Question 1

Worked Solution

Train P: accelerates from rest over 300 m to 30 m s^{-1}, then constant speed for T s, then decelerates at 1.25 m s^{-2} to rest. Total distance 1500 m.

Part (a): Acceleration of P during first 300 m

Using $v^2 = u^2 + 2as$ with $u = 0$, $v = 30$, $s = 300$:

$$900 = 2a(300) \implies a = \frac{900}{600} = 1.5 \text{ m s}^{-2}$$

Acceleration = 1.5 m s^{-2}

Part (b): Value of T

Time for acceleration phase: $t_1 = \frac{v}{a} = \frac{30}{1.5} = 20$ s.

Distance in deceleration phase (from $v = 30$ to $v = 0$ at 1.25 m s^{-2}):

$$s_{\text{dec}} = \frac{v^2}{2 \times 1.25} = \frac{900}{2.5} = 360 \text{ m}$$

Distance at constant speed:

$$30T = 1500 - 300 - 360 = 840 \implies T = 28 \text{ s}$$

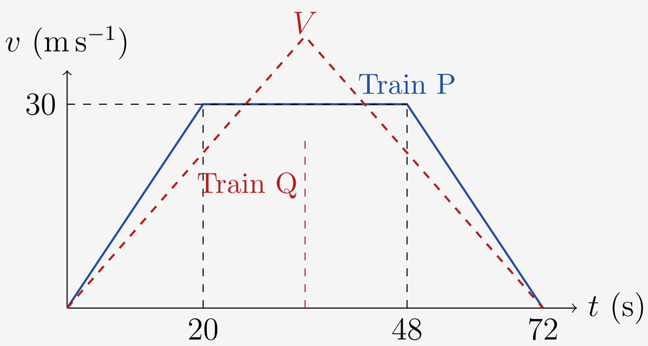
$T = 28$ s

Part (c): Velocity–time graph for train Q

Train Q completes the same journey (1500 m) in the same total time. Total time for P:

$$t_{\text{total}} = t_1 + T + t_{\text{dec}} = 20 + 28 + \frac{30}{1.25} = 20 + 28 + 24 = 72 \text{ s}$$

Q accelerates from 0 to V then immediately decelerates to 0, forming a triangle on the v–t graph with base 72 s and peak V.



Part (d): Value of V

Time for Q to accelerate from 0 to V: $t_1' = \frac{V}{a_1'}$.

Since Q uses same total time 72 s with a triangle shape, the peak occurs at the midpoint only if accelerations are equal; in general use area = 1500 m:

$$\text{Area of triangle} = \frac{1}{2} \times 72 \times V = 1500 \implies V = \frac{3000}{72} = \frac{125}{3} \approx 41.67 \text{ m s}^{-1}$$

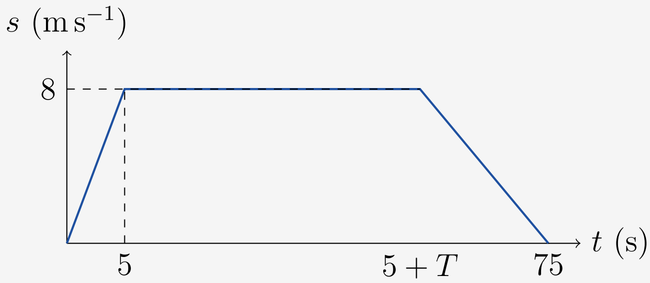
$V = \dfrac{125}{3} \approx 41.7 \text{ m s}^{-1}$

Question 2

Worked Solution

Athlete: accelerates from rest for 5 s reaching 8 $\mathrm{m\,s^{-1}}$, maintains 8 $\mathrm{m\,s^{-1}}$ for T s, then decelerates to rest. Total distance 500 m in 75 s.

Part (a): Speed–time graph



Part (b): Value of T

Area under graph = total distance = 500 m. The graph is a trapezium with parallel sides $(T + 75)$ and T (the two horizontal lengths at speed 8) – more straightforwardly:

$$\text{Area} = \frac{1}{2}(T + 75) \times 8 = 500$$

Wait – the parallel sides of the trapezium are the top (length T) and the base (length 75), height 8:

