Kinematics With Variable Acceleration (Sheet 2)

Q1.

Question	Scheme	Marks	AOs
(a)	Differentiate s wrt t	M1	3.1a
	$(v=)\ t^2-5t+6$	A1	1.1b
	Equate their v to 0 and solve	M1	1.1b
	$t=2$ or 3	A1	1.1b
	$(a=)\ 2t-5$	B1ft	2.1
	$a=$ 1 and -1 (m s^{-2}) isw (A0 if extras)	A1	1.1b
		(6)	
(b)	Attempt to find values of s for $t=2$, 3 *and* 4 oe Correct values are $\left(s_2=\frac{14}{3}, s_3=\frac{9}{2}\ \textit{and}\ s_4=\frac{16}{3}\right)$ Could be implied by correct values for: s_2 , (s_3-s_2) and (s_4-s_3) which are $\frac{14}{3}$, $(-\frac{1}{6})$ and $\frac{5}{6}$	DM1	1.1b
	Total distance travelled $=s_2+(s_2-s_3)+s_4-s_3$ **OR** $s_2-(s_3-s_2)+s_4-s_3$ **OR** $\left[\frac{1}{3}t^3-\frac{5}{2}t^2+6t\right]_0^2-\left[\frac{1}{3}t^3-\frac{5}{2}t^2+6t\right]_2^3+\left[\frac{1}{3}t^3-\frac{5}{2}t^2+6t\right]_3^4$ **OR** $\frac{14}{3}-(-\frac{1}{6})+\frac{5}{6}$ **OR** $s_2+2(s_2-s_3)+s_4-s_2$ $(=2s_2-2s_3+s_4)$ oe	M1	2.1
	$5\frac{2}{3}$ oe (m) Accept 5.7 or better	A1	1.1b
		(3)	
			(9 marks)

Q2.

Question	Scheme	Marks	AOs
(a)	Substitution of both $t = 0$ and $t = 10$	M1	2.1
	$s = 0$ for both $t = 0$ and $t = 10$	A1	1.1b
	Explanation ($s > 0$ for $0 < t < 10$) since $s = \frac{1}{10}t^2(t-10)^2$	A1	2.4
		(3)	
(b)	Differentiate displacement s w.r.t. t to give velocity, v	M1	1.1a
	$v = \frac{1}{10}(4t^3 - 60t^2 + 200t)$	A1	1.1b
	Interpretation of 'rest' to give $v = \frac{1}{10}(4t^3 - 60t^2 + 200t) = \frac{2}{5}t(t-5)(t-10) = 0$	M1	1.1b
	$t = 0, 5, 10$	A1	1.1b
	Select $t = 5$ and substitute their $t = 5$ into s	M1	1.1a
	Distance = 62.5 m	A1 ft	1.1b
		(6)	
			(9 marks)

Notes

(a) M1 for substituting $t = 0$ and $t = 10$ into s expression
A1 for noting that $s = 0$ at both times
A1 Since s is a perfect square, $s > 0$ for all other t- values.
(b) 1st M1 for differentiating s w.r.t. t to give v (powers of t reducing by 1)
1st A1 for a correct v expression in any form
2nd M1 for equating v to 0 and factorising
2nd A1 for correct t values
3rd M1 for substituting their intermediate t value into s
3rd A1 ft following an incorrect t-value.

Q3.

Question Number	Scheme	Marks	
a	At rest when $v=0$: $\left(2t^2-9t+4\right)=0$	M1	
	$=(2t-1)(t-4)$,	DM1	Solve for t. Dependent on the previous M1
	$t=\frac{1}{2}$, 4	A1	Incorrect answers with no method shown score M0A0
		[3]	
b	$a=\frac{\mathrm{d}v}{\mathrm{d}t}=4t-9$	M1	Differentiate v to obtain a (at least one power of t going down)
		A1	Correct derivative
	$t=5$, $a=11\,(\mathrm{m\,s^{-2}})$	A1	
		[3]	
c	$s=\int v\mathrm{d}t=\frac{2}{3}t^3-\frac{9}{2}t^2+4t\,(+C)$	M1	Integrate v to obtain s (at least one power of t going up)
		A1	
	Use of $t=0$, $t=\frac{1}{2}$, $t=4$, $t=5$ (and $t=0, s=15$) as limits in integrals	DM1	Correct strategy for their limits - requires subtraction of the negative distance. Dependent on the previous M1 and at least one positive solution for t in (0,5) from (a)
	$\left[\frac{2}{3}t^3-\frac{9}{2}t^2+4t(+15)\right]_0^{\frac{1}{2}}$ $-\left[\frac{2}{3}t^3-\frac{9}{2}t^2+4t(+15)\right]_{\frac{1}{2}}^{4}$ $+\left[\frac{2}{3}t^3-\frac{9}{2}t^2+4t(+15)\right]_4^5$	A1	NB: $\int_0^5 v\mathrm{d}t$ scores M0A0A0

