Connected Particles — Q1 Full Solution

A-Level Mechanics, Year 1

Setup

Let the tension in the rope be T N and the acceleration of the system be a m s^{-2}. Since the rope is light and inextensible and the pulley is smooth, both particles share the same magnitude of acceleration.

Part (a) — Show that the acceleration of Q is 4.2 m s^{-2}

Equation of motion for Q (taking downwards as positive):

$$0.6g - T = 0.6a \qquad (1)$$

Equation of motion for P (taking direction of motion toward pulley as positive):

$$T = 0.8a \qquad (2)$$

Substituting (2) into (1):

$$0.6g - 0.8a = 0.6a$$
$$0.6g = 1.4a$$
$$a = \frac{0.6 \times 9.8}{1.4} = \frac{5.88}{1.4}$$

$$\boxed{a = 4.2 \text{ m s}^{-2}} \checkmark$$

Part (b) — Find the time for P to hit the pulley

The motion occurs in two distinct phases. P starts 1.5 m from the pulley; Q starts 0.4 m above the floor.

Phase 1: Q falls 0.4 m (and P travels 0.4 m), with $a = 4.2$ m s^{-2}

Using $s = ut + \frac{1}{2}at^2$ with $u = 0$, $s = 0.4$ m:

$$0.4 = \tfrac{1}{2} \times 4.2 \times t_1^2$$
$$t_1^2 = \frac{0.8}{4.2} \implies t_1 = 0.436 \text{ s}$$

Speed of P (and Q) at the moment Q hits the floor:

$$v = at_1 = 4.2 \times 0.436 = 1.833 \text{ m s}^{-1}$$
$$\left(\text{or equivalently: } v = \sqrt{2 \times 4.2 \times 0.4} = \sqrt{3.36} = 1.833 \text{ m s}^{-1}\right)$$

Phase 2: Q hits the floor and does not rebound — string goes slack

Once Q hits the floor the string becomes slack. The table is smooth, so there is no friction acting on P. Therefore P continues at **constant velocity** $v = 1.833 \text{ m s}^{-1}$.

Remaining distance for P to reach the pulley:

$$1.5 - 0.4 = 1.1 \text{ m}$$

$$t_2 = \frac{1.1}{1.833} = 0.600 \text{ s}$$

Total time

$$t = t_1 + t_2 = 0.436 + 0.600 = \boxed{1.0 \text{ s}}$$

Part (c) — State one limitation of the model

Any *one* of the following limitations of the modelling assumptions stated in the question is acceptable:

- The rope may not be perfectly **light** (i.e. it has mass in reality).
- The rope may not be perfectly **inextensible** (it may stretch slightly).
- The pulley may not be perfectly **smooth** (friction at the pulley would affect the tension).
- The pulley may not be perfectly **small**.
- The balls may not behave as true **particles** (they have size and may rotate).

Note: *Air resistance* and *the table being smooth* are **not** accepted answers for this part, as these are not limitations of the model *as stated in the question*.

Connected Particles — Q2 Full Solution

A-Level Mechanics, Year 1

Setup

Ball P has mass $2m$ and hangs at height $2h$. Ball Q has mass $5m$ and hangs at height h. Since Q is heavier, Q descends and P rises when released. Let the tension be T and the common magnitude of acceleration be a.

Part (a) — Equations of motion

(i) Equation of motion for P (taking upwards as positive for P):

$$T - 2mg = 2ma \tag{1}$$

(ii) Equation of motion for Q (taking downwards as positive for Q):

$$5mg - T = 5ma \tag{2}$$

Part (b) — Height above ground at which P first comes to instantaneous rest

Step 1: Find the acceleration

Adding equations (1) and (2):

$$3mg = 7ma \implies a = \frac{3g}{7}$$

Step 2: Find the speed of P when Q hits the ground

Q falls a distance h before hitting the ground. Using $v^2 = u^2 + 2as$ with $u = 0$:

$$v^2 = 2 \times \frac{3g}{7} \times h = \frac{6gh}{7}$$

Step 3: Motion of P after Q hits the ground

Q does not rebound, so the string goes slack. At this instant P is moving upwards with speed $v = \sqrt{\frac{6gh}{7}}$ and is now at height $2h + h = 3h$ (it has risen by h while Q fell).

