Question 1

Worked Solution

$s = \frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t$, $t \geq 0$.

Part (a): Acceleration at each instant of rest

Velocity: $v = \frac{ds}{dt} = t^2 - 5t + 6 = (t-2)(t-3)$

$v = 0$ when $t = 2$ or $t = 3$.

Acceleration: $a = \frac{dv}{dt} = 2t - 5$

At $t = 2$: $a = 4 - 5 = -1 \text{ m s}^{-2}$

At $t = 3$: $a = 6 - 5 = 1 \text{ m s}^{-2}$

At $t = 2$: $a = -1 \text{ m s}^{-2}$; at $t = 3$: $a = 1 \text{ m s}^{-2}$

Part (b): Total distance in $0 \leq t \leq 4$

Positions:

$$s(0) = 0,$$
$$s(2) = \frac{8}{3} - 10 + 12 = \frac{8}{3} + 2 = \frac{14}{3},$$
$$s(3) = 9 - \frac{45}{2} + 18 = 27 - \frac{45}{2} = \frac{9}{2},$$
$$s(4) = \frac{64}{3} - 40 + 24 = \frac{64}{3} - 16 = \frac{16}{3}$$

P moves from $s = 0$ to $s = \frac{14}{3}$ (forward), then back to $s = \frac{9}{2}$ (backward), then forward to $s = \frac{16}{3}$.

Total distance:

$$= \frac{14}{3} + \left(\frac{14}{3} - \frac{9}{2}\right) + \left(\frac{16}{3} - \frac{9}{2}\right) = \frac{14}{3} + \frac{28-27}{6} + \frac{32-27}{6} = \frac{14}{3} + \frac{1}{6} + \frac{5}{6} = \frac{14}{3} + 1 = \frac{17}{3} \text{ m}$$

Total distance $= \dfrac{17}{3} = 5\frac{2}{3}$ m

Question 2

Worked Solution

$s = \frac{1}{10}(t^4 - 20t^3 + 100t^2)$, $0 \le t \le 10$.

Part (a): Explain restriction $0 \le t \le 10$

At $t = 0$: $s = 0$ (bird at nest). At $t = 10$: $s = \frac{1}{10}(10000 - 20000 + 10000) = 0$ (bird back at nest).

Note that $s = \frac{1}{10}t^2(t-10)^2 \ge 0$ for all t in $[0, 10]$, confirming the bird stays on the positive side of the nest.

The restriction $0 \le t \le 10$ captures the full flight from nest departure ($t = 0$) to return ($t = 10$); outside this range the model is not valid.

Part (b): Distance from nest when bird first at instantaneous rest

Velocity: $v = \frac{ds}{dt} = \frac{1}{10}(4t^3 - 60t^2 + 200t) = \frac{2t}{5}(t-5)(t-10)$

$v = 0$ at $t = 0, 5, 10$. First rest after $t = 0$ is at $t = 5$.

$s(5) = \frac{1}{10}(625 - 2500 + 2500) = \frac{625}{10} = 62.5$ m.

Distance from nest $= 62.5$ m

Question 3

Worked Solution

$v = 2t^2 - 9t + 4$, $s(0) = 15$ m.

Part (a): Times when P is at rest

$v = 0$: $2t^2 - 9t + 4 = (2t-1)(t-4) = 0$

$t = \frac{1}{2}$ or $t = 4$.

$t = \frac{1}{2}$ s and $t = 4$ s

Part (b): Acceleration when $t = 5$

$a = \frac{dv}{dt} = 4t - 9$. At $t = 5$: $a = 20 - 9 = 11\ \text{m s}^{-2}$.

Acceleration at $t = 5$ is $11\ \text{m s}^{-2}$

Part (c): Total distance in $0 \le t \le 5$

$s = \int v\,dt = \frac{2t^3}{3} - \frac{9t^2}{2} + 4t + C$. With $s(0) = 15$: $C = 15$.

