 - \frac{320}{3} + \frac{10}{3} = 160 - \frac{310}{3} = \frac{480-310}{3} = \frac{170}{3} \approx 56.7$ m.

Displacement from A at $t = 20$ is ≈ 56.7 m

Question 4

Worked Solution

$v = t^2 - 9$. Particle passes through fixed point O when $t = 2$.

Part (i): Displacement from O when $t = 0$

$s = \int v\,dt = \frac{t^3}{3} - 9t + C$.

At $t = 2$, $s = 0$: $0 = \frac{8}{3} - 18 + C \implies C = 18 - \frac{8}{3} = \frac{46}{3}$.

At $t = 0$: $s = \frac{46}{3}$ m.

> Displacement $= \dfrac{46}{3} \approx 15.3$ m (in positive direction from O)

Part (ii): Distance from $t = 0$ until direction changes

Direction changes when $v = 0$: $t^2 = 9 \implies t = 3$ (taking $t > 0$; $t = 0$ is initial instant not a change of direction in the motion as $v(0) = -9 < 0$, so the particle is always moving in the negative direction from $t = 0$ until $t = 3$).

At $t = 0$: $s = \frac{46}{3}$. At $t = 3$: $s = 9 - 27 + \frac{46}{3} = -18 + \frac{46}{3} = \frac{-54+46}{3} = -\frac{8}{3}$.

Distance $= \frac{46}{3} - (-\frac{8}{3}) = \frac{54}{3} = 18$ m.

> Distance $= 18$ m

Part (iii): Distance from O when acceleration is 10 m s^{-2}

$a = \frac{dv}{dt} = 2t$. Setting $a = 10$: $t = 5$.

$s(5) = \frac{125}{3} - 45 + \frac{46}{3} = \frac{171}{3} - 45 = 57 - 45 = 12$ m.

> Distance from $O = 12$ m

Question 5

Worked Solution

$v = 0.6t^2 + 3$ m s^{-1}. Passes through fixed point A at $t = 0$.

Part (i): Velocity when passing through A

At $t = 0$: $v = 0 + 3 = 3$ m s^{-1}.

Velocity at A = 3 m s^{-1}

Part (ii): Displacement from A when $t = 1.5$

$x = \int_0^{1.5} (0.6t^2 + 3)\,dt = [0.2t^3 + 3t]_0^{1.5} = 0.2(3.375) + 4.5 = 0.675 + 4.5 = 5.175$ m.

Displacement = 5.175 m

Part (iii): Velocity when acceleration is 6 m s^{-2}

$a = \frac{dv}{dt} = 1.2t$. Setting $a = 6$: $t = 5$.

$v(5) = 0.6(25) + 3 = 15 + 3 = 18$ m s^{-1}.

Velocity = 18 m s^{-1}

Question 6

Worked Solution

Particle P passes through O at $t = 0$ with velocity $2\ \mathrm{m\,s^{-1}}$. Acceleration for t seconds after passing O: $a = (4 + 12t)\ \mathrm{m\,s^{-2}}$.

Part (i): Velocity when $t = 3$

$v = \int (4 + 12t)\,dt = 4t + 6t^2 + C$. At $t = 0$, $v = 2$: $C = 2$.

$v(3) = 12 + 54 + 2 = 68\ \mathrm{m\,s^{-1}}$.

Velocity at $t = 3$ is $68\ \mathrm{m\,s^{-1}}$

Part (ii): Distance OP when $t = 3$

$s = \int (4t + 6t^2 + 2)\,dt = 2t^2 + 2t^3 + 2t + K$. At $t = 0$, $s = 0$: $K = 0$.

$s(3) = 18 + 54 + 6 = 78$ m.

Distance $OP = 78$ m

Question 7

Worked Solution

Two particles A and B on same straight line, both at point S when $t = 0$.

Particle A: initially at rest, acceleration $0.18t$ m s^{-2}.

Particle B: initial velocity U m s^{-1}, distance from S for $0 \leq t \leq 5$: $(Ut + 0.08t^3)$ m, then constant velocity 9 m s^{-1} from $t = 5$ to $t = 15$, then decelerates for 1 s to collide with A at $t = 16$.

Part (i): Time when A and B have same velocity

Velocity of A: $v_A = \int 0.18t\,dt = 0.09t^2$.

Velocity of B (for $0 \leq t \leq 5$): $v_B = \frac{d}{dt}(Ut + 0.08t^3) = U + 0.24t^2$.

Set equal: $0.09t^2 = U + 0.24t^2 \implies (0.09 - 0.24)t^2 = U \implies -0.15t^2 = U$.

Since $U > 0$, they cannot be equal for $0 \leq t \leq 5$. For $t > 5$, $v_B = 9$ m s^{-1} and $v_A = 0.09t^2$.

$0.09t^2 = 9 \implies t^2 = 100 \implies t = 10$ s.

Same velocity at $t = 10$ s

Part (ii): Value of U and verify B is 25 m from S at $t = 5$

At $t = 5$, $v_B = 9$ m s^{-1} (transitions to constant velocity):

$$9 = U + 0.24(25) = U + 6 \implies U = 3 \text{ m s}^{-1}$$

Distance of B at $t = 5$: $3(5) + 0.08(125) = 15 + 10 = 25$ m ✓

$U = 3$ m s^{-1}; B is 25 m from S at $t = 5$ (verified)

Part (iii): Velocity of B when $t = 16$

At $t = 15$, B has travelled: $25 + 9(10) = 115$ m from S. A has travelled $x_A(16)$:

$x_A = \int_0^{16} 0.09t^2\,dt = 0.03t^3\big|_0^{16} = 0.03(4096) = 122.88$ m from S.

B decelerates uniformly from $t = 15$ to $t = 16$ (1 s), covering $122.88 - 115 = 7.88$ m.

Using $s = \frac{1}{2}(u + v)t$ where $u = 9$, $s = 7.88$, $t = 1$:

$$7.88 = \frac{1}{2}(9 + v)(1) \implies v = 2(7.88) - 9 = 15.76 - 9 = 6.76 \text{ m s}^{-1}$$

Velocity of B at $t = 16$ is 6.76 m s^{-1}

Question 8

Worked Solution

Toy car at A (3 m from O) at $t = 0$. $v = 2 + 12t - 3t^2$ m s^{-1}.

Find distance from O when acceleration is zero.

$a = \frac{dv}{dt} = 12 - 6t = 0 \implies t = 2$ s.

$x = 3 + \int_0^2 (2 + 12t - 3t^2)\,dt = 3 + [2t + 6t^2 - t^3]_0^2 = 3 + (4 + 24 - 8) = 3 + 20 = 23$ m.

Distance from O = 23 m

End of Worked Solutions