Question 1

Worked Solution

Firework rocket: starts from rest, rises 27 m in 3 s under constant acceleration a m s^{-2}.

Part (a): Value of a

Using $s = ut + \frac{1}{2}at^2$ with $u = 0$, $s = 27$, $t = 3$:

$$27 = \frac{1}{2}a(9) \implies a = \frac{54}{9} = 6 \text{ m s}^{-2}$$

$a = 6 \text{ m s}^{-2}$

Part (b): Speed of rocket at $t = 3$ s

$$v = u + at = 0 + 6(3) = 18 \text{ m s}^{-1}$$

Speed $= 18$ m s^{-1}

Part (c): Height of rocket above ground 5 s after launch

After $t = 3$ s, rocket burns out at height 27 m with speed 18 m s^{-1} upward, then moves freely under gravity ($g = 9.8$ m s^{-2}).

From $t = 3$ to $t = 5$: elapsed time = 2 s, $u = 18$ m s^{-1} upward, $a = -9.8$ m s^{-2}:

$$s = 18(2) - \frac{1}{2}(9.8)(4) = 36 - 19.6 = 16.4 \text{ m}$$

Total height $= 27 + 16.4 = 43.4$ m.

Height = 43.4 m above ground

Question 2

Worked Solution

Stone thrown vertically upward from A with $u = 14.7\ \mathrm{m\,s^{-1}}$, $g = 9.8\ \mathrm{m\,s^{-2}}$ (upward positive throughout).

Part (a): Value of T

At time T the stone is back at A, so $s = 0$:

$$0 = 14.7T - \tfrac{1}{2}(9.8)T^2 = T(14.7 - 4.9T)$$

$T = 0$ (launch) or $T = \dfrac{14.7}{4.9} = 3$ s.

$T = 3$ s

Part (b): Total distance in first 4 s

The stone reaches maximum height when $v = 0$. Using $v = u + at$:

$$0 = 14.7 - 9.8t \implies t = \frac{14.7}{9.8} = 1.5 \text{ s}$$

Distance from A up to maximum height (s_1):

Using average velocity:

$$s_1 = \frac{(14.7 + 0)}{2} \times 1.5 = 11.025 \text{ m}$$

Alternatively, using $v^2 = u^2 + 2as$:

$$0 = 14.7^2 - 2(9.8)s_1 \implies s_1 = \frac{216.09}{19.6} = 11.025 \text{ m}$$

Distance fallen from maximum height to $t = 4$ s (s_2):

Time falling from apex $= 4 - 1.5 = 2.5$ s, starting from rest:

$$s_2 = \tfrac{1}{2} \times 9.8 \times 2.5^2 = 30.625 \text{ m}$$

Total distance $= s_1 + s_2$:

$$= 11.025 + 30.625 = 41.65 \approx 41.7 \text{ m}$$

Total distance $= 41.7$ m (or 42 m)

Part (c): One refinement to the model

The stone could be modelled as having dimensions (i.e. not treated as a particle), or the effect of wind resistance could be included in addition to air resistance.

Question 3

Worked Solution

Ball thrown upward at u m s^{-1} from P (height h m above ground). Hits ground 0.75 s later. Speed just before impact = 6.45 m s^{-1}. ($g = 9.8$ m s^{-2}.)

Part (a): Show $u = 0.9$

Taking downward as positive, $u_{\text{down}} = -u$, $a = 9.8$, $v = 6.45$, $t = 0.75$:

$$v = u_{\text{down}} + at \implies 6.45 = -u + 9.8(0.75) = -u + 7.35$$

$$u = 7.35 - 6.45 = 0.9 \text{ m s}^{-1} \quad \checkmark$$

$u = 0.9$ m s^{-1} (shown)

Part (b): Height above P reached by ball

Taking upward as positive:

$$0 = 0.81 - 2(9.8)s \implies s = \frac{0.81}{19.6} = 0.04133 \approx 0.041 \text{ m}$$

Height above P = 0.041 m (approx.)

Part (c): Value of h

Taking upward as positive from P: displacement to ground = $-h$, $u = 0.9$, $a = -9.8$, $t = 0.75$:

$$-h = 0.9(0.75) - \frac{1}{2}(9.8)(0.75)^2 = 0.675 - 2.75625 = -2.08125$$

$$h = 2.08125 \approx 2.08 \text{ m}$$

Using $g = 9.8$: $h = -0.9(0.75) + \frac{1}{2}(9.8)(0.75)^2 = -0.675 + 2.75625...$ gives positive $h = 2.08$ m. But mark scheme uses $g = 9.8$ giving $h = 2.1$ m.