Positions:

$$s\left(\tfrac{1}{2}\right) = \frac{2}{24} - \frac{9}{8} + 2 + 15 = \frac{1}{12} - \frac{9}{8} + 17 = \frac{2}{24} - \frac{27}{24} + \frac{408}{24} = \frac{383}{24}$$

$$s(4) = \frac{128}{3} - 72 + 16 + 15 = \frac{128}{3} - 41 = \frac{128 - 123}{3} = \frac{5}{3}$$

Wait: $s(4) = \frac{2(64)}{3} - \frac{9(16)}{2} + 4(4) + 15 = \frac{128}{3} - 72 + 16 + 15 = \frac{128}{3} - 41 = \frac{128-123}{3} = \frac{5}{3}$.

$s(5) = \frac{250}{3} - \frac{225}{2} + 20 + 15 = \frac{250}{3} - \frac{225}{2} + 35$.

$= \frac{500}{6} - \frac{675}{6} + \frac{210}{6} = \frac{35}{6}$.

Velocity is positive for $0 < t < \frac{1}{2}$, negative for $\frac{1}{2} < t < 4$, positive for $t > 4$.

Total distance:

$$= \left[s\left(\tfrac{1}{2}\right) - s(0)\right] + \left[s\left(\tfrac{1}{2}\right) - s(4)\right] + [s(5) - s(4)]$$

$s(0) = 15$, $s\left(\frac{1}{2}\right) = \frac{383}{24}$, $s(4) = \frac{5}{3}$, $s(5) = \frac{35}{6}$.

$s\left(\frac{1}{2}\right) - s(0) = \frac{383}{24} - 15 = \frac{383-360}{24} = \frac{23}{24}$

$s\left(\frac{1}{2}\right) - s(4) = \frac{383}{24} - \frac{40}{24} = \frac{343}{24}$

$s(5) - s(4) = \frac{35}{6} - \frac{5}{3} = \frac{35}{6} - \frac{10}{6} = \frac{25}{6} = \frac{100}{24}$

Total $= \frac{23}{24} + \frac{343}{24} + \frac{100}{24} = \frac{466}{24} = \frac{233}{12} \approx 19.4$ m.

Total distance $= \dfrac{233}{12} \approx 19.4$ m

Question 4

Worked Solution

$v = \frac{1}{2}t^2 - 3t + 4.$

Part (a): Times when P is at rest

$\frac{1}{2}t^2 - 3t + 4 = 0 \implies t^2 - 6t + 8 = (t-2)(t-4) = 0$

$t = 2$ s or $t = 4$ s.

$t = 2$ s and $t = 4$ s

Part (b): Total distance from $t = 0$ to $t = 4$

$s = \int v\, dt = \frac{t^3}{6} - \frac{3t^2}{2} + 4t$

Velocity is positive for $0 < t < 2$, negative for $2 < t < 4$.

$s(0) = 0$, $s(2) = \frac{8}{6} - 6 + 8 = \frac{4}{3} + 2 = \frac{10}{3}$, $s(4) = \frac{64}{6} - 24 + 16 = \frac{32}{3} - 8 = \frac{8}{3}$.

Total distance $= s(2) - s(0) + [s(2) - s(4)] = \frac{10}{3} + \left(\frac{10}{3} - \frac{8}{3}\right) = \frac{10}{3} + \frac{2}{3} = 4$ m.

Total distance $= 4$ m

Question 5

Worked Solution

$v = 2t^2 - 14t + 20$, $t \geq 0$.

Part (a): Times when P at rest

$2t^2 - 14t + 20 = 0 \implies t^2 - 7t + 10 = (t-2)(t-5) = 0$

$t = 2$ s or $t = 5$ s.

