Kinematics with Variable Acceleration MS (From OCR 4728)

Q1, (Jan 2006, Q3)

(i)	$a = 7.2 - 0.9t$	M1 A1		For using $a = dv/dt$
	$T = 8$ See also special case in appendix.	M1 A1	[4]	For attempting to solve $a(t) = 0$
(ii)	$v(T) = 28.8$ See also special case in appendix.	B1	[1]	AG (From $7.2\times 8 - 0.45\times 8^2$)
(iii)	$s = 3.6t^2 - 0.15t^3 \quad (+C)$	M1 A1		For using $s = \int v dt$
	$s = 153.6\ (+C)$	DM1 A1		For finding $s(T$ or 31) or using limits (0) to T or (0) to 31 (dep on integration)
	s at constant speed = 662.4	B1ft		Condone $+C$
	Displacement is 816 m	A1ft	[6]	For using (31 – cv T) $\times 28.8$ cv 153.6 + cv 662.4 (non-zero numerical)

Q2, (Jan 2007, Q5)

(i)	$1.8t^2/2 \quad (+C)$	M*1		For using $v = \int a dt$
	(t = 0,v = 0) C = 0	B1		May be awarded in (ii). Accept c written and deleted.
	Expression is $1.8t^2/2$	A1	3	also for $1.8t^2 + c$
(ii)		M1		For using $s = \int v dt$
	$0.9t^3/3 \quad (+K)$	A1		**SR** Award B1 for (s = 0, t = 0) K = 0 if not already given in (i), or +K included and limits used.
	0.3 x 64	M1		For using limits 0 to 4 (or equivalent)
	19.2m AG	A1	4	
(iii)	$u = 0.9 \times 4^2$	D*		For using 'u' = v(4)
		M1		
		M1		For using $s = ut + \frac{1}{2} \times 7.2t^2$ with non-zero u
	$s = 14.4 \times 3 + \frac{1}{2}\ 7.2 \times 3^2$	A1		(s = 75.6)
	19.2 + 75.6	M1		For adding distances for the two distinct stages
	Displacement is 94.8m	A1	5	
	OR			
	$v = \int 7.2 dt$	D*		For finding v(4)
	t = 0, v = 14.4, c = 14.4	M1		Integration and finding non-zero integration constant Nb Using t=4, v=14.4 gives c = -14.4
	$s = \int 7.2t + 14.4 dt$			
	t = 0, s = 0, k = 0			Integration and finding integration constant. Nb t=4 with s=19.2 and v=7.2t-14.4 gives k=19.2
		M1		Substituting t = 3 (OR 7 into $s = 3.6t^2 - 14.4t + 19.2$)
	$s=3.6x3^2+14.4x3$	A1		(s=75.6) (OR $s = 3.6 \times 7^2 - 14.4x7 + 19.2$)
	19.2 + 75.6 = 94.8	M1		Adding two distinct stages OR $s = 3.6 \times 7^2 - 14.4x7 + 19.2 = 94.8$ final M1A1
	Displacement is 94.8m	A1		

Q3, (Jun 2007, Q6)

6(i)		M1	For differentiating s
	$v = 0.004t^3 - 0.12t^2 + 1.2t$	A1	Condone the inclusion of +c
	$v(10) = 4 - 12 + 12 = 4ms^{-1}$ (AG)	A1	Correct formula for v (no +c) and t=10
		[3]	stated sufficient
(ii)		M1	For integrating a
	$v = 0.8t - 0.04t^2$ (+ C)	A1	
	8 - 4 + C = 4	M1*	Only for using v(10) = 4 to find C
	$v = 0.8x20 - 0.04x20^2$ (+ C)	M1	
	v(20) = 16 – 16 = 0 (AG)	DA1	Dependant on M1*
		[5]	
(iii)		M1	For integrating v
	$S = 0.4t^2 - 0.04t^3/3$ (+K)	A1	Accept $0.4t^2 - 0.013t^3$ (+ ct +K, must be linear)
	s(10) = 10 – 40 + 60 = 30	B1	
		M1	For using S(10) = 30 to find K
	40 – 40/3 + K = 30 ➔ K = 10/3	A1	Not if S includes ct term
	S(20) = 160 – 320/3 + 10/3 = 56.7m	B1	Accept 56.6 to 56.7, Adding 30 subsequently is not isw,
	OR	[6]	hence B0
	s(10) = 10 – 40 + 60 = 30	B1	
		M1	For integrating v
	$S = 0.4t^2 - 0.04t^3/3$	A1	Accept $0.4t^2 - 0.013t^3$ (+ ct +K, must be linear)
		M1	Using limits of 10 and 20 (limits 0, 10 M0A0B0)
	S(20) - S(10) = 26.6, 26.7	A1	For 53.3 - 26.7 or better (Note S(10) = 26.7 is fortuitously correct M0A0B0)
	displacement is 56.7m	B1	Accept 56.6 to 56.7

Q4, (Jan 2011, Q6)

