Questions

Q1.

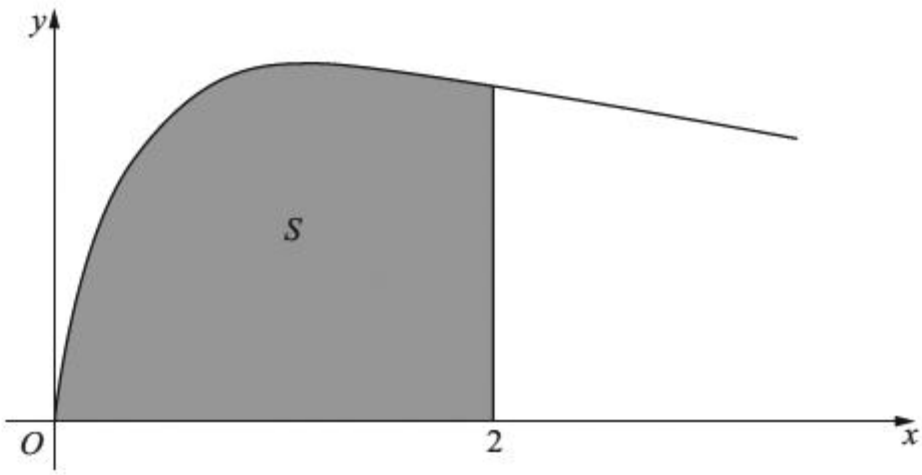

Figure 1

Figure 1 shows the curve with equation

$$y = \sqrt{\left(\frac{2x}{3x^2 + 4}\right)}, \quad x \geqslant 0$$

The finite region S, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 2$

The region S is rotated 360° about the x-axis.

Use integration to find the exact value of the volume of the solid generated, giving your answer in the form $k \ln a$, where k and a are constants.

(5)

(Total 5 marks)

(Q01 6666/01, Jan 2012)

Q2.

(a) Express $\frac{25}{x^2(2x+1)}$ in partial fractions.

(4)

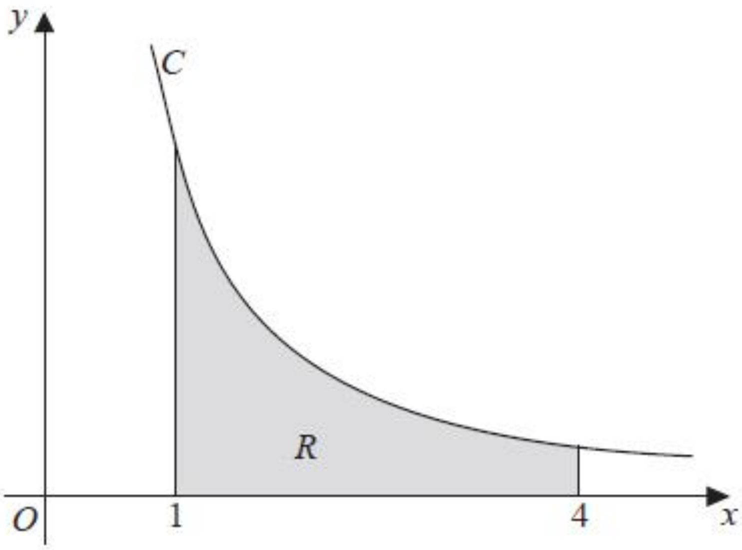

Figure 2

Figure 2 shows a sketch of part of the curve C with equation $y = \frac{5}{x\sqrt{(2x+1)}}$, $x > 0$

The finite region R is bounded by the curve C, the x-axis, the line with equation $x = 1$ and the line with equation $x = 4$

This region is shown shaded in Figure 2

The region R is rotated through 360° about the x-axis.

(b) Use calculus to find the exact volume of the solid of revolution generated, giving your answer in the form $a + b\ln c$, where a, b and c are constants.

(6)

(Total 10 marks)

(Q01 6666/01/R, June 2014)

Q3.