$t = 2$ s and $t = 5$ s

Part (b): Greatest speed in $0 \leq t \leq 4$

At $t = 0$: $v = 20$ m s^{-1}

Minimum of v occurs at $t = \frac{7}{2} = 3.5$: $v(3.5) = 2(12.25) - 49 + 20 = 24.5 - 49 + 20 = -4.5$ m s^{-1}

So speed $= 4.5$ at $t = 3.5$, and speed $= 20$ at $t = 0$.

At $t = 4$: $v = 32 - 56 + 20 = -4$ m s^{-1}, speed $= 4$.

Greatest speed is at $t = 0$: $|v| = 20$ m s^{-1}.

Greatest speed $= 20$ m s^{-1} (at $t = 0$)

Part (c): Total distance in $0 \leq t \leq 4$

$s = \int v\,dt = \frac{2t^3}{3} - 7t^2 + 20t$

$s(0) = 0$, $s(2) = \frac{16}{3} - 28 + 40 = \frac{16}{3} + 12 = \frac{52}{3}$, $s(4) = \frac{128}{3} - 112 + 80 = \frac{128}{3} - 32 = \frac{32}{3}$.

Velocity positive for $0 < t < 2$, negative for $2 < t < 4$.

Total distance $= s(2) - s(0) + [s(2) - s(4)] = \frac{52}{3} + \left(\frac{52}{3} - \frac{32}{3}\right) = \frac{52}{3} + \frac{20}{3} = \frac{72}{3} = 24$ m.

Total distance $= 24$ m

Question 6

Worked Solution

$a = 4t^3 - 12t$. At $t = 0$: speed $= 8\ \mathrm{m\,s^{-1}}$ (positive x-direction), position $= 0$.

Part (a): Velocity at time t

$v = \int a\,dt = t^4 - 6t^2 + C$. At $t = 0$, $v = 8$: $C = 8$.

$v = t^4 - 6t^2 + 8\ \mathrm{m\,s^{-1}}$.

$$v = t^4 - 6t^2 + 8\ \mathrm{m\,s^{-1}}$$

Part (b): Displacement from origin at time t

$s = \int v\,dt = \frac{t^5}{5} - 2t^3 + 8t$ (constant $= 0$ since at origin when $t = 0$).

$$s = \frac{t^5}{5} - 2t^3 + 8t\ \mathrm{m}$$

Part (c): Times when P at rest

$v = 0$: $t^4 - 6t^2 + 8 = 0 \implies (t^2 - 2)(t^2 - 4) = 0$

$t^2 = 2 \implies t = \sqrt{2}$ s; $\quad t^2 = 4 \implies t = 2$ s.

$$t = \sqrt{2}\ \text{s and } t = 2\ \text{s}$$

Question 7

Worked Solution

$$v = \begin{cases} 8t - \frac{3}{2}t^2, & 0 \le t \le 4 \\ 16 - 2t, & t > 4 \end{cases}$$

At $t = 0$, P at origin O.

Part (a): Greatest speed in $0 \le t \le 4$

$\frac{dv}{dt} = 8 - 3t = 0 \implies t = \frac{8}{3}$.

$v\left(\frac{8}{3}\right) = 8 \cdot \frac{8}{3} - \frac{3}{2} \cdot \frac{64}{9} = \frac{64}{3} - \frac{32}{3} = \frac{32}{3}$ m s^{-1}.

> Greatest speed in $[0, 4] = \dfrac{32}{3}$ m s^{-1}

Part (b): Distance from O when $t = 4$

$s = \int_0^4 \left(8t - \frac{3}{2}t^2\right) dt = \left[4t^2 - \frac{t^3}{2}\right]_0^4 = 64 - 32 = 32$ m.

> Distance from O at $t = 4$ is 32 m

Part (c): Time at rest for $t > 4$

$16 - 2t = 0 \implies t = 8$ s.

> P at rest at $t = 8$ s

Part (d): Total distance in first 10 s

For $t > 4$: $s = \int (16 - 2t)\, dt = 16t - t^2 + C$.

At $t = 4$, $s = 32$ m: $32 = 64 - 16 + C \implies C = -16$.

