## *Summations - Method Of Differences Exam Questions (Edexcel)*

**Q1.**

(a) Express $\dfrac{5r+4}{r(r+1)(r+2)}$ in partial fractions.

(4)

(b) Hence, or otherwise, show that

$$\sum_{r=1}^{n} \frac{5r+4}{r(r+1)(r+2)} = \frac{7n^2+11n}{2(n+1)(n+2)}.$$

(5)

**(Total 9 marks)**

**(Q05 6674/01, Jan 2008)**

**Q2.**

Prove that, for $n \in \mathbb{Z}$, $n \geqslant 0$

$$\sum_{r=0}^{n} \frac{1}{(r+1)(r+2)(r+3)} = \frac{(n+a)(n+b)}{c(n+2)(n+3)}$$

where $a$, $b$ and $c$ are integers to be found.

**(Total for question = 5 marks)**

**(Q04 9FM0/01, June 2019)**

**Q3.**

(a) Use the method of differences to prove that for $n > 2$

$$\sum_{r=2}^{n} \ln\left(\frac{r+1}{r-1}\right) \equiv \ln\left(\frac{n(n+1)}{2}\right)$$

(4)

(b) Hence find the exact value of

$$\sum_{r=51}^{100} \ln\left(\frac{r+1}{r-1}\right)^{35}$$

Give your answer in the form $a \ln\left(\dfrac{b}{c}\right)$ where $a$, $b$ and $c$ are integers to be determined.

(3)

**(Total for question = 7 marks)**

**(Q04 9FM0/01, June 2022)**

**Q4.**

Prove that

$$\sum_{r=1}^{n} \frac{1}{(r+1)(r+3)} = \frac{n(an+b)}{12(n+2)(n+3)}$$

where $a$ and $b$ are constants to be found.

**(5)**

**(Total for question = 5 marks)**

**(Q01 9FM0/01, Specimen papers )**

**Q5.**

(a) Show that, for $r > 0$

$$\frac{1}{r^2} - \frac{1}{(r+1)^2} \equiv \frac{2r+1}{r^2(r+1)^2}$$

**(1)**

(b) Hence prove that, for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2} = \frac{n(n+2)}{(n+1)^2}$$

**(3)**

(c) Show that, for $n \in \mathbb{N}$, $n > 1$

$$\sum_{r=n}^{3n} \frac{6r+3}{r^2(r+1)^2} = \frac{an^2+bn+c}{n^2(3n+1)^2}$$

where $a$, $b$ and $c$ are constants to be found.

**(3)**

**(Total for question = 7 marks)**

**(Q09 6668/01, June 2017)**

**Q6.**

(a) Show that, for $r > 0$

$$r - 3 + \frac{1}{r+1} - \frac{1}{r+2} = \frac{r^3 - 7r - 5}{(r+1)(r+2)}$$

(2)

(b) Hence prove, using the method of differences, that

$$\sum_{r=1}^{n} \frac{r^3 - 7r - 5}{(r+1)(r+2)} = \frac{n(n^2 + an + b)}{2(n+2)}$$

where $a$ and $b$ are constants to be found.

(5)

**(Total for question = 7 marks)**

**(Q08 6668/01, June 2016)**

**Q7.**

(a) Express $\frac{2}{4r^2 - 1}$ in partial fractions.

(2)

(b) Hence use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{4r^2 - 1} = \frac{n}{2n+1}$$

(3)

**(Total 5 marks)**

**(Q08 6668/01/R, June 2014)**

**Q8.**

(a) Express $\frac{2}{(r+1)(r+3)}$ in partial fractions.

(2)

(b) Hence show that

$$\sum_{r=1}^{n} \frac{2}{(r+1)(r+3)} = \frac{n(5n+13)}{6(n+2)(n+3)}$$

(4)

(c) Evaluate $\sum_{r=10}^{100} \frac{2}{(r+1)(r+3)}$, giving your answer to 3 significant figures.

(2)

**(Total 8 marks)**

**(Q07 6668/01/R, June 2013)**

**Q9.**

(a) Express $\dfrac{1}{r(r+2)}$ in partial fractions.

**(2)**

(b) Hence prove, by the method of differences, that

$$\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{n(an+b)}{4(n+1)(n+2)}$$

where $a$ and $b$ are constants to be found.

**(6)**

(c) Hence show that

$$\sum_{r=n+1}^{2n} \frac{1}{r(r+2)} = \frac{n(4n+5)}{4(n+1)(n+2)(2n+1)}$$

**(3)**

**(Total 11 marks)**

**(Q11 6668/01, June 2012)**

**Q10.**

(a) Express $\dfrac{2}{(2r+1)(2r+3)}$ in partial fractions.

**(2)**

(b) Using your answer to (a), find, in terms of $n$,

$$\sum_{r=1}^{n} \frac{3}{(2r+1)(2r+3)}$$

Give your answer as a single fraction in its simplest form.

**(3)**

**(Total 5 marks)**

**(Q07 6668/01, June 2013)**