P then moves freely under gravity (decelerating), travelling a further distance H upward before coming to instantaneous rest. Using $v^2 = u^2 - 2gH$:

$$0 = \frac{6gh}{7} - 2gH \implies H = \frac{3h}{7}$$

Step 4: Total height above the ground

$$\text{Total height} = 3h + H = 3h + \frac{3h}{7} = \frac{21h + 3h}{7}$$

$$\boxed{\text{Height} = \frac{24h}{7}}$$

Part (c) — Limitation of modelling balls as particles

Since the balls are modelled as particles, their size is ignored. In reality the balls have finite size, so the distance Q falls to the ground would not be exactly h — it would be reduced by the radius of the ball. This would affect the speed at which the string goes slack and hence the final height of P.

Part (d) — Effect of the string not being inextensible

If the string is not inextensible, it will stretch when the system is released. This means the accelerations of the two balls would **not have equal magnitude** — the string would absorb some of the motion, so the actual accelerations would be smaller than predicted by the model.

Connected Particles — Q3 Full Solution

A-Level Mechanics, Year 1

Setup

Particle A has mass $5m$ and particle B has mass km where $k < 5$. Since $k < 5$, A is heavier and descends. The acceleration of A as it descends is given as $\frac{1}{4}g$. Let T be the tension in the string.

Part (a) — Show that the tension is $\frac{15}{4}mg$

Equation of motion for A (taking downwards as positive):

$$5mg - T = 5m \times \frac{1}{4}g = \frac{5mg}{4}$$

$$T = 5mg - \frac{5mg}{4} = \frac{20mg - 5mg}{4}$$

$$\boxed{T = \frac{15}{4}mg} \checkmark$$

Part (b) — Find the value of k

Equation of motion for B (taking upwards as positive):

$$T - kmg = km \times \frac{1}{4}g$$

$$\frac{15}{4}mg = kmg + \frac{kmg}{4} = kmg\left(1 + \frac{1}{4}\right) = \frac{5kmg}{4}$$

$$\frac{15}{4}mg = \frac{5k}{4}mg \implies 15 = 5k$$

$$\boxed{k = 3}$$

Part (c) — How the smooth pulley assumption was used

Since the pulley is smooth, the tension in the string is the **same on both sides** of the pulley. This allowed us to use the single value T in both equations of motion.

Part (d) — Greatest height reached by B above the plane

Step 1: Speed of A (and B) when A hits the plane

A descends from rest with acceleration $\frac{1}{4}g$ for $t = 1.2$ s.

$$v = u + at = 0 + \frac{1}{4}g \times 1.2 = 0.3g \approx 2.94 \text{ m s}^{-1}$$

Distance A has descended (= distance B has risen):

$$s_1 = \frac{1}{2} \times \frac{1}{4}g \times 1.2^2 = \frac{1}{8}g \times 1.44 = 0.18g \approx 1.764 \text{ m}$$

Step 2: Motion of B after A hits the plane

When A hits the plane it is immediately brought to rest, so the string goes slack. B is now moving **upwards** at $v = 0.3g$ m s^{-1} and moves freely under gravity.

Additional distance B travels upward before stopping:

$$0 = v^2 - 2gs_2 \implies s_2 = \frac{(0.3g)^2}{2g} = \frac{0.09g^2}{2g} = \frac{0.09g}{2} \approx 0.441 \text{ m}$$

Step 3: Total height of B above the plane

B started at the same height as A (both on the plane level at the start, but the initial distance A was above the plane equals s_1 by symmetry of the setup). The greatest height reached by B is:

$$S = 2s_1 + s_2 = 2 \times 0.18g + \frac{0.09g}{2} = 0.36g + 0.045g$$

$$S = 0.405g \approx \boxed{4.0 \text{ m}}$$

Connected Particles — Q4 Full Solution

A-Level Mechanics, Year 1

Setup

A vertical light rod PQ has:

- Particle of mass 0.5 kg at P (top)
- Particle of mass 0.75 kg at Q (bottom)

A force of 15 N is applied **upwards** at Q. Let:

- T = thrust in the rod (positive = compression/thrust upward on P, downward on Q)
- a = upward acceleration of the whole system

Solution — Find the thrust in the rod

Step 1: Equation of motion for the whole system (upwards positive)

Taking the entire system ($0.5 + 0.75 = 1.25$ kg) with the applied force 15 N upward:

$$15 - 0.5g - 0.75g = 1.25a$$

$$15 - 1.25g = 1.25a$$
$$15 - 1.25 \times 9.8 = 1.25a$$
$$15 - 12.25 = 1.25a \implies 2.75 = 1.25a$$
$$a = 2.2 \text{ m s}^{-2} \text{ (upwards)}$$

Step 2: Equation of motion for particle P alone (upwards positive)

The only forces on P are its weight ($0.5g$ downward) and the thrust T from the rod (upward on P, since the rod is in compression — it pushes P up):

$$T - 0.5g = 0.5a$$

$$T = 0.5g + 0.5a = 0.5 \times 9.8 + 0.5 \times 2.2$$
$$T = 4.9 + 1.1$$

$$\boxed{T = 6 \text{ N}}$$

The thrust in the rod is 6 N.

Check using particle Q:
Forces on Q: 15 N up, $0.75g$ down, thrust T down (rod pushes Q downward).

$$15 - T - 0.75g = 0.75a \implies 15 - 6 - 7.35 = 0.75 \times 2.2 = 1.65 \checkmark$$

Connected Particles — Q5 Full Solution

A-Level Mechanics, Year 1

Setup

- Scale pan: mass 0.5 kg
- Brick: mass 1.5 kg
- Total system mass: 2.0 kg
- Constant upward acceleration: $a = 0.5 \text{ m s}^{-2}$
- Tension in rope AB: T N
- Normal reaction of scale pan on brick: R N

Part (a) — Find the tension in rope AB

Applying Newton's second law to the **whole system** (pan + brick, total mass 2.0 kg), taking upwards as positive:

$$T - 0.5g - 1.5g = 2.0 \times 0.5$$

$$T - 2.0g = 1.0$$

$$T = 2.0 \times 9.8 + 1.0 = 19.6 + 1.0$$

$$\boxed{T = 20.6 \text{ N}}$$

Part (b) — Force exerted on the scale pan by the brick

Applying Newton's second law to the **brick alone** (mass 1.5 kg), taking upwards as positive. The forces on the brick are: R upward (reaction from pan) and $1.5g$ downward.

$$R - 1.5g = 1.5 \times 0.5$$

$$R = 1.5g + 0.75 = 1.5 \times 9.8 + 0.75 = 14.7 + 0.75$$

$$\boxed{R = 15.5 \text{ N}}$$

Check using the scale pan alone:
Forces on pan: T up, $0.5g$ down, R down (brick pushes down on pan by Newton's

3rd Law).

$$T - R - 0.5g = 0.5 \times 0.5 \implies 20.6 - 15.5 - 4.9 = 0.25 \checkmark$$

Connected Particles — Q6 Full Solution

A-Level Mechanics, Year 1

Setup

- Particle A: mass $3m$, moves **upward**
- Particle B: mass $2m$, with C (mass $2m$) attached $\Rightarrow$ combined mass $4m$, moves **downward**
- String tension: T; common acceleration magnitude: a