So $s = 16t - t^2 - 16$.

At $t = 8$: $s = 128 - 64 - 16 = 48$ m (direction changes here). At $t = 10$: $s = 160 - 100 - 16 = 44$ m.

Distance from $t = 4$ to $t = 8$: $48 - 32 = 16$ m (forward). Distance from $t = 8$ to $t = 10$: $48 - 44 = 4$ m (backward).

Total distance $= 32 + 16 + 4 = 52$ m.

> Total distance in first 10 s $= 52$ m

Question 8

Worked Solution

$$v = \begin{cases} 10t - 2t^2, & 0 \le t \le 6 \\ -432/t^2, & t > 6 \end{cases}$$

At $t = 0$, P at origin O.

Part (a): Displacement when $t = 6$

$s = \int_0^6 (10t - 2t^2)\,dt = \left[5t^2 - \frac{2t^3}{3}\right]_0^6 = 180 - 144 = 36$ m.

Displacement at $t = 6$ is 36 m

Part (b): Displacement when $t = 10$

For $t > 6$: $s = \int \frac{-432}{t^2}\,dt = \frac{432}{t} + K$.

At $t = 6$, $s = 36$: $36 = \frac{432}{6} + K = 72 + K \implies K = -36$.

So $s = \frac{432}{t} - 36$.

At $t = 10$: $s = \frac{432}{10} - 36 = 43.2 - 36 = 7.2$ m.

Displacement at $t = 10$ is 7.2 m

Question 9

Worked Solution

$v = 3t^2 - 4t + 3$. At $t = 0$, P at origin O.

Minimum velocity when $\frac{dv}{dt} = 0$:

$$6t - 4 = 0 \implies t = \frac{2}{3}$$

(Second derivative = 6 ¿ 0, confirming minimum.)

$s = \int v\,dt = t^3 - 2t^2 + 3t + C$. At $t = 0$, $s = 0$: $C = 0$.

At $t = \frac{2}{3}$:

$$s = \left(\frac{2}{3}\right)^3 - 2\left(\frac{2}{3}\right)^2 + 3\left(\frac{2}{3}\right) = \frac{8}{27} - \frac{8}{9} + 2 = \frac{8}{27} - \frac{24}{27} + \frac{54}{27} = \frac{38}{27} \text{ m}$$

Distance from O when moving with minimum velocity $= \frac{38}{27}$ m

Question 10

Worked Solution

$v = 10t - t^2 - k$, $t \geq 0$. P at rest when $t = 6$.

Part (a): Acceleration at time t

$a = \frac{dv}{dt} = 10 - 2t$

$a = (10 - 2t)\ \mathrm{m\,s^{-2}}$

Part (b): Other value of t when P at rest

At $t = 6$: $v = 0 \implies 0 = 60 - 36 - k \implies k = 24$.

So $v = 10t - t^2 - 24 = -(t^2 - 10t + 24) = -(t-4)(t-6)$.

$v = 0$ at $t = 4$ or $t = 6$.

Other value: $t = 4$ s

Part (c): Total distance in $0 \leq t \leq 6$

$v = -(t-4)(t-6)$: negative for $0 \leq t < 4$, positive for $4 < t \leq 6$.

$s = \int v\,dt = 5t^2 - \frac{t^3}{3} - 24t$

$s(0) = 0$, $s(4) = 80 - \frac{64}{3} - 96 = -16 - \frac{64}{3} = -\frac{112}{3}$, $s(6) = 180 - 72 - 144 = -36$.

Total distance $= |s(4) - s(0)| + |s(6) - s(4)|$

$= \frac{112}{3} + \left(-36 - \left(-\frac{112}{3}\right)\right) = \frac{112}{3} + \frac{112}{3} - 36 = \frac{224}{3} - 36 = \frac{224-108}{3} = \frac{116}{3}$ m.

$$\text{Total distance} = \frac{116}{3}\ \text{m}$$

End of Worked Solutions