	$(0, \frac{23}{24}, -\frac{40}{3}, \frac{-55}{6})$ $=\frac{23}{24}+\frac{343}{24}+\frac{100}{24}=19.4$ (m) $\left(15,\ 15\frac{23}{24}\left(\frac{383}{24}\right),\ \frac{5}{3},\ 5.8\dot{3}\left(\frac{35}{6}\right)\right)$	A1 [5]	$19\frac{5}{12}\left(\frac{233}{12}\right)$ or better
		(11)	

Q4.

Question Number	Scheme	Marks	Notes
(a)	$\frac{1}{2}t^2-3t+4=0$	M1	Set $v=0$
	$t^2-6t+8=0$		
	$(t-2)(t-4)=0$	DM1	Solve for v
	$t=2$ s *or* 4 s	A1 A1	
		(4)	
(b)	$\int \frac{1}{2}t^2-3t+4\,dt$	M1	Integration – majority of powers increasing
	$=\frac{1}{6}t^3-\frac{3}{2}t^2+4t(+C)$	A1	Correct (+C not required)
	$s=\int_0^2 \frac{1}{2}t^2-3t+4\,dt-\int_2^4 \frac{1}{2}t^2-3t+4\,dt$	DM1	Correct strategy for finging the required distance. Follow their "2". Subtraction/swap limits/modulus signs
	$=\left[\frac{1}{6}t^3-\frac{3}{2}t^2+4t\right]_0^2-\left[\frac{1}{6}t^3-\frac{3}{2}t^2+4t\right]_2^4$		
	$=\frac{8}{6}-6+8-(\frac{64}{6}-24+16-(\frac{8}{6}-6+8))$	A1	Correct unsimplified
	$=\frac{10}{3}-\frac{8}{3}+\frac{10}{3}$		
	$=4$	A1	
		(5)	
		[9]	

Q5.

Question Number	Scheme	Marks	Notes
a	$v=0=2t^2-14t+20$ $=2\ t-2\ \ t-5$ $t=2$ or $t=5$	M1 M1 A1 [3]	Set $v=0$ Solve for t
	There are many different approaches to part (b). The allocation of the two M marks is M1: A method to find the time when the velocity is a minimum M1: Evaluate the speed at that time		
e.g. b	$t=0$, $v=20$ (m s^{-1}) $a=4t-14=0$ $t=\frac{7}{2}$, $v=2\times\frac{3}{2}\times\frac{-3}{2}=\frac{-9}{2}$ Max speed = 20 ms^{-1}	B1 M1 M1A1 A1 [5]	 Must see ±4.5 Clearly stated & correct conclusion. Depends on the two M marks. From correct solution only.
balt1	$t=0$, $v=20$ (m s^{-1}) Sketch with symmetry about their $t=3.5$ v(their 3.5) -4.5 Max speed = 20 ms^{-1}	B1 M1 M1 A1 A1 [5]	 Evaluate v at min. Correct work Clearly stated & correct conclusion. Depends on the two M marks. From correct solution only.
b alt 2	$t=0$, $v=20$ (m s^{-1}) Justification of minimum or tabulate sufficient values to confirm location Evaluate v at min. Correct work Correct conclusion. Depends on the two M marks	B1 M1 M1 A1 A1 [5]	 Clearly stated & from correct solution only.

c	$\int 2t^2 - 14t + 20\,dt = \frac{2}{3}t^3 - 7t^2 + 20t\,(+C)$	**M1 A1**
	Distance $= \left[\frac{2}{3}t^3 - 7t^2 + 20t\right]_0^2 - \left[\frac{2}{3}t^3 - 7t^2 + 20t\right]_2^4$	**M1 A1**
	$= 2 \times \left[\frac{2}{3}t^3 - 7t^2 + 20t\right]^2 - \left[\frac{2}{3}t^3 - 7t^2 + 20t\right]_4$	
	$= 2 \times \left[\frac{16}{3} - 7\times4 + 40\right] - \left[\frac{2\times64}{3} - 7\times16 + 80\right] = 24$ (m)	**A1** **(5)** **[13]**

Q6.

