Kinematics With Variable Acceleration (Sheet 2)

Q1.
A fixed point O lies on a straight line.

A particle P moves along the straight line.

At time t seconds, $t \geq 0$, the distance, s metres, of P from O is given by

$$s = \frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t$$

(a) Find the acceleration of P at each of the times when P is at instantaneous rest.

(6)

(b) Find the total distance travelled by P in the interval $0 \leq t \leq 4$

(3)

(Total for question = 9 marks)

Q2.
A bird leaves its nest at time $t = 0$ for a short flight along a straight line.

The bird then returns to its nest.

The bird is modelled as a particle moving in a straight horizontal line.

The distance, s metres, of the bird from its nest at time t seconds is given by

$$s = \frac{1}{10}(t^4 - 20t^3 + 100t^2), \text{ where } 0 \leqslant t \leqslant 10$$

(a) Explain the restriction, $0 \leq t \leq 10$

(3)

(b) Find the distance of the bird from the nest when the bird first comes to instantaneous rest.

(6)

(Total for question = 9 marks)

Q3.
A particle P moves on the positive x–axis. The velocity of P at time t seconds is $(2t^2 - 9t + 4)$ m s^{-1}. When $t = 0$, P is 15 m from the origin O.

Find

(a) the values of t when P is instantaneously at rest,

(3)

(b) the acceleration of P when $t = 5$

(3)

(c) the total distance travelled by P in the interval $0 \leq t \leq 5$

(5)

(Total for question = 11 marks)

Q4.

A particle P moves along a straight line in such a way that at time t seconds its velocity v m s^{-1} is given by

$$v = \frac{1}{2}t^2 - 3t + 4$$

Find

(a) the times when P is at rest,

(4)

(b) the total distance travelled by P between $t = 0$ and $t = 4$.

(5)

(Total 9 marks)

Q5.

A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where

$$v = 2t^2 - 14t + 20, \qquad t \geq 0$$

Find

(a) the times when P is instantaneously at rest,

(3)

(b) the greatest speed of P in the interval $0 \leq t \leq 4$

(5)

(c) the total distance travelled by P in the interval $0 \leq t \leq 4$

(5)

(Total 13 marks)

Q6.

A particle moves along the x-axis. At time $t = 0$ the particle passes through the origin with speed 8 m s^{-1} in the positive x-direction. The acceleration of the particle at time t seconds, $t\geqslant$ is $(4t^3 - 12t)$ m s^{-2} in the positive x-direction.

Find

(a) the velocity of the particle at time t seconds,

(3)

(b) the displacement of the particle from the origin at time t seconds,

(2)

(c) the values of t at which the particle is instantaneously at rest.

(3)

(Total 8 marks)

Q7.

A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where v is given by

$$v = \begin{cases} 8t - \frac{3}{2}t^2, & 0 \leqslant t \leqslant 4, \\ 16 - 2t, & t > 4. \end{cases}$$

When $t = 0$, P is at the origin O.

Find

(a) the greatest speed of P in the interval $0 \leq t \leq 4$,

(4)

(b) the distance of P from O when $t = 4$,

(3)

(c) the time at which P is instantaneously at rest for $t > 4$,

(1)

(d) the total distance travelled by P in the first 10 s of its motion.

(8)

(Total 16 marks)

Q8.

A particle P moves along the x-axis in a straight line so that, at time t seconds, the velocity of P is v m s^{-1}, where

$$v = \begin{cases} 10t - 2t^2, & 0 \leqslant t \leqslant 6, \\ \frac{-432}{t^2}, & t > 6. \end{cases}$$

At $t = 0$, P is at the origin O. Find the displacement of P from O when

(a) $t = 6$,

(3)

(b) $t = 10$.

(5)

(Total 8 marks)

Q9.

A particle P moves along the x-axis. At time t seconds the velocity of P is v m s^{-1} in the positive x-direction, where $v = 3t^2 - 4t + 3$. When $t = 0$, P is at the origin O. Find the distance of P from O when P is moving with minimum velocity.

(8)

(Total 8 marks)

Q10.

A particle P moves along a straight line.

At time t seconds, the velocity v m s^{-1} of P is modelled as

$$v = 10t - t^2 - k \qquad t \geq 0$$

where k is a constant.

(a) Find the acceleration of P at time t seconds.

(2)

The particle P is instantaneously at rest when $t = 6$

(b) Find the other value of t when P is instantaneously at rest.

(4)

(c) Find the total distance travelled by P in the interval $0 \leq t \leq 6$

(4)

(Total for question = 10 marks)