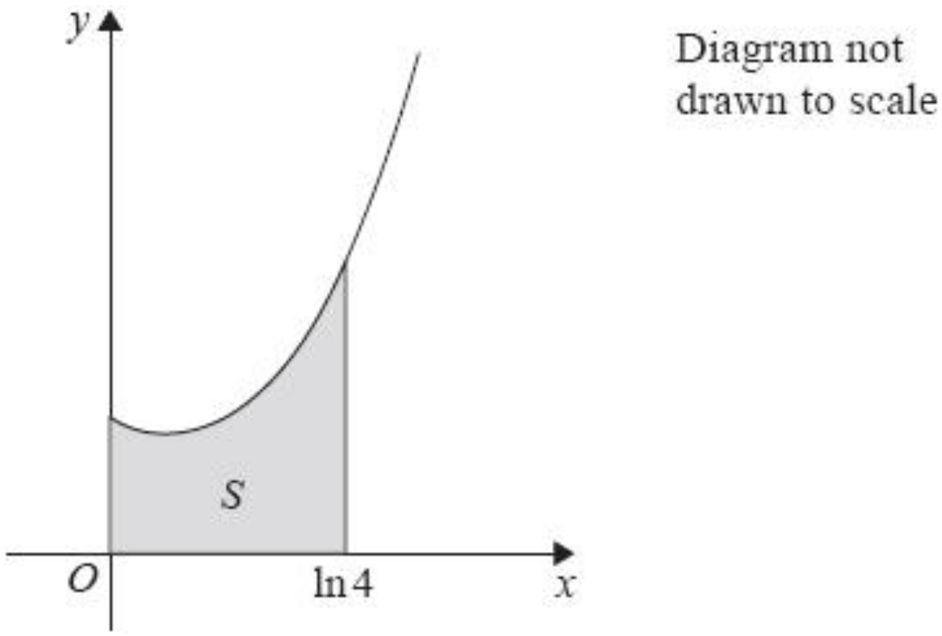

Figure 2

The finite region S, shown shaded in Figure 2, is bounded by the y-axis, the x-axis, the line with equation $x = \ln 4$ and the curve with equation

$$y = e^x + 2e^{-x}, \qquad x \geq 0$$

The region S is rotated through 2π radians about the x-axis.

Use integration to find the exact value of the volume of the solid generated.
Give your answer in its simplest form.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(7)

(Total for question = 7 marks)

(Q01 6666/01, June 2017)

Q4.

