Question 1

Worked Solution

Part (i) — Magnitude and bearing of resultant

$\mathbf{X}$ acts along a bearing of 090° (due East), so it is purely horizontal: $\mathbf{X} = 12\mathbf{i}$

$\mathbf{Y}$ acts along a bearing of 000° (due North), so it is purely vertical: $\mathbf{Y} = 15\mathbf{j}$

Magnitude of resultant:

$$R = \sqrt{12^2 + 15^2} = \sqrt{144 + 225} = \sqrt{369}$$

$$R \approx 19.2 \quad \text{N}$$

Bearing of resultant:

The resultant has an eastward component of 12 N and a northward component of 15 N. The angle θ east of north is:

$$\tan\theta = \frac{12}{15} \implies \theta = \arctan\left(\frac{12}{15}\right) = \arctan(0.8)$$

$$\text{Bearing} = 038.7^\circ$$

Part (ii) — Equilibrium force E

For three forces to be in equilibrium, the third force must be equal and opposite to the resultant of the other two.

$$E = 19.2 \quad \text{N}, \quad \text{bearing} = 180^\circ + 38.7^\circ = 219^\circ$$

Question 2

Worked Solution

Part (i) — Calculate x

The two perpendicular forces have magnitudes x N and $3x$ N. Their resultant has magnitude 6 N.

By Pythagoras:

$$x^2 + (3x)^2 = 6^2$$

$$x^2 + 9x^2 = 36$$

$$10x^2 = 36 \implies x^2 = 3.6$$

$$x = \sqrt{3.6} \approx 1.90$$

Part (ii) — Angle with the smaller force

The smaller force is x N (horizontal). The angle θ the resultant makes with it satisfies:

$$\tan\theta = \frac{3x}{x} = 3$$

$$\theta = \arctan(3)$$

$$\theta \approx 71.6°$$

Question 3

Worked Solution

Part (i) — Calculate F and the angle

Two perpendicular forces F N (vertical) and 8 N (horizontal) have a resultant of magnitude 17 N.

By Pythagoras:

$$F^2 + 8^2 = 17^2$$

$$F^2 = 289 - 64 = 225$$

$$F = 15 \text{ N}$$

Angle the resultant makes with the 8 N force:

$$\cos\alpha = \frac{8}{17} \implies \alpha = \arccos\left(\frac{8}{17}\right)$$

$$\alpha \approx 61.9^\circ$$

Part (ii) — Equilibrium with third force E

For the three forces to be in equilibrium, E must be equal in magnitude to the resultant of F and 8 N, and act in the opposite direction.

$$E = 17 \text{ N}$$

The resultant of F and 8 N acts at 61.9° above the horizontal (i.e. 61.9° from the 8 N force). So E acts in the opposite direction, meaning the angle between E and the 8 N force is:

$$180^\circ - 61.9^\circ = 118.1^\circ \approx 118^\circ$$

Question 4

Worked Solution

Part (i) — Expression for R

The three forces are in equilibrium, so their vector sum is zero:

$$(-3\mathbf{i} + 4\mathbf{j}) + (21\mathbf{i} - 7\mathbf{j}) + \mathbf{R} = \mathbf{0}$$

$$(18\mathbf{i} - 3\mathbf{j}) + \mathbf{R} = \mathbf{0}$$

$$\mathbf{R} = -18\mathbf{i} + 3\mathbf{j}$$

Part (ii) — Magnitude of R and angle with i

Magnitude:

$$|\mathbf{R}| = \sqrt{(-18)^2 + 3^2} = \sqrt{324 + 9} = \sqrt{333}$$

$$|\mathbf{R}| \approx 18.2 \text{ N}$$

Angle with the **i** direction:

The vector $\mathbf{R} = -18\mathbf{i} + 3\mathbf{j}$ lies in the second quadrant (negative **i**, positive **j** component).

The reference angle is:

$$\arctan\left(\frac{3}{18}\right) = \arctan\left(\frac{1}{6}\right) \approx 9.46°$$

So the angle measured from the positive **i** direction (anticlockwise):

$$180° - 9.46°$$

$$\text{Angle with } \mathbf{i} \text{ direction} \approx 171°$$

Question 5

Worked Solution

Given: $\mathbf{F} = (3.5\mathbf{i} + 12\mathbf{j})$ N, where $\mathbf{i}$ is east and $\mathbf{j}$ is north.