Part (a) — Show acceleration of A is $\frac{g}{7}$; find tension

(i) Show acceleration is $\frac{g}{7}$

Equation of motion for the B+C side (downward positive, total mass $4m$):

$$4mg - T = 4ma \tag{1}$$

Equation of motion for A (upward positive, mass $3m$):

$$T - 3mg = 3ma \tag{2}$$

Adding (1) and (2):

$$4mg - 3mg = 7ma \implies mg = 7ma$$

$$\boxed{a = \frac{g}{7}} \checkmark$$

(ii) Find the tension

Substituting $a = \dfrac{g}{7}$ into equation (2):

$$T = 3mg + 3m \cdot \frac{g}{7} = 3mg\left(1 + \frac{1}{7}\right) = 3mg \cdot \frac{8}{7}$$

$$\boxed{T = \frac{24mg}{7} \approx 33.6m \text{ N}}$$

Part (b) — Speed of A when it is 0.7 m above its original position

Using $v^2 = u^2 + 2as$ with $u = 0$, $s = 0.7$ m, $a = \frac{g}{7}$:

$$v^2 = 2 \times \frac{g}{7} \times 0.7 = \frac{2 \times 9.8 \times 0.7}{7} = \frac{13.72}{7} = 1.96$$

$$\boxed{v = 1.4\ \mathrm{m\,s^{-1}}}$$

Part (c) — Acceleration of A after C separates from B

After C falls away, the system is now A (mass $3m$) vs. B alone (mass $2m$). A is still moving upward and B upward (since A was rising, B was falling, but now B has mass $2m$). Let the new acceleration magnitude be a'.

Equation of motion for B (downward positive, mass $2m$):

$$2mg - T' = 2ma' \tag{3}$$

Equation of motion for A (upward positive, mass $3m$):

$$T' - 3mg = 3ma' \tag{4}$$

Adding (3) and (4):

$$2mg - 3mg = 5ma' \implies -mg = 5ma' \implies a' = -\frac{g}{5}$$

The negative sign means the net acceleration of A is now **downward** (decelerating), since B (mass $2m$) is now lighter than A (mass $3m$).

$$\boxed{a' = \frac{g}{5} \text{ downward on } A}$$

Part (d) — Greatest height reached by A above its original position

After C separates, A has speed $1.4\ \mathrm{m\,s^{-1}}$ upward and decelerates at $\frac{g}{5}$. Additional distance s travelled before coming to rest:

$$0 = 1.4^2 - 2 \times \frac{g}{5} \times s \implies s = \frac{1.96 \times 5}{2g} = \frac{9.8}{2 \times 9.8} = 0.5\ \text{m}$$

Total height above original position:

$$0.7 + 0.5 = 1.2\ \text{m}$$

Greatest height = 1.2 m above original position

Connected Particles — Q7 Full Solution

A-Level Mechanics, Year 1

Setup

- Lift mass: 200 kg, descending with constant acceleration a m s^{-2} downward
- Crate mass: 55 kg, inside the lift
- Upward resistance on lift: 150 N
- Normal reaction on crate from floor: $R = 473$ N
- Cable tension: T N (upward on lift)

Taking **downward as positive** for the direction of motion.

Part (a) — Find the acceleration of the lift

Apply Newton's second law to the **crate alone** (mass 55 kg). Forces on the crate: $55g$ downward, 473 N upward (normal reaction from floor).

$$55g - 473 = 55a$$

$$55 \times 9.8 - 473 = 55a$$

$$539 - 473 = 55a \implies 66 = 55a$$

$$\boxed{a = 1.2 \text{ m s}^{-2} \text{ (downward)}}$$

Part (b) — Force exerted on the lift by the cable

Apply Newton's second law to the **whole system** (lift + crate, total mass 255 kg). Forces: $255g$ downward, T upward, 150 N resistance upward.

$$255g - T - 150 = 255a$$

$$255 \times 9.8 - T - 150 = 255 \times 1.2$$

$$2499 - T - 150 = 306$$

$$T = 2499 - 150 - 306 = 2043$$

$$\boxed{T = 2040 \text{ N} \quad \text{(or 2000 N using } g = 10)}$$

Alternative — using the lift alone:
Forces on lift: $200g$ downward, T upward, 150 N resistance upward, 473 N downward (crate pushes down on lift floor by Newton's 3rd Law).

$$200g + 473 - T - 150 = 200 \times 1.2$$

$$1960 + 473 - T - 150 = 240 \implies T = 2043 \approx 2040 \text{ N}\checkmark$$

Connected Particles — Q8 Full Solution

A-Level Mechanics, Year 1

Setup

- Car: mass 1000 kg, resistance 800 N, driving force 3200 N
- Caravan: mass 750 kg, resistance R N
- Tow-bar tension: T N
- Acceleration: $a = 0.88\ \mathrm{m\,s^{-2}}$

Taking the direction of motion as positive throughout.