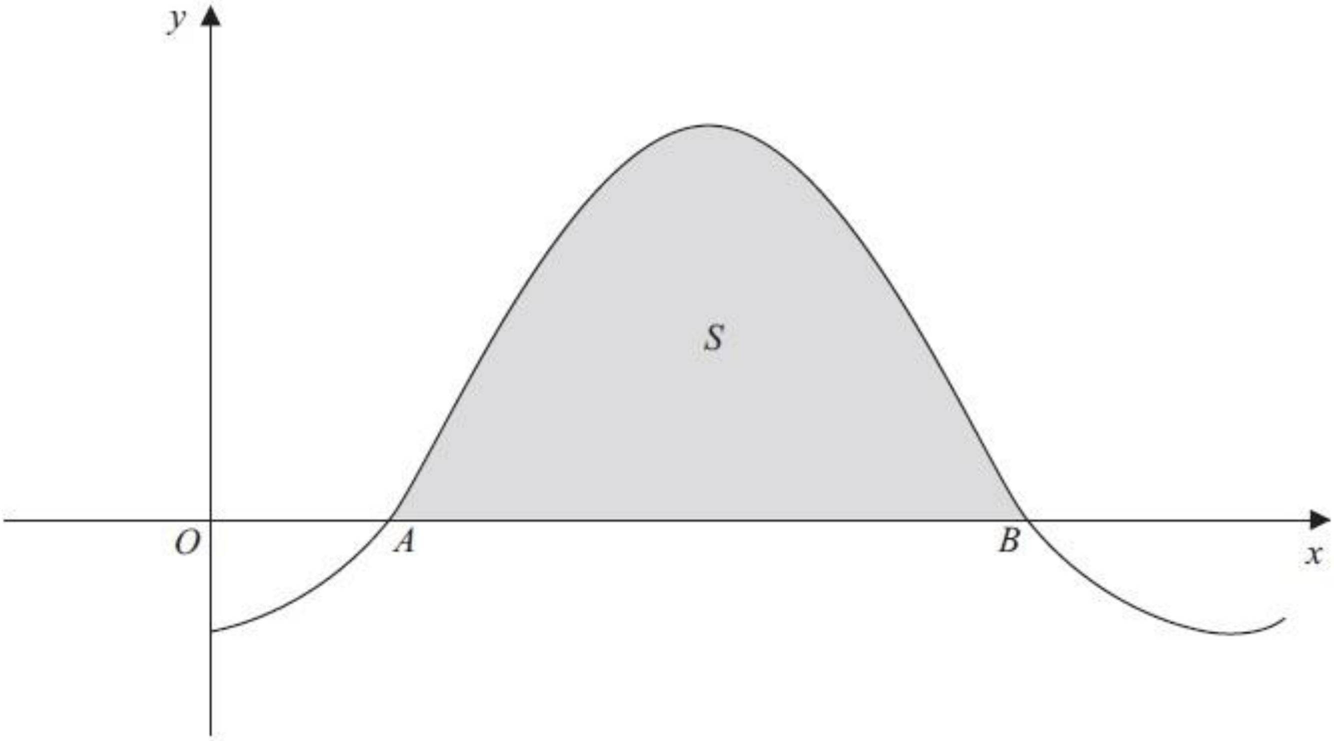

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = 1 - 2\cos x$, where x is measured in radians. The curve crosses the x-axis at the point A and at the point B.

(a) Find, in terms of π, the x coordinate of the point A and the x coordinate of the point B.

(3)

The finite region S enclosed by the curve and the x-axis is shown shaded in Figure 3. The region S is rotated through 2π radians about the x-axis.

(b) Find, by integration, the exact value of the volume of the solid generated.

(6)

(Total 9 marks)

(Q01 9FM0/01, Jan 2013)

Q5.

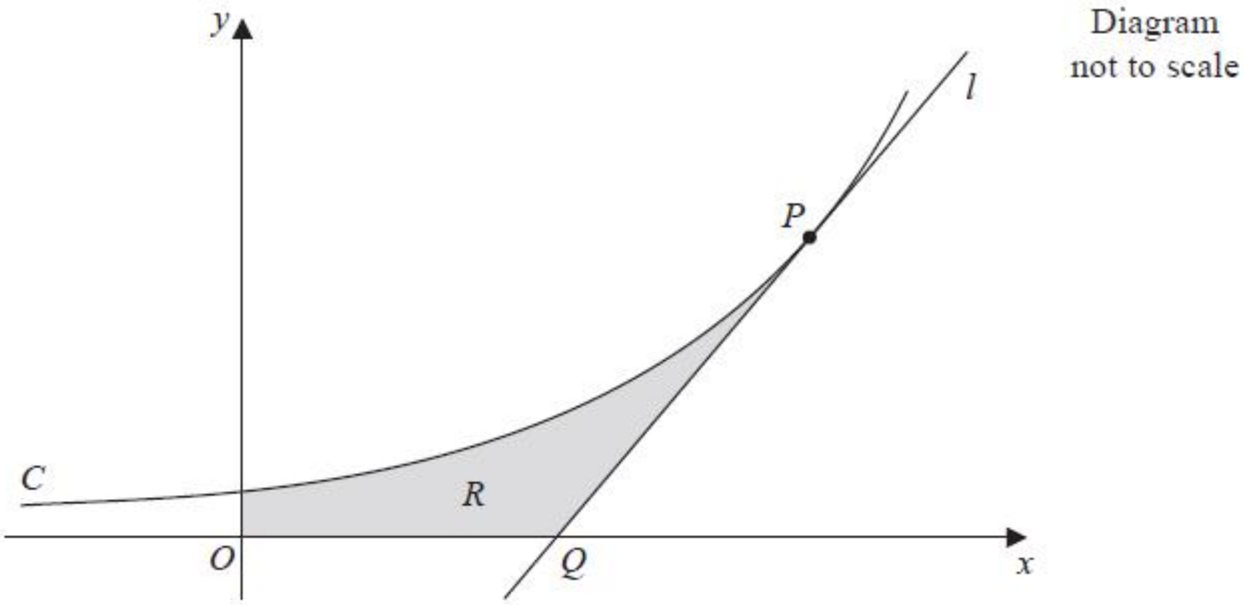

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = 3^x$$

The point P lies on C and has coordinates (2, 9).