Part (i) — Magnitude and bearing of F

Magnitude:

$$|\mathbf{F}| = \sqrt{3.5^2 + 12^2} = \sqrt{12.25 + 144} = \sqrt{156.25} = 12.5$$

$$|\mathbf{F}| = 12.5 \text{ N}$$

Bearing:

The eastward component is 3.5 N and the northward component is 12 N. The angle east of north:

$$\theta = \arctan\left(\frac{3.5}{12}\right) \approx 16.3^\circ$$

$$\text{Bearing of } \mathbf{F} \approx 016.3^\circ$$

Part (ii) — G and F in same direction

$\mathbf{G} = (7\mathbf{i} + 24\mathbf{j})$ N

Check if $\mathbf{G}$ is a scalar multiple of $\mathbf{F}$:

$$\frac{7}{3.5} = 2 \qquad \frac{24}{12} = 2$$

Both ratios equal 2, so $\mathbf{G} = 2\mathbf{F}$. Therefore $\mathbf{G}$ and $\mathbf{F}$ are in the same direction.

$|\mathbf{G}| = 2|\mathbf{F}| = 25$ N, so $\mathbf{G}$ has twice the magnitude of $\mathbf{F}$.

Part (iii) — Find q

$\mathbf{F}_1 = (9\mathbf{i} - 18\mathbf{j})$ N, $\mathbf{F}_2 = (12\mathbf{i} + q\mathbf{j})$ N

$$\mathbf{F}_1 + \mathbf{F}_2 = (21\mathbf{i} + (q - 18)\mathbf{j}) \text{ N}$$

For $\mathbf{F}_1 + \mathbf{F}_2$ to be in the direction of $\mathbf{F} = (3.5\mathbf{i} + 12\mathbf{j})$, the ratios of components must be equal:

$$\frac{q-18}{21} = \frac{12}{3.5} = \frac{24}{7}$$

$$q - 18 = 21 \times \frac{24}{7} = 72$$

$$q = 90$$

Question 6

Worked Solution

A particle of mass 1.5 kg has force $\mathbf{F} = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$ N acting on it.

Part (i) — Acceleration as a vector

Using Newton's Second Law, $\mathbf{F} = m\mathbf{a}$:

$$\begin{pmatrix} 6 \\ 9 \end{pmatrix} = 1.5\,\mathbf{a} \implies \mathbf{a} = \frac{1}{1.5}\begin{pmatrix} 6 \\ 9 \end{pmatrix}$$

$$\mathbf{a} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \text{ m s}^{-2}$$

Part (ii) — Angle with direction $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

The direction $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is the positive x-axis (horizontal). The acceleration $\mathbf{a} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$ makes an angle:

$$\theta = \arctan\left(\frac{6}{4}\right) = \arctan(1.5)$$

$$\theta \approx 56.3^\circ$$

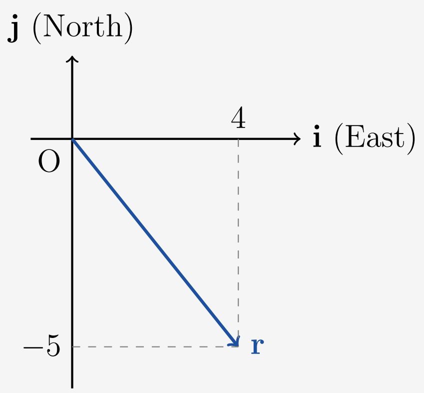
Question 7

Worked Solution

Position vector $\mathbf{r} = 4\mathbf{i} - 5\mathbf{j}$, where $\mathbf{i}$ is east and $\mathbf{j}$ is north.

Part (i) — Sketch

The vector $\mathbf{r} = 4\mathbf{i} - 5\mathbf{j}$ lies 4 units east and 5 units south of the origin O — it points into the fourth quadrant (positive $\mathbf{i}$, negative $\mathbf{j}$).



Part (ii) — Magnitude and bearing

Magnitude:

$$|\mathbf{r}| = \sqrt{4^2 + (-5)^2} = \sqrt{16 + 25} = \sqrt{41}$$

$$\boxed{|\mathbf{r}| = \sqrt{41} \approx 6.40}$$

Bearing:

The vector points east and south. The angle south of east:

$$\arctan\left(\frac{5}{4}\right) \approx 51.3° \text{ south of east}$$

Converting to a bearing (measured clockwise from north):

$$\text{Bearing} = 90° + 51.3° = 141.3°$$

$$\boxed{\text{Bearing} \approx 141°}$$

Note: The mark scheme uses $180° - \arctan\left(\frac{4}{5}\right) \approx 141°$, *which is equivalent.*

Part (iii) — Vector with same direction, three times magnitude

Simply multiply $\mathbf{r}$ by 3:

$$\boxed{3\mathbf{r} = 12\mathbf{i} - 15\mathbf{j}}$$

Question 8

Worked Solution

Given: $\mathbf{p} = 8\mathbf{i} + \mathbf{j}$ and $\mathbf{q} = 4\mathbf{i} - 7\mathbf{j}$.