$$\frac{1}{2}(T + 75) \times 8 = 500$$

$$4(T + 75) = 500 \implies T + 75 = 125 \implies T = 50 \text{ s}$$

$T = 50$ s

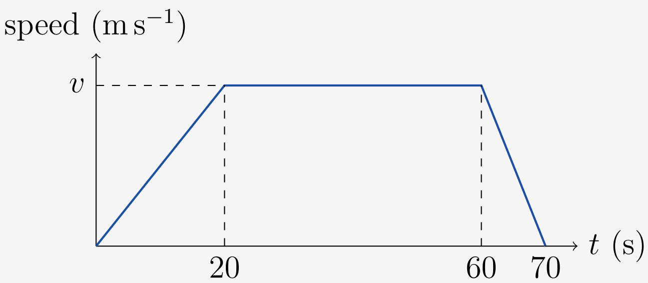
Question 3

Worked Solution

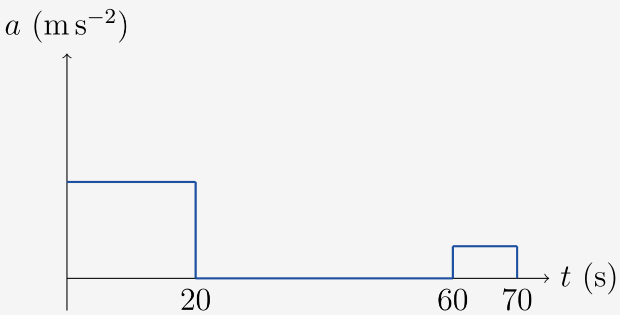
Car: accelerates from rest for 20 s to v m s^{-1}, constant speed for 40 s, decelerates for 10 s to rest. Total distance 880 m.

Part (a): Sketches

(i) Speed–time graph:



(ii) Acceleration–time graph:



Part (b): Value of v

Area under speed–time graph (trapezium with parallel sides 40 and 70, height v):

$$\frac{1}{2}(40+70)\times v = 880 \implies \frac{110v}{2} = 880 \implies v = \frac{880\times 2}{110} = 16 \text{ m s}^{-1}$$

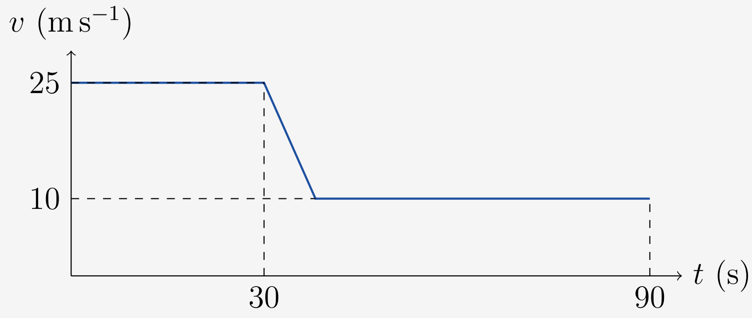
$v = 16$ m s^{-1}

Question 4

Worked Solution

Car: constant 25 $\mathrm{m\,s^{-1}}$ for 30 s, decelerates to 10 $\mathrm{m\,s^{-1}}$, then constant 10 $\mathrm{m\,s^{-1}}$ until B. Total time 90 s, AB = 1410 m.

Part (a): Speed–time graph sketch



Part (b): Deceleration

Let t = duration of deceleration phase.

Distance:

$$30 \times 25 + \frac{1}{2}(25 + 10)t + 10(60 - t) = 1410$$

$$750 + \frac{35t}{2} + 600 - 10t = 1410$$

$$1350 + 7.5t = 1410 \implies 7.5t = 60 \implies t = 8 \text{ s}$$

Deceleration:

$$a = \frac{25 - 10}{8} = \frac{15}{8} = 1.875 \ \mathrm{m\,s^{-2}}$$

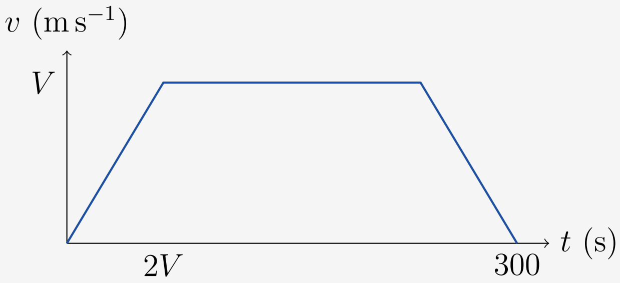
Deceleration = 1.875 $\mathrm{m\,s^{-2}}$

Question 5

Worked Solution

Train: accelerates from rest at $0.5\ \text{m s}^{-2}$ to $V\ \text{m s}^{-1}$, constant speed, then decelerates at $0.25\ \text{m s}^{-2}$ to rest. Total time 5 min = 300 s. Distance AB = 6300 m.