6 i	$x = \int t^2 - 9\ dt$	M1*	Uses integration of v(t)
	$x = t^3/3 - 9t$ (+c)	A1	Award if +c omitted
	Finds x(2)	D*M1	Allow + c or c omitted
	Displacement = 15⅓ m	B1	Accept 15.3, 46/3. Must be +ve
	OR	[4]	
	$x(2) = [t^3/3 - 9t]_0^2$	D*M1	Uses limits $[]_0^2$ on integrated x(t)
	Displacement = 15⅓ m	B1	Must be +ve
ii	t=0 s=0 or s=46/3 hence x(0) or c= 0 or 46/3	B1*	Needs explanation, may be seen in part i
	Solves $t^2 - 9 = 0$	M1*	May be implied
	t = (±)3	A1	Value of t when direction of motion changes
	$x(3) = 3^3/3 - 9x3$ (+ 15.3)	D*M1	Substitutes cv(t) >2 in integrated x(t)
	x(3) = -18 (or -2.67)	M1	Evaluates c – 18 may be implied award if ..
	Dist = 18 m	D*B1	Accept 18(.0) [c=0 assumed]
		[6]	
iii	$a = d(t^2 - 9)/dt$	M1*	Uses differentiation of v(t)
	a = 2t	A1	
	10 = 2t	D*M1	
	t = 5	A1	
	$x(5) (= 5^3/3 - 9x5 + 15.3) = 12$ m	A1	
	OR	[5]	
	$[t^3/3 - 9t]_2^5 = 12$ m	A1	

Q5, (Jun 2014, Q3)

(i)		$3\ \text{ms}^{-1}$	B1 [1]	
(ii)		$x = \int (0.6t^2 + 3)\,dt$ $x = 0.6t^3/3 + 3t\ (+ c)$ Substitutes 1.5 in expression for x $x(1.5) = 5.175$ m	M1* A1 D*M1 A1 [4]	Integrates v Accept with/without + c Needs integration and 2 terms in t Only without +c. Accept 5.17, 5.18
(iii)		$a = \text{d}(0.6t^2 + 3)/\text{d}t$ $6 = 2 \times 0.6t$ $v(5) = 18\ \text{ms}^{-1}$	M1* D*M1 A1 [3]	Differentiates v Plus attempt to solve $a(t) = 6$

Q6, (Jun 2015, Q6)

(i)		$v = \int 4 + 12t\ \text{dt}$ $v = 4t + 12t^2/2\ (+ c)$ $(t = 0, v = 2)\ c = 2$ and $v(3) = 4 \times 3 + 12 \times 3^2/2\ (+ 2)$ $v = 68\ \text{m s}^{-1}$	M1* A1 D*M1 A1 **[4]**	Integrates acceleration Award without (+ c) Evaluates constant
(ii)		$\int 4t + 6t^2 (+2)\text{d}t$ $x = 4t^2/2 + 6t^3/3 + 2t\ (+ d)$ $x(3) = 4 \times 3^2/2 + 6 \times 3^3/3\ (+ 3 \times 2)$ $x = 78$ m	M1* A1ft D*M1 A1 **[4]**	Integrates velocity accept omission of d for all subsequent marks
(iii)		$k = 132$	B1ft **[1]**	ft cv(78) + 54
(iv)		$v = \text{d}(k - 2t^3)/\text{d}t$ $v = -2 \times 3t^2$ $v(3) = -6 \times 3^2\ (= -54)$ $68m - 54m = 2mv$ $v = 7\ \text{m s}^{-1}$	M1* A1 D*M1 M1 A1 **[5]**	Differentiates displacement Award even if k wrong earlier Substitutes $t = 3$ Conservation of momentum, must have $2m$, cv(68)

Q7, (Jun 16, Q7)

i		A: $v=\int 0.18t\ dt$ $v=0.18/2\ t^2$ (+c) $9=0.09t^2$ $t = 10$	**M1*** **A1** **D*M1** **A1** **[4]**
ii		B: $v = d(Ut+0.08t^3)/dt$ $v = U+0.24t^2$ $9=U+0.24x5^2$ $U = 3$ $SB(5)= 3x5+0.08x5^3$ $SB(5) = 25$ m AG	**M1*** **D*M1** **A1** **A1** **[4]**
iii		A: $x=\int 0.09t^2 dt$ $x=0.09t^3/3$ $x(16)=0.03x16^3$ x=122.88 (may be implied by later work) $122.88=25+10x9+(9+v)(x1)/2$ $v=6.76$ m s^{-1} *OR* $122.88-25-10x9 =9x1+/-ax1^2/2$ Deceleration = 2.24 m s^{-2} $v = 9 – 2.24x1$ $v = 6.76$ m s^{-1}	**M1*** **D*M1** **A1** **M1** **A1** **[5]** **M1** **A1**

Q8, (OCR 4761, Jan 2008, Q5)

$a = 12 - 6t$	M1 A1	Differentiation, at least one term correct.
a = 0 gives t = 2	F1	Follow **their** a
$x = \int (2 + 12t - 3t^2)\mathrm{d}x$	M1	Integration indefinite or definite, at least one term correct.
$2t + 6t^2 - t^3 + C$	A1	Correct. Need not be simplified. Allow as definite integral. Ignore C or limits
x = 3 when t = 0	M1	Allow x = ± 3 or argue it is $\int_0^2$ from A then ± 3
so $3 = C$ and $x = 2t + 6t^2 - t^3 + 3$	A1	Award if seen WWW or $x = 2t + 6t^2 - t^3$ seen with +3 added later.
$x(2) = 4 + 24 - 8 + 3 = 23$ m	B1	FT **their** t and **their** x if obtained by integration but not if -3 obtained instead of +3. [If 20 m seen WWW for displacement award SC6] [Award SC1 for position if constant acceleration used for displacement and then +3 applied]
	8	