Question 1

Worked Solution

Object projected upward at $7\ \mathrm{m\,s^{-1}}$. $g = 9.8\ \mathrm{m\,s^{-2}}$.

Part (i): Speed when 2.1 m above projection point

Taking upward as positive: $u = 7$, $a = -9.8$, $s = 2.1$:

$$v^2 = 49 - 2(9.8)(2.1) = 49 - 41.16 = 7.84 \implies v = 2.8\ \mathrm{m\,s^{-1}}$$

Speed $= 2.8\ \mathrm{m\,s^{-1}}$

Part (ii): Greatest height above projection point

At greatest height $v = 0$:

$$0 = 49 - 2(9.8)s \implies s = \frac{49}{19.6} = 2.5\ \mathrm{m}$$

Greatest height $= 2.5$ m

Part (iii): Time when travelling downward at 5.7 $\mathrm{m\,s^{-1}}$

Travelling downward: $v = -5.7\ \mathrm{m\,s^{-1}}$:

$$-5.7 = 7 - 9.8t \implies t = \frac{12.7}{9.8} \approx 1.30\ \mathrm{s}$$

$t \approx 1.30$ s

Question 2

Worked Solution

Car of mass 900 kg: driving force 600 N, resisting force 240 N. $g = 9.8 \text{ m s}^{-2}$.

Part (i): Show acceleration is 0.4 m s^{-2}

Net force $= 600 - 240 = 360$ N. By Newton's second law:

$$a = \frac{F}{m} = \frac{360}{900} = 0.4 \text{ m s}^{-2} \quad \checkmark$$

Acceleration $= 0.4 \text{ m s}^{-2}$ (shown)

Part (ii): Time and distance to increase from 5 to 9 m s^{-1}

Using $v = u + at$: $9 = 5 + 0.4t \implies t = \frac{4}{0.4} = 10$ s.

Using $s = \frac{1}{2}(u+v)t = \frac{1}{2}(14)(10) = 70$ m.

Time $= 10$ s; Distance $= 70$ m

Question 3

Worked Solution

Particle P projected upward at 8.4 m s^{-1} from ground. Particle Q projected upward at u m s^{-1} from 2 m above ground. Both reach maximum height 3.6 m above ground. Projected simultaneously.

Part (i): Show greatest height above ground reached by P is 3.6 m

Taking upward as positive: $u_P = 8.4$, $v = 0$, $a = -9.8$:

$$0 = 8.4^2 - 2(9.8)s_P \implies s_P = \frac{70.56}{19.6} = 3.6 \text{ m} \quad \checkmark$$

Greatest height of P above ground = 3.6 m (shown)

Part (ii): Value of u

Q is projected from 2 m above ground and reaches maximum height 3.6 m above ground, so Q rises $3.6 - 2 = 1.6$ m above its launch point:

$$0 = u^2 - 2(9.8)(1.6) \implies u^2 = 31.36 \implies u = 5.6 \text{ m s}^{-1}$$

$u = 5.6 \text{ m s}^{-1}$

Part (iii): Show that when P and Q are at same height, they have same speed and are moving in opposite directions

Time for P to reach max height: $t_P = \frac{8.4}{9.8} = \frac{6}{7}$ s.

Time for Q to reach max height: $t_Q = \frac{5.6}{9.8} = \frac{4}{7}$ s.

Since $t_P > t_Q$, at the time they are at the same height (between t_Q and t_P), Q has already started descending while P is still ascending.

At any height s above ground (with P at s going up and Q at s going down):

For P (from ground): $v_P^2 = 8.4^2 - 2(9.8)(s) = 70.56 - 19.6s$

For Q (from 2 m): $v_Q^2 = 5.6^2 - 2(9.8)(s - 2) = 31.36 - 19.6s + 39.2 = 70.56 - 19.6s$

So $v_P^2 = v_Q^2$, meaning the speeds are equal. Since P is still ascending ($v_P > 0$) and Q has passed its peak and is descending ($v_Q < 0$), they are moving in opposite directions. ✓

Question 4

Worked Solution

Particle P projected upward, reaches greatest height 0.5 s after projection. $g = 9.8$ $\text{m}\,\text{s}^{-2}$.

Part (i): Speed of projection

$v = 0$ at greatest height: $0 = U - 9.8(0.5) \implies U = 4.9\ \text{m}\,\text{s}^{-1}$.