Part (a): Speed–time graph sketch



Part (b): Times in terms of V

(i) Accelerating: $t_1 = \frac{V}{0.5} = 2V$ s

(ii) Decelerating: $t_2 = \frac{V}{0.25} = 4V$ s

(iii) Constant speed: $t_3 = 300 - 2V - 4V = 300 - 6V$ s

(i) $2V$ s (ii) $4V$ s (iii) $(300 - 6V)$ s

Part (c): Value of V

Total distance = area under graph:

$$\frac{1}{2}(2V)(V) + V(300 - 6V) + \frac{1}{2}(4V)(V) = 6300$$

$$V^2 + 300V - 6V^2 + 2V^2 = 6300$$

$$-3V^2 + 300V = 6300$$

$$V^2 - 100V + 2100 = 0$$

$$(V - 30)(V - 70) = 0 \implies V = 30 \text{ or } V = 70$$

Since $V < 50$:

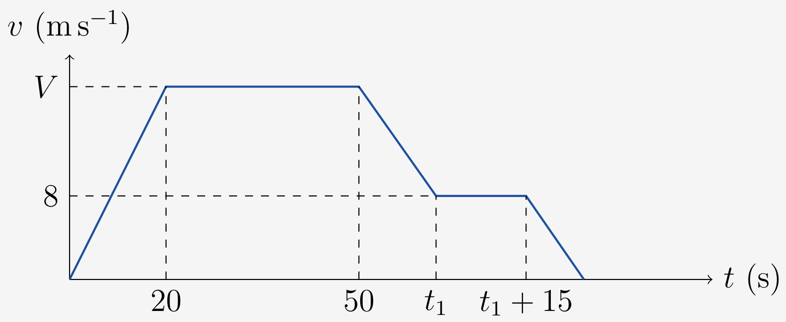
$V = 30\ \text{m s}^{-1}$

Question 6

Worked Solution

Car: from rest, accelerates to V m s^{-1} in 20 s (travels 140 m), constant V for 30 s, decelerates at $\frac{1}{2}$ m s^{-2} to 8 m s^{-1}, constant 8 m s^{-1} for 15 s, decelerates at $\frac{1}{3}$ m s^{-2} to rest.

Part (a): Speed–time graph sketch



Part (b): Value of V

In first 20 s: distance $= \frac{1}{2}(20)V = 140 \implies V = 14$ m s^{-1}

$V = 14$ m s^{-1}

Part (c): Total time

Time to decelerate from 14 to 8 m s^{-1} at 0.5 m s^{-2}: $t_1 = \frac{14-8}{0.5} = 12$ s

Time to decelerate from 8 to 0 at $\frac{1}{3}$ m s^{-2}: $t_2 = \frac{8}{1/3} = 24$ s

Total time $= 20 + 30 + 12 + 15 + 24 = 101$ s

Total time $= 101$ s

Part (d): Total distance

$$d = 140 + 30(14) + \frac{1}{2}(14+8)(12) + 15(8) + \frac{1}{2}(8)(24)$$
$$= 140 + 420 + 132 + 120 + 96 = 908 \text{ m}$$

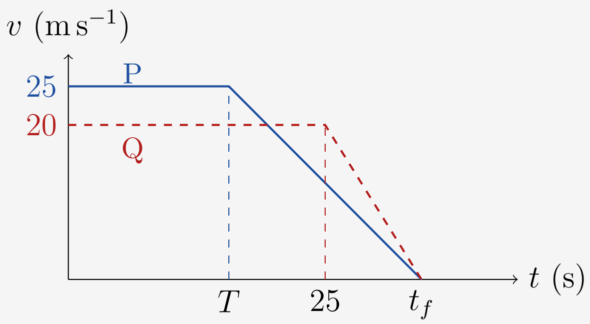
Total distance $= 908$ m

Question 7

Worked Solution

Car P: constant 25 $\mathrm{m\,s^{-1}}$ from $t = 0$, decelerates from $t = T$ to rest at X (800 m from start).
Car Q: constant 20 $\mathrm{m\,s^{-1}}$ from $t = 0$, decelerates from $t = 25$ s to rest at X at same instant as P.

Part (a): Speed–time graph sketch



Part (b): Value of T

For Q: Q travels at constant 20 $\mathrm{m\,s^{-1}}$ until $t = 25$, then decelerates to 0. Total distance = 800 m.