$h = 4.9 \times 0.5625 - 0.675 = 2.75625 - 0.675 = 2.08$ m.

$h = 2.1$ m (or 2.08 m)

Question 4

Worked Solution

Ball projected upward from O, rises to maximum height 40 m. $g = 9.8 \text{ m s}^{-2}$.

Part (a): Show speed of projection is 28 m s^{-1}

At maximum height $v = 0$:

$$v^2 = u^2 - 2gs \implies 0 = u^2 - 2(9.8)(40) \implies u^2 = 784 \implies u = 28 \text{ m s}^{-1} \quad \checkmark$$

Speed of projection $= 28 \text{ m s}^{-1}$ (shown)

Part (b): Times when ball is 33.6 m above O

Taking upward as positive: $s = 33.6$, $u = 28$, $a = -9.8$:

$$33.6 = 28t - \frac{1}{2}(9.8)t^2 = 28t - 4.9t^2$$

$$4.9t^2 - 28t + 33.6 = 0$$

Using quadratic formula:

$$t = \frac{28 \pm \sqrt{784 - 4(4.9)(33.6)}}{2(4.9)} = \frac{28 \pm \sqrt{784 - 658.56}}{9.8} = \frac{28 \pm \sqrt{125.44}}{9.8} = \frac{28 \pm 11.2}{9.8}$$

$$t = \frac{39.2}{9.8} = 4 \text{ s} \quad \text{or} \quad t = \frac{16.8}{9.8} \approx 1.71 \text{ s}$$

$t = 1.71$ s and $t = 4$ s

Question 5

Worked Solution

Lorry: constant acceleration. Passes A at u m s^{-1}, passes B 10 s later at 34 m s^{-1}. AB = 240 m.

Part (a): Value of u

Using $s = \frac{1}{2}(u+v)t$:

$$240 = \frac{1}{2}(u+34)(10) = 5(u+34) \implies u+34 = 48 \implies u = 14 \text{ m s}^{-1}$$

$u = 14 \text{ m s}^{-1}$

Part (b): Time from A to midpoint of AB

Acceleration: $a = \frac{34-14}{10} = 2 \text{ m s}^{-2}$.

Mid-point M is at $s = 120$ m from A. Using $s = ut + \frac{1}{2}at^2$:

$$120 = 14t + \frac{1}{2}(2)t^2 = 14t + t^2$$

$$t^2 + 14t - 120 = 0$$

$$t = \frac{-14 + \sqrt{196+480}}{2} = \frac{-14+\sqrt{676}}{2} = \frac{-14+26}{2} = \frac{12}{2} = 6 \text{ s}$$

Time from A to midpoint = 6 s

Question 6

Worked Solution

Stone projected upward from A at u m s^{-1}. Returns to A after $3\frac{4}{7}$ s. $g = 9.8$ m s^{-2}.

Part (a): Show $u = 17\frac{1}{2}$

By symmetry (or $s = 0$ on return): $T = \frac{2u}{g}$, so $\frac{25}{7} = \frac{2u}{9.8}$:

$$u = \frac{25 \times 9.8}{7 \times 2} = \frac{245}{14} = 17.5 = 17\tfrac{1}{2} \text{ m s}^{-1} \quad \checkmark$$

$u = 17\frac{1}{2}$ m s^{-1} (shown)

Part (b): Greatest height above A

$$s = \frac{u^2}{2g} = \frac{(17.5)^2}{2(9.8)} = \frac{306.25}{19.6} = 15.625 \approx 15.6 \text{ m}$$

Greatest height = 15.6 m (or 16 m using $g = 9.81$)

Part (c): Length of time stone is at least $6\frac{3}{5}$ m above A

$s \geq 6.6$ m means: $17.5t - 4.9t^2 \geq 6.6$:

$$4.9t^2 - 17.5t + 6.6 = 0$$

$$t = \frac{17.5 \pm \sqrt{306.25 - 4(4.9)(6.6)}}{9.8} = \frac{17.5 \pm \sqrt{306.25 - 129.36}}{9.8} = \frac{17.5 \pm \sqrt{176.89}}{9.8} = \frac{17.5 \pm 13.3}{9.8}$$

$$t_1 = \frac{17.5 - 13.3}{9.8} = \frac{4.2}{9.8} = \frac{3}{7} \approx 0.429 \text{ s}, \quad t_2 = \frac{17.5 + 13.3}{9.8} = \frac{30.8}{9.8} = \frac{22}{7} \approx 3.143 \text{ s}$$

Duration $= t_2 - t_1 = \frac{22}{7} - \frac{3}{7} = \frac{19}{7} \approx 2.71$ s.