The line l is a tangent to C at P. The line l cuts the x-axis at the point Q.

(a) Find the exact value of the x coordinate of Q.

(4)

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the x-axis, the y-axis and the line l. This region R is rotated through 360° about the x-axis.

(b) Use integration to find the exact value of the volume of the solid generated.

Give your answer in the form $\frac{p}{q}$ where p and q are exact constants.

[*You may assume the formula* $V = \frac{1}{3}\pi r^2 h$ *for the volume of a cone.*]

(6)

(Total for question = 10 marks)

(Q02 6666/01, June 2015)

Q6.

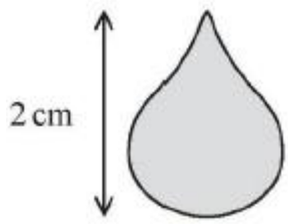

Figure 2

Figure 2 shows the image of a gold pendant which has height 2 cm. The pendant is modelled by a solid of revolution of a curve C about the y-axis. The curve C has parametric equations

$$x = \cos\theta + \frac{1}{2}\sin 2\theta, \quad y = -(1+\sin\theta) \qquad 0 \leqslant \theta \leqslant 2\pi$$

(a) Show that a Cartesian equation of the curve C is

$$x^2 = -\left(y^4 + 2y^3\right)$$

(4)

(b) Hence, using the model, find, in cm^3, the volume of the pendant.

(4)

(Total for question = 8 marks)

(Q07 9FM0/01, Specimen papers)

Q7.

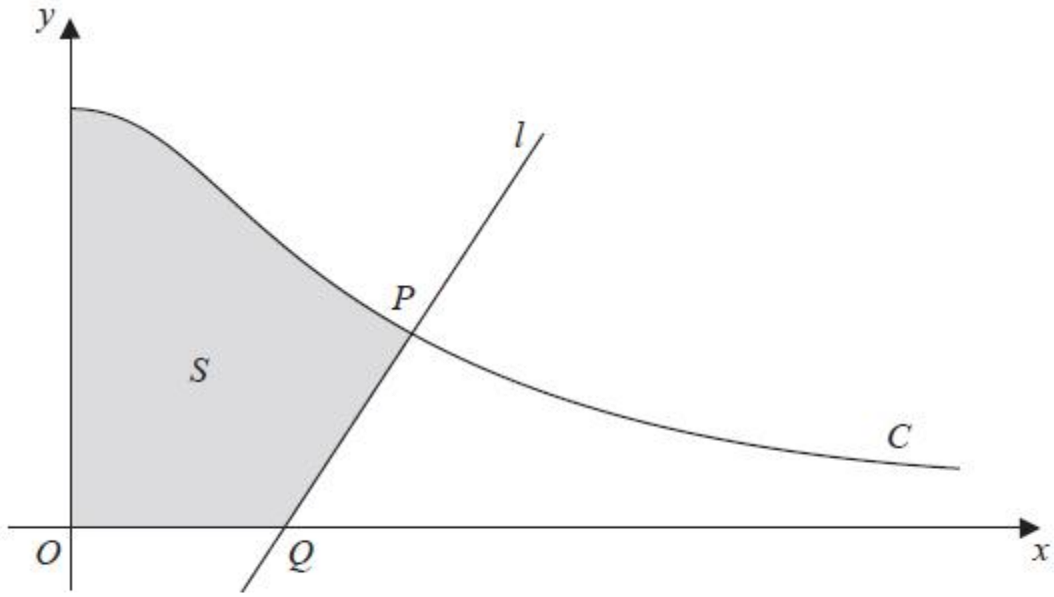

Figure 4

Figure 4 shows a sketch of part of the curve C with parametric equations

$$x = 3\tan\theta, \quad y = 4\cos^2\theta, \quad 0 \leq \theta < \tfrac{\pi}{2}$$

The point P lies on C and has coordinates (3, 2).