Let t_f be the final time when both stop.

$$20 \cdot \frac{t_f + 25}{2} = 800 \implies t_f + 25 = 80 \implies t_f = 55 \text{ s}$$

For P: P travels at 25 $\mathrm{m\,s^{-1}}$ from 0 to T, then decelerates to 0 at $t = 55$ s. Total distance = 800 m.

$$25 \cdot \frac{T + 55}{2} = 800 \implies T + 55 = 64 \implies T = 9 \text{ s}$$

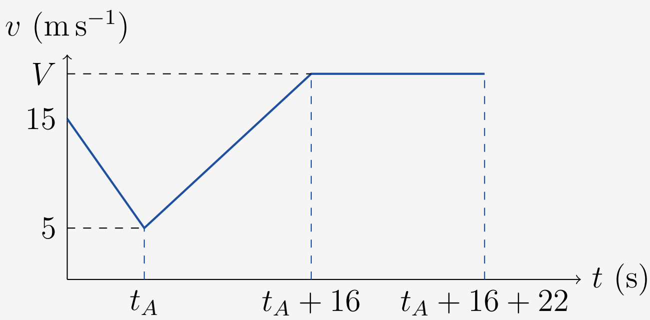
$T = 9$ s

Question 8

Worked Solution

Car: at A speed 15 $\mathrm{m\,s^{-1}}$, decelerates uniformly to 5 $\mathrm{m\,s^{-1}}$ over 120 m (to W), then accelerates uniformly for 16 s to V $\mathrm{m\,s^{-1}}$, then constant V for 22 s to B. AB = 1000 m.

Part (a): Speed–time graph sketch



Part (b): Time from A to B

Time from A to W: using $\frac{1}{2}(15+5)t = 120 \implies 10t = 120 \implies t = 12$ s.

Total time: $12 + 16 + 22 = 50$ s.

Time from A to B = 50 s

Part (c): Value of V

Total distance AB = 1000 m. Distance A to W = 120 m. Remaining distance W to B = 880 m.

$$\frac{1}{2}(5+V)(16) + 22V = 880$$
$$8(5+V) + 22V = 880$$
$$40 + 8V + 22V = 880$$
$$30V = 840 \implies V = 28\ \mathrm{m\,s^{-1}}$$

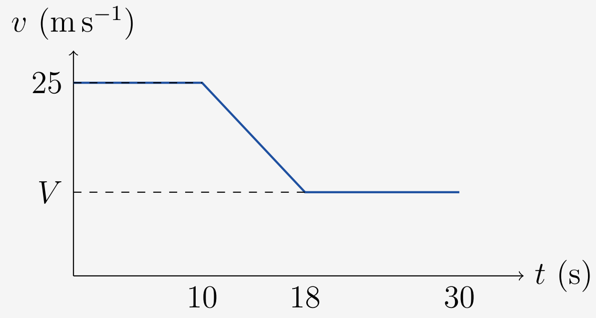
$V = 28\ \mathrm{m\,s^{-1}}$

Question 9

Worked Solution

Car: at A ($t = 0$), speed 25 m s^{-1}; constant speed to $t = 10$ s; decelerates uniformly 8 s to speed V m s^{-1} at $t = 18$ s; constant V to B at $t = 30$ s. AB = 526 m.

Part (a): Speed–time graph sketch



Part (b): Value of V

Total area = 526 m:

$$25(10) + \frac{1}{2}(25 + V)(8) + V(12) = 526$$

$$250 + 4(25 + V) + 12V = 526$$

$$250 + 100 + 4V + 12V = 526$$

$$16V = 176 \implies V = 11 \text{ m s}^{-1}$$

$V = 11 \text{ m s}^{-1}$

Part (c): Deceleration between $t = 10$ s and $t = 18$ s

$$a = \frac{v - u}{t} = \frac{11 - 25}{8} = \frac{-14}{8} = -1.75 \text{ m s}^{-2}$$

Deceleration $= 1.75 \text{ m s}^{-2}$

Question 10

Worked Solution

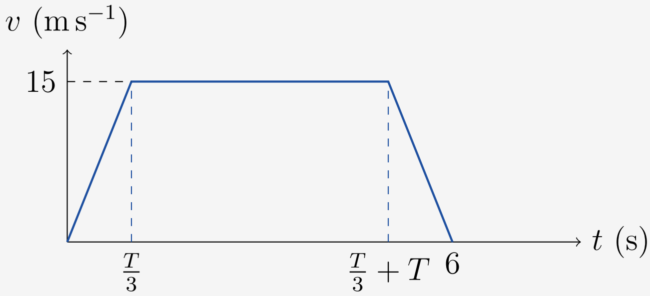
Car A to B (AB = 885 m): accelerates from rest at a m s^{-2} for $\frac{1}{3}T$ s to 15 m s^{-1}, constant 15 m s^{-1} for T s, decelerates at 2.5 m s^{-2} to rest at B.