Part (a) — Show that $R = 860$

Apply Newton's second law to the **whole system** (car + caravan, total mass 1750 kg). The tow-bar forces are internal and cancel, leaving only the driving force and both resistances:

$$3200 - 800 - R = 1750 \times 0.88$$
$$2400 - R = 1540$$
$$R = 2400 - 1540$$

$$\boxed{R = 860 \text{ N}}\checkmark$$

Part (b) — Find the tension in the tow-bar

Apply Newton's second law to the **caravan alone** (mass 750 kg). Forces: tension T forward, resistance 860 N backward.

$$T - 860 = 750 \times 0.88$$
$$T = 660 + 860$$

$$\boxed{T = 1520 \text{ N}}$$

Alternative — using the car alone:
Forces on car: 3200 N forward, 800 N resistance backward, T backward (reaction from tow-bar).

$$3200 - 800 - T = 1000 \times 0.88 \implies 2400 - T = 880 \implies T = 1520 \text{ N}\checkmark$$

Connected Particles — Q9 Full Solution

A-Level Mechanics, Year 1

Setup

- Particle P: mass 0.5 kg, starts 3.15 m above ground, descends
- Particle Q: mass m kg where $m < 0.5$, ascends
- P hits the ground after $t = 1.5$ s from rest

Part (a) — Show acceleration of P is 2.8 m s^{-2}

Using $s = ut + \frac{1}{2}at^2$ with $u = 0$, $s = 3.15$ m, $t = 1.5$ s:

$$3.15 = \frac{1}{2} \times a \times 1.5^2 = \frac{1}{2} \times a \times \frac{9}{4}$$

$$3.15 = \frac{9a}{8} \implies a = \frac{3.15 \times 8}{9} = \frac{25.2}{9}$$

$$\boxed{a = 2.8 \text{ m s}^{-2}} \checkmark$$

Part (b) — Find the tension in the string as P descends

Equation of motion for P (downward positive):

$$0.5g - T = 0.5 \times 2.8$$

$$4.9 - T = 1.4$$

$$\boxed{T = 3.5 \text{ N}}$$

Part (c) — Show that $m = \frac{5}{18}$

Equation of motion for Q (upward positive):

$$T - mg = ma$$

$$3.5 - 9.8m = 2.8m$$

$$3.5 = 12.6m \implies m = \frac{3.5}{12.6} = \frac{35}{126} = \frac{5}{18}$$

$$\boxed{m = \frac{5}{18}} \checkmark$$

Part (d) — How the inextensible string assumption was used

Since the string is inextensible, both particles have the **same speed and the same magnitude of acceleration** at every instant. This allowed us to use a single value of a in both equations of motion.

Part (e) — Time between P hitting the ground and string becoming taut again

Step 1: Speed of Q when P hits the ground

Q has been accelerating upward at 2.8 m s^{-2} for 1.5 s:

$$v = u + at = 0 + 2.8 \times 1.5 = 4.2 \text{ m s}^{-1} \text{ (upward)}$$

Step 2: Motion of Q after string goes slack

When P hits the ground the string becomes slack. Q now moves freely under gravity (decelerating upward, then falling back down). The string becomes taut again when Q returns to the same height it was at when the string went slack (since P is stationary on the ground and the string length is fixed).

So the string becomes taut when Q returns to its position at the instant P hit the ground. This means Q travels up and comes back to the same point — the time for this is found using $v = u + at$ where the velocity returns to -4.2 m s^{-1} (downward):

$$-4.2 = 4.2 - 9.8t$$

$$9.8t = 8.4 \implies t = \frac{8.4}{9.8} = \frac{6}{7}$$

$$\boxed{t = \frac{6}{7} \approx 0.857 \text{ s}}$$

Connected Particles — Q10 Full Solution

A-Level Mechanics, Year 1

Setup

- Particle A: mass 0.4 kg, descends
- Particle B: mass 0.3 kg, ascends
- Both start at height 1 m above the floor, from rest
- String breaks after 0.5 s of motion

Part (a) — Tension in the string immediately after release

Let T be the tension and a the common acceleration magnitude.