The line l is the normal to C at P. The normal cuts the x-axis at the point Q.

(a) Find the x coordinate of the point Q.

(6)

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the x-axis, the y-axis and the line l. This shaded region is rotated 2π radians about the x-axis to form a solid of revolution.

(b) Find the exact value of the volume of the solid of revolution, giving your answer in the form $p\pi + q\pi^2$, where p and q are rational numbers to be determined.

[*You may use the formula* $V = \frac{1}{3}\pi r^2 h$ *for the volume of a cone.*]

(9)

(Total 15 marks)

(Q01 6666/01, June 2014)

Q8.

(a) Using the identity $\cos 2\theta = 1 - 2\sin^2\theta$, find

$$\int \sin^2\theta \, d\theta .$$

(2)

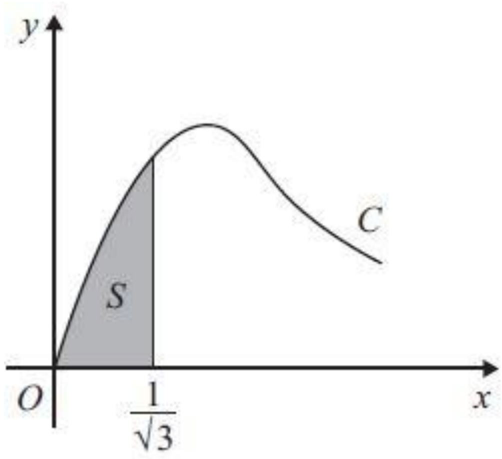

Figure 4

Figure 4 shows part of the curve C with parametric equations

$$x = \tan\theta, \qquad y = 2\sin 2\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The finite shaded region S shown in Figure 4 is bounded by C, the line $x = \frac{1}{\sqrt{3}}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Show that the volume of the solid of revolution formed is given by the integral

$$k\int_0^{\frac{\pi}{6}} \sin^2\theta \, d\theta$$

where k is a constant.

(5)

(c) Hence find the exact value for this volume, giving your answer in the form $p\pi^2 + q\pi\sqrt{3}$, where p and q are constants.

(3)

(Total 10 marks)

(Q02 6666/01, June 2009)

Q9.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

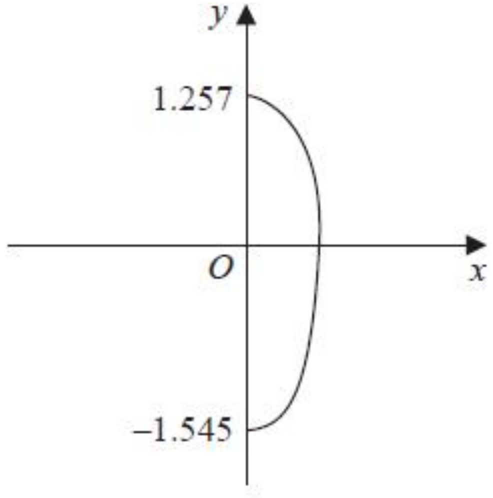

Figure 2

John picked 100 berries from a plant.

The largest berry picked was approximately 2.8 cm long.

The shape of this berry is modelled by rotating the curve with equation

$$16x^2 + 3y^2 - y\cos\left(\frac{5}{2}y\right) = 6 \qquad x \geqslant 0$$

shown in Figure 2, about the y-axis through 2π radians, where the units are cm.

Given that the y intercepts of the curve are −1.545 and 1.257 to four significant figures,

(a) use algebraic integration to determine, according to the model, the volume of this berry.

(6)

Given that the 100 berries John picked were then squeezed for juice,

(b) use your answer to part (a) to decide whether, in reality, there is likely to be enough juice to fill a 200 cm^3 cup, giving a reason for your answer.

(2)

(Total for question = 8 marks)

(Q07 9FM0/02, June 2023)