Part (a): Time for deceleration

$$0 = 15 - 2.5t \implies t = 6 \text{ s}$$

Time decelerating = 6 s

Part (b): Speed–time graph sketch



Part (c): Value of T

Total area = 885 m:

$$\frac{1}{2} \cdot \frac{T}{3} \cdot 15 + T \cdot 15 + \frac{1}{2}(6)(15) = 885$$

$$\frac{15T}{6} + 15T + 45 = 885$$

$$\frac{5T}{2} + 15T = 840$$

$$\frac{35T}{2} = 840 \implies T = \frac{840 \times 2}{35} = 48 \text{ s}$$

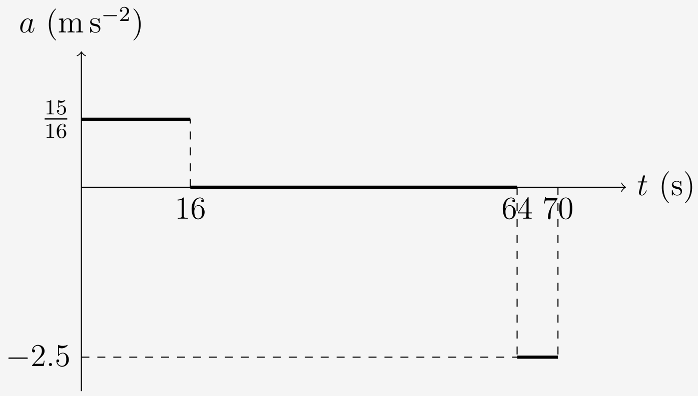
$T = 48$ s

Part (d): Value of a

$$a = \frac{v}{t} = \frac{15}{T/3} = \frac{15 \times 3}{48} = \frac{45}{48} = \frac{15}{16} \approx 0.9375 \text{ m s}^{-2}$$

$$a = \frac{15}{16}\ \mathrm{m\,s^{-2}} \approx 0.9375\ \mathrm{m\,s^{-2}}$$

Part (e): Acceleration–time graph sketch



Question 11

Worked Solution

Ball projected upward at U m s^{-1} from A, 16.8 m above ground. Speed just before hitting ground = 19 m s^{-1}. $g = 10$ m s^{-2}.

Part (a): Show $U = 5$

Taking upward as positive, displacement to ground = -16.8 m.

Using $v^2 = u^2 + 2as$ (taking downward as positive for hitting ground):

$$19^2 = U^2 + 2(10)(16.8)$$

$$361 = U^2 + 336 \implies U^2 = 25 \implies U = 5 \text{ m s}^{-1} \quad \checkmark$$

$U = 5$ m s^{-1} (shown)

Part (b): Value of T

Taking upward as positive: $u = 5$, $a = -10$, final velocity = -19 m s^{-1}:

$$-19 = 5 - 10T \implies T = \frac{24}{10} = 2.4 \text{ s}$$

$T = 2.4$ s

Part (c): Time when ball is 1.2 m below A

Displacement = -1.2 m (downward from A):

$$-1.2 = 5t - \frac{1}{2}(10)t^2 = 5t - 5t^2$$

$$5t^2 - 5t - 1.2 = 0$$

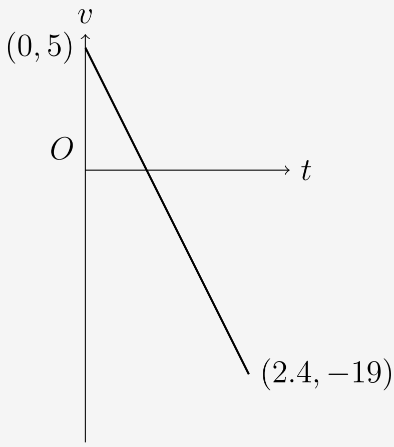
Using quadratic formula:

$$t = \frac{5 \pm \sqrt{25 + 4(5)(1.2)}}{10} = \frac{5 \pm \sqrt{25 + 24}}{10} = \frac{5 \pm 7}{10}$$

Taking positive root: $t = \frac{12}{10} = 1.2$ s.

Time = 1.2 s

Part (d): Velocity–time graph for $0 \le t \le T$



Start point $(0, 5)$; end point $(2.4, -19)$.

Part (e): Effect of air resistance on U

With air resistance, the ball loses more energy on the way down, so the speed just before impact would be less than 19 m s^{-1}. To achieve the same final speed of 19 m s^{-1}, the ball would need to be projected with a greater initial speed. Therefore the new value of U would be **greater** than 5 m s^{-1}.

Part (f): One further refinement

Model the ball as having a non-zero size (not as a particle) to account for the effect of spin; or use a more accurate value of g.

End of Worked Solutions