(a)	$a = 4t^3 - 12t$ Convincing attempt to integrate $v = t^4 - 6t^2\,(+c)$ Use initial condition to get $v = t^4 - 6t^2 + 8\,(\text{ms}^{-1})$.	M1 A1 A1 (3)
(b)	Convincing attempt to integrate $s = \frac{t^5}{5} - 2t^3 + 8t\,(+0)$ Integral of their v	M1 A1ft (2)
(c)	Set their $v = 0$ Solve a quadratic in t^2 $(t^2-2)(t^2-4) = 0 \Rightarrow$ at rest when $t = \sqrt{2},\ t = 2$	M1 DM1 A1 (3) **[8]**

Q7.

Question Number	Scheme	Marks
(a)	$0 \le t \le 4$: $a = 8 - 3t$ $a = 0 \Rightarrow t = 8/3$ s $\rightarrow v = 8.\frac{8}{3} - \frac{3}{2}\left(\frac{8}{3}\right)^2 = \frac{32}{3}$ (m/s) second M1 dependent on the first, and third dependent on the second.	M1 DM1 DM1 A1 (4)
(b)	$s = 4t^2 - t^3/2$ $t = 4$: $s = 64 - 64/2 = \underline{32 \text{ m}}$	M1 M1 A1 (3)
(c)	$t > 4$: $v = 0 \Rightarrow t = \underline{8 \text{ s}}$	B1 (1)
(d)	*Either* $t > 4$ $s = 16t - t^2$ $(+ C)$ $t = 4, s = 32 \rightarrow C = -16 \Rightarrow s = 16t - t^2 - 16$ $t = 10 \rightarrow s = 44$ m But direction changed, so: $t = 8, s = 48$ Hence total dist travelled $= 48 + 4 = \underline{52 \text{ m}}$ *Or (probably accompanied by a sketch?)* t=4 v=8, t=8 v=0, so area under line $= \frac{1}{2} \times (8-4) \times 8$ t=8 v=0, t=10 v=-4, so area above line $= \frac{1}{2} \times (10-8) \times 4$ $\therefore$ total distance = 32(from b) + 16 + 4 = $\underline{52 \text{ m}}$	M1 M1 A1 M1 A1 M1 DM1 A1 (8) M1A1A1 M1A1A1 M1A1 (8)

Q8.

(a)	$v = 10t - 2t^2$, $s = \int v\,dt$	M1
	$= 5t^2 - \frac{2t^3}{3}(+C)$	A1
	$t = 6 \Rightarrow s = 180 - 144 = \underline{36}$ (m)	A1 (3)
(b)	$\underline{s} = \int v\,dt = \frac{-432t^{-1}}{-1}(+K) = \frac{432}{t}(+K)$	B1
	$t = 6$, $s =$ "36" $\Rightarrow 36 = \frac{432}{6} + K$	M1*
	$\Rightarrow K = -36$	A1
	At $t = 10$, $s = \frac{432}{10} - 36 = \underline{7.2}$ (m)	d*M1 A1 (5) [8]

Q9.

Question Number	Scheme	Marks
	$\frac{dv}{dt} = 6t - 4$	M1 A1
	$6t - 4 = 0 \Rightarrow t = \frac{2}{3}$	M1 A1
	$s = \int 3t^2 - 4t + 3\,dt = t^3 - 2t^2 + 3t\,(+c)$	M1 A1
	$t = \frac{2}{3} \Rightarrow s = -\frac{16}{27} + 2$ so distance is $\frac{38}{27}$ m	M1 A1 [8]

Q10.

Question	Scheme	Marks	AOs
(a)	Differentiate v w.r.t. t	M1	3.1a
	$a=\frac{dv}{dt}=10-2t$ isw	A1	1.1b
		(2)	
(b)	Solve problem using $v=0$ when $t=6$	M1	3.1a
	$0=10t-t^2-24$	A1	1.1b
	Solve quadratic oe to find other value of t	M1	1.1b
	$t=4$	A1	1.1b
		(4)	
(c)	Integrate v or $-v$ w.r.t. t	M1	3.1a
	$5t^2-\frac{1}{3}t^3-24t$	A1	1.1b
	Total distance $=-\left[5t^2-\frac{1}{3}t^3-24t\right]_0^4+\left[5t^2-\frac{1}{3}t^3-24t\right]_4^6$	M1	2.1
	$\frac{116}{3}$ (m)	A1	1.1b
		(4)	
			(10 marks)