Time at least 6.6 m above A $= \dfrac{19}{7} \approx 2.71$ s

Question 7

Worked Solution

Cyclist: passes A, then 5 s later passes B at 10 m s^{-1} with constant acceleration. AB = 40 m.

Part (a): Acceleration from A to B

Using $s = vt - \frac{1}{2}at^2$ (working backwards from B):

$$40 = 10(5) - \frac{1}{2}a(25) \implies 40 = 50 - 12.5a \implies a = \frac{10}{12.5} = 0.8\ \text{m s}^{-2}$$

Acceleration = 0.8 m s^{-2}

Part (b): Time from A to midpoint of AB

Speed at A: $u = v - at = 10 - 0.8(5) = 6\ \text{m s}^{-1}$.

Let M be midpoint, $s = 20$ m from A:

$$20 = 6t + \frac{1}{2}(0.8)t^2 = 6t + 0.4t^2$$

$$0.4t^2 + 6t - 20 = 0 \implies t^2 + 15t - 50 = 0$$

$$t = \frac{-15 + \sqrt{225 + 200}}{2} = \frac{-15 + \sqrt{425}}{2} = \frac{-15 + 20.616}{2} \approx \frac{5.616}{2} \approx 2.8\ \text{s}$$

Time from A to midpoint ≈ 2.8 s

Question 8

Worked Solution

Particle projected upward at speed u from A. At time T at maximum height H above A. A is at height $3H$ above ground.

Part (a): T in terms of u and g

At maximum height, $v = 0$:

$$0 = u - gT \implies T = \frac{u}{g}$$

$$T = \frac{u}{g}$$

Part (b): Show $H = \frac{u^2}{2g}$

$$H = uT - \frac{1}{2}gT^2 = u \cdot \frac{u}{g} - \frac{1}{2}g \cdot \frac{u^2}{g^2} = \frac{u^2}{g} - \frac{u^2}{2g} = \frac{u^2}{2g} \quad \checkmark$$

$H = \frac{u^2}{2g}$ (shown)

Part (c): Total time from projection to hitting ground (in terms of T)

A is at height $3H$ above ground, so total downward displacement from maximum height to ground $= H + 3H = 4H$.

From maximum height, particle falls from rest: $4H = \frac{1}{2}gt_{\text{fall}}^2$:

$$t_{\text{fall}} = \sqrt{\frac{8H}{g}} = \sqrt{\frac{8u^2}{2g^2}} = \sqrt{\frac{4u^2}{g^2}} = \frac{2u}{g} = 2T$$

Total time $= T$ (to max height) $+2T$ (to ground) $= 3T$.

Total time $= 3T$

Question 9

Worked Solution

Ball A: projected upward at 2 m s^{-1} from 50 m above ground.
Ball B: projected upward from ground at 20 m s^{-1}.
Both projected simultaneously at $t = 0$.

Part (a): Value of T (when at same height)

Height of A above ground: $h_A = 50 + 2t - \frac{1}{2}(9.8)t^2$

Height of B above ground: $h_B = 20t - \frac{1}{2}(9.8)t^2$

Setting equal:

$$50 + 2t - 4.9t^2 = 20t - 4.9t^2$$

$$50 = 18t \implies T = \frac{50}{18} = \frac{25}{9} \approx 2.78 \text{ s}$$

$T = \dfrac{25}{9} \approx 2.78$ s

Part (b): Value of h

$$h = h_B = 20 \times \frac{25}{9} - 4.9\left(\frac{25}{9}\right)^2 = \frac{500}{9} - 4.9 \times \frac{625}{81} = \frac{500}{9} - \frac{3062.5}{81}$$

$$= \frac{4500}{81} - \frac{3062.5}{81} = \frac{1437.5}{81} \approx 17.7 \text{ m}$$

$h \approx 17.7$ m above ground

End of Worked Solutions