Second Order Differential Equations (From OCR 4727)

Q1, (Jun 2007, Q3)

Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = e^{3x}. \qquad [6]$$

Q2, (Jan 2008, Q2)

Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 4x. \qquad [7]$$

Q3, (Jun 2008, Q8)

(i) Find the complementary function of the differential equation

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x. \qquad [2]$$

(ii) It is given that $y = p(\ln \sin x)\sin x + qx\cos x$, where p and q are constants, is a particular integral of this differential equation.

(a) Show that $p - 2(p+q)\sin^2 x \equiv 1$. [6]

(b) Deduce the values of p and q. [2]

(iii) Write down the general solution of the differential equation. State the set of values of x, in the interval $0 \leqslant x \leqslant 2\pi$, for which the solution is valid, justifying your answer. [3]

Q4, (Jan 2009, Q4)

Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 65\sin 2x. \qquad \textbf{[9]}$$

Q5, (Jun 2009, Q5)

The variables x and y satisfy the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x}.$$

(i) Find the complementary function. **[3]**

(ii) Explain briefly why there is no particular integral of either of the forms $y = ke^{3x}$ or $y = kxe^{3x}$. **[1]**

(iii) Given that there is a particular integral of the form $y = kx^2e^{3x}$, find the value of k. **[5]**

Q6, (Jan 2010, Q6)

The variables x and y satisfy the differential equation

$$\frac{d^2y}{dx^2} + 16y = 8\cos 4x.$$

(i) Find the complementary function of the differential equation. **[2]**

(ii) Given that there is a particular integral of the form $y = px\sin 4x$, where p is a constant, find the general solution of the equation. **[6]**

(iii) Find the solution of the equation for which $y = 2$ and $\frac{dy}{dx} = 0$ when $x = 0$. **[4]**

Q7, (Jun 2010, Q6)

(i) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 17y = 17x + 36.$$ **[7]**

(ii) Show that, when x is large and positive, the solution approximates to a linear function, and state its equation. **[2]**

Q8, (Jan 2011, Q5)

(i) Find the general solution of the differential equation

$$3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 2y = -2x + 13.$$ **[7]**

(ii) Find the particular solution for which $y = -\frac{7}{2}$ and $\frac{dy}{dx} = 0$ when $x = 0$. **[5]**

(iii) Write down the function to which y approximates when x is large and positive. **[1]**

Q9, (Jan 2012, Q5)

The variables x and y satisfy the differential equation

$$2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 2y = 5e^{-2x}.$$

(i) Find the complementary function of the differential equation. **[2]**

(ii) Given that there is a particular integral of the form $y = pxe^{-2x}$, find the constant p. **[4]**

(iii) Find the solution of the equation for which $y = 0$ and $\frac{dy}{dx} = 4$ when $x = 0$. **[5]**

Q10, (Jun 2016, Q5)

Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 85\cos x.$$ **[8]**

Q11, (Jun 2012, Q6)

The variables x and y satisfy the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} = 12e^{2x}.$$

(i) Find the general solution of the differential equation. **[6]**

(ii) It is given that the curve which represents a particular solution of the differential equation has gradient 6 when $x = 0$, and approximates to $y = e^{2x}$ when x is large and positive. Find the equation of the curve. **[4]**

Q12, (Jun 2014, Q5)

Solve the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-x}$$

subject to the conditions $y = \frac{dy}{dx} = 0$ when $x = 0$. **[10]**

Q13, (Jan 2013, Q6)

The differential equation $\frac{d^2y}{dx^2} + 4y = \sin kx$ is to be solved, where k is a constant.

(i) In the case $k = 2$, by using a particular integral of the form $ax\cos 2x + bx\sin 2x$, find the general solution. **[7]**

(ii) Describe briefly the behaviour of y when $x \to \infty$. **[2]**

(iii) In the case $k \neq 2$, explain whether y would exhibit the same behaviour as in part **(ii)** when $x \to \infty$. **[2]**

Q14, (Jun 2015, Q1)

Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = \sin x.$$ **[8]**

Q15, (Jun 2017, Q3)

Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 25\sin x.$$ **[8]**