Part (i) — Show p and q are equal in magnitude

$$|\mathbf{p}| = \sqrt{8^2 + 1^2} = \sqrt{64 + 1} = \sqrt{65}$$
$$|\mathbf{q}| = \sqrt{4^2 + (-7)^2} = \sqrt{16 + 49} = \sqrt{65}$$

Since $|\mathbf{p}| = |\mathbf{q}| = \sqrt{65}$, the vectors are equal in magnitude. □

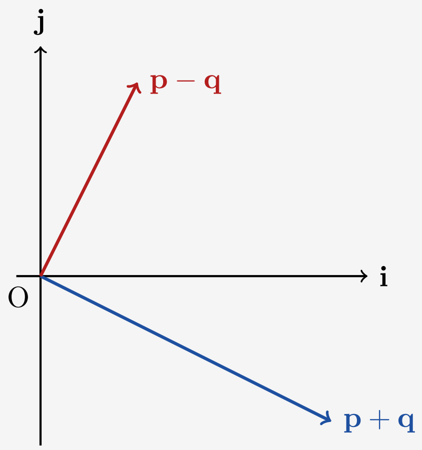
Part (ii) — Show p + q is parallel to 2i − j

$$\mathbf{p} + \mathbf{q} = (8 + 4)\mathbf{i} + (1 - 7)\mathbf{j} = 12\mathbf{i} - 6\mathbf{j} = 6(2\mathbf{i} - \mathbf{j})$$

Since $\mathbf{p} + \mathbf{q}$ is a scalar multiple of $(2\mathbf{i} - \mathbf{j})$, it is parallel to $2\mathbf{i} - \mathbf{j}$. □

Part (iii) — Draw p + q and p − q; find the angle between them

$$\mathbf{p} + \mathbf{q} = 12\mathbf{i} - 6\mathbf{j}$$
$$\mathbf{p} - \mathbf{q} = (8 - 4)\mathbf{i} + (1 + 7)\mathbf{j} = 4\mathbf{i} + 8\mathbf{j}$$



From the diagram, we see that the two vectors are perpendicular. We could also further verify this by calculating the gradient of each vector and multiplying them together to obtain -1.

Angle between $\mathbf{p} + \mathbf{q}$ and $\mathbf{p} - \mathbf{q} = 90°$

Question 9

Worked Solution

Given: $\mathbf{p} = 12\mathbf{i} - 5\mathbf{j}$ and $\mathbf{q} = 16\mathbf{i} + 1.5\mathbf{j}$ (in newtons).

Part (i) — Show p + q is parallel to 8i − j

$$\mathbf{p} + \mathbf{q} = (12 + 16)\mathbf{i} + (-5 + 1.5)\mathbf{j} = 28\mathbf{i} - 3.5\mathbf{j}$$

Factor out 3.5:

$$\mathbf{p} + \mathbf{q} = 3.5(8\mathbf{i} - \mathbf{j})$$

Since $\mathbf{p} + \mathbf{q}$ is a scalar multiple of $8\mathbf{i} - \mathbf{j}$, it is parallel to $8\mathbf{i} - \mathbf{j}$. □

Part (ii) — Show 3p + 10q acts in the horizontal direction

$$3\mathbf{p} = 3(12\mathbf{i} - 5\mathbf{j}) = 36\mathbf{i} - 15\mathbf{j}$$
$$10\mathbf{q} = 10(16\mathbf{i} + 1.5\mathbf{j}) = 160\mathbf{i} + 15\mathbf{j}$$
$$3\mathbf{p} + 10\mathbf{q} = (36 + 160)\mathbf{i} + (-15 + 15)\mathbf{j} = 196\mathbf{i}$$

The $\mathbf{j}$ component is zero, so the force acts entirely in the $\mathbf{i}$ (horizontal) direction. □

Part (iii) — Equilibrium under kp, 3q and weight w; show k = −4, find mass

For equilibrium, the sum of all forces is zero:

$$k\mathbf{p} + 3\mathbf{q} + \mathbf{w} = \mathbf{0}$$

Compute $k\mathbf{p} + 3\mathbf{q}$:

$$k\mathbf{p} = k(12\mathbf{i} - 5\mathbf{j}) = 12k\mathbf{i} - 5k\mathbf{j}$$
$$3\mathbf{q} = 3(16\mathbf{i} + 1.5\mathbf{j}) = 48\mathbf{i} + 4.5\mathbf{j}$$
$$k\mathbf{p} + 3\mathbf{q} = (12k + 48)\mathbf{i} + (-5k + 4.5)\mathbf{j}$$

Weight acts downward: $\mathbf{w} = -mg\mathbf{j}$. Since weight has no $\mathbf{i}$ component, equilibrium in the $\mathbf{i}$ direction requires:

$$12k + 48 = 0 \implies k = -4 \square$$

Substituting $k = -4$ into the $\mathbf{j}$ component for equilibrium:

$$(-5)(-4) + 4.5 - mg = 0$$

$$20 + 4.5 = mg \implies mg = 24.5$$
$$m = \frac{24.5}{9.8} = 2.5$$

$$m = 2.5 \quad \text{kg}$$

End of Worked Solutions — ALevelMathsRevision.com