Equation of motion for A (downward positive):

$$0.4g - T = 0.4a \tag{1}$$

Equation of motion for B (upward positive):

$$T - 0.3g = 0.3a \tag{2}$$

Adding (1) and (2):

$$0.4g - 0.3g = 0.7a \implies 0.1g = 0.7a \implies a = \frac{g}{7} = 1.4\ \mathrm{m\,s^{-2}}$$

Substituting into (2):

$$T = 0.3g + 0.3 \times \frac{g}{7} = 0.3g\left(\frac{8}{7}\right) = \frac{2.4g}{7}$$

$$T = \frac{2.4 \times 9.8}{7} = \frac{23.52}{7}$$

$$\boxed{T = \frac{12g}{35} \approx 3.36\ \mathrm{N}}$$

Part (b) — Acceleration of A immediately after release

The acceleration was already found in part (a) when solving the simultaneous equations. Returning to equation (1):

$$0.4g - T = 0.4a$$

Substituting the tension $T = \dfrac{12g}{35}$ found in part (a):

$$0.4g - \frac{12g}{35} = 0.4a$$

$$\frac{14g}{35} - \frac{12g}{35} = 0.4a$$

$$\frac{2g}{35} = 0.4a \implies a = \frac{2g}{35 \times 0.4} = \frac{2g}{14} = \frac{g}{7}$$

$$a = \frac{9.8}{7} = 1.4\ \mathrm{m\,s^{-2}}$$

Alternatively, from adding the two equations of motion directly:

$$0.4g - 0.3g = (0.4 + 0.3)a \implies 0.1 \times 9.8 = 0.7a \implies a = \frac{0.98}{0.7} = 1.4\ \mathrm{m\,s^{-2}}$$

$$\boxed{a = \frac{g}{7} = 1.4\ \mathrm{m\,s^{-2}} \text{ downward (for } A\text{), upward (for } B\text{)}}$$

Part (c) — Further time until B hits the floor after string breaks

Step 1: Velocity of B when the string breaks (at $t = 0.5$ s)

B has been moving upward with acceleration $\dfrac{g}{7} = 1.4\ \mathrm{m\,s^{-2}}$:

$$v_B = 0 + 1.4 \times 0.5 = 0.7\ \mathrm{m\,s^{-1}} \text{ (upward)}$$

Step 2: Height of B when string breaks

B started at height 1 m and has risen:

$$s = \frac{1}{2} \times 1.4 \times 0.5^2 = \frac{1}{2} \times 1.4 \times 0.25 = 0.175 \text{ m}$$

So B is at height $1 + 0.175 = 1.175$ m above the floor.

Step 3: Time for B to fall to the floor

After the string breaks, B moves freely under gravity. Taking **downward as positive**, B has initial velocity $-0.7\ \mathrm{m\,s^{-1}}$ (i.e. $0.7\ \mathrm{m\,s^{-1}}$ upward) and must travel 1.175 m downward to reach the floor:

$$1.175 = -0.7t + \frac{1}{2} \times 9.8 \times t^2$$
$$4.9t^2 - 0.7t - 1.175 = 0$$

Using the quadratic formula:

$$t = \frac{0.7 \pm \sqrt{0.49 + 4 \times 4.9 \times 1.175}}{2 \times 4.9} = \frac{0.7 \pm \sqrt{0.49 + 23.03}}{9.8} = \frac{0.7 \pm \sqrt{23.52}}{9.8}$$
$$t = \frac{0.7 \pm 4.8497}{9.8}$$

Taking the positive root:

$$t = \frac{0.7 + 4.8497}{9.8} = \frac{5.5497}{9.8}$$

$$\boxed{t \approx 0.566\ \mathrm{s} \approx 0.57\ \mathrm{s}}$$