Speed of projection $= 4.9\ \text{m}\,\text{s}^{-1}$

Part (ii): Greatest height above projection point

$$s = \frac{U^2}{2g} = \frac{4.9^2}{2(9.8)} = \frac{24.01}{19.6} = 1.225\ \text{m}$$

Greatest height $= 1.225$ m

Part (iii): Speed just before hitting ground (projection point is 0.539 m above ground)

Total fall from max height to ground $= 1.225 + 0.539 = 1.764$ m.

$$v^2 = 2(9.8)(1.764) = 34.5744 \implies v \approx 5.88\ \text{m}\,\text{s}^{-1}$$

Speed before hitting ground $\approx 5.88\ \text{m}\,\text{s}^{-1}$

Question 5

Worked Solution

Particle P projected vertically downward at 3.5 m s^{-1} from A, 5 m above horizontal ground. $g = 9.8\ \text{m s}^{-2}$.

Part (i): Speed just before hitting ground

Taking downward as positive: $u = 3.5$, $a = 9.8$, $s = 5$:

$$v^2 = 3.5^2 + 2(9.8)(5) = 12.25 + 98 = 110.25 \implies v = 10.5\ \text{m s}^{-1}$$

Speed = 10.5 m s^{-1}

Part (ii): Speed just after leaving ground (rebounds, passes A after 0.87 s)

After rebounding, P moves upward and passes through A (5 m above ground) after 0.87 s.

Taking upward as positive from ground: $s = 5$ m, $t = 0.87$ s, $a = -9.8$:

$$5 = u(0.87) - \frac{1}{2}(9.8)(0.87)^2 = 0.87u - 3.71169$$

$$u = \frac{5 + 3.71169}{0.87} = \frac{8.71169}{0.87} \approx 10.0\ \text{m s}^{-1}$$

Speed just after leaving ground $\approx 10.0\ \text{m s}^{-1}$

Question 6

Worked Solution

Particle P projected vertically downward at 14 $\mathrm{m\,s^{-1}}$ from 30 m above ground. $g = 9.8$ $\mathrm{m\,s^{-2}}$.

Part (i): Speed when reaching ground

$$v^2 = 14^2 + 2(9.8)(30) = 196 + 588 = 784 \implies v = 28\ \mathrm{m\,s^{-1}}$$

Speed $= 28\ \mathrm{m\,s^{-1}}$

Part (ii): Distance in first 0.4 s

$$s = 14(0.4) + \frac{1}{2}(9.8)(0.4)^2 = 5.6 + 0.784 = 6.384\ \mathrm{m}$$

Distance $= 6.384$ m (accept 6.38 m)

Part (iii): Time to travel final 15 m

Total distance 30 m; first 15 m from top, then final 15 m.

Time to travel first 15 m: $15 = 14t + 4.9t^2 \implies 4.9t^2 + 14t - 15 = 0$:

$$t = \frac{-14 + \sqrt{196 + 294}}{9.8} = \frac{-14 + \sqrt{490}}{9.8} = \frac{-14 + 22.136}{9.8} \approx \frac{8.136}{9.8} \approx 0.8302\ \mathrm{s}$$

Total time to reach ground:

$$28 = 14 + 9.8t_{\text{total}} \implies t_{\text{total}} = \frac{14}{9.8} \approx 1.4286\ \mathrm{s}$$

Time for final 15 m: $1.4286 - 0.8302 \approx 0.598$ s.

Time for final 15 m ≈ 0.598 s

Question 7

Worked Solution

Stone released from rest on bridge, falls into lake. Speed when entering lake = 14 $\mathrm{m\,s^{-1}}$. $g = 9.8\ \mathrm{m\,s^{-2}}$.

Part (i): Distance fallen and time to enter lake

Distance: $v^2 = 2gs \implies s = \frac{196}{19.6} = 10$ m.

Time: $v = gt \implies t = \frac{14}{9.8} = \frac{10}{7} \approx 1.43$ s.

Distance = 10 m; Time ≈ 1.43 s

Part (ii): Acceleration through lake

Lake depth 15 m; speed enters lake at 14 $\mathrm{m\,s^{-1}}$, reaches bed at 20 $\mathrm{m\,s^{-1}}$ (no sudden velocity change):

$$v^2 = u^2 + 2as \implies 400 = 196 + 2a(15) \implies 30a = 204 \implies a = 6.8\ \mathrm{m\,s^{-2}}$$

Acceleration through lake = 6.8 $\mathrm{m\,s^{-2}}$ (downward)

End of Worked Solutions