## *Second Order Differential Equations Exam Questions (Edexcel)*

**Q1.**

(a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 26\sin 3x$$

(8)

(b) Find the particular solution of this differential equation for which $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$

(5)

**(Total for question = 13 marks)**

**(Q12 6668/01, June 2017)**

**Q2.**

(a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 27e^{-x}$$

(6)

(b) Find the particular solution that satisfies $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$

(6)

**(Total 12 marks)**

**(Q12 6668/01, June 2014)**

**Q3.**

(a) Find the value of $\lambda$ for which $\lambda t^2e^{3t}$ is a particular integral of the differential equation

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = 6e^{3t}, \qquad t \geqslant 0$$

(5)

(b) Hence find the general solution of this differential equation.

(3)

Given that when $t = 0$, $y = 5$ and $\frac{dy}{dt} = 4$

(c) find the particular solution of this differential equation, giving your solution in the form $y = f(t)$.

(5)

**(Total 13 marks)**

**(Q10 6668/01/R, June 2013)**

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**Q4.**

Find the general solution of the differential equation

$$\frac{d^2x}{dt^2}+5\frac{dx}{dt}+6x=2\cos t-\sin t$$

(9)

**(Total 9 marks)**

**(Q10 6668/01, June 2012)**

**Q5.**

The differential equation

$$\frac{d^2x}{dt^2}+6\frac{dx}{dt}+9x=\cos 3t, \quad t \geqslant 0$$

describes the motion of a particle along the $x$-axis.

(a) Find the general solution of this differential equation.

(8)

(b) Find the particular solution of this differential equation for which, at $t = 0$,

$$x=\frac{1}{2} \text{ and } \frac{dx}{dt}=0.$$

(5)

On the graph of the particular solution defined in part (b), the first turning point for $t > 30$ is the point $A$.

(c) Find approximate values for the coordinates of $A$.

(2)

**(Total 15 marks)**

**(Q12 6668/01, June 2011)**

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**Q6.**

A scientist is investigating the concentration of antibodies in the bloodstream of a patient following a vaccination.

The concentration of antibodies, $x$, measured in micrograms (μg) per millilitre (ml) of blood, is modelled by the differential equation

$$100\frac{d^2x}{dt^2} + 60\frac{dx}{dt} + 13x = 26$$

where $t$ is the number of weeks since the vaccination was given.

(a) Find a general solution of the differential equation.

**(4)**

Initially,

- there are no antibodies in the bloodstream of the patient
- the concentration of antibodies is estimated to be increasing at 10 μg/ml per week

(b) Find, according to the model, the maximum concentration of antibodies in the bloodstream of the patient after the vaccination.

**(8)**

A second dose of the vaccine has to be given to try to ensure that it is fully effective. It is only safe to give the second dose if the concentration of antibodies in the bloodstream of the patient is less than 5 μg/ml.

(c) Determine whether, according to the model, it is safe to give the second dose of the vaccine to the patient exactly 10 weeks after the first dose.

**(2)**

**(Total for question = 14 marks)**

**(Q03 9FM0/02, Oct 2020)**

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**Q7.**

A company plans to build a new fairground ride. The ride will consist of a capsule that will hold the passengers and the capsule will be attached to a tall tower. The capsule is to be released from rest from a point half way up the tower and then made to oscillate in a vertical line.

The vertical displacement, $x$ metres, of the top of the capsule below its initial position at time $t$ seconds is modelled by the differential equation,

$$m\frac{d^2x}{dt^2}+4\frac{dx}{dt}+x=200\cos t, \quad t\geqslant 0$$

where $m$ is the mass of the capsule including its passengers, in thousands of kilograms.

The maximum permissible weight for the capsule, including its passengers, is 30 000N.

Taking the value of $g$ to be 10 ms$^{-2}$ and assuming the capsule is at its maximum permissible weight,

(a) (i) explain why the value of $m$ is 3

(ii) show that a particular solution to the differential equation is

$$x = 40\sin t - 20\cos t$$

(iii) hence find the general solution of the differential equation.

**(8)**

(b) Using the model, find, to the nearest metre, the vertical distance of the top of the capsule from its initial position, 9 seconds after it is released.

**(4)**

**(Total for question = 12 marks)**

**(Q09 9FM0/01, Specimen papers )**

**Q8.**

An engineer is investigating the motion of a sprung diving board at a swimming pool.
Let $E$ be the position of the end of the diving board when it is at rest in its equilibrium position and when there is no diver standing on the diving board.
A diver jumps from the diving board.
The vertical displacement, $h$ cm, of the end of the diving board above $E$ is modelled by the differential equation

$$4\frac{d^2h}{dt^2}+4\frac{dh}{dt}+37h=0$$

where $t$ seconds is the time after the diver jumps.

(a) Find a general solution of the differential equation.

**(2)**

When $t = 0$, the end of the diving board is 20 cm below $E$ and is moving upwards with a speed of 55 cm s$^{-1}$.

(b) Find, according to the model, the maximum vertical displacement of the end of the diving board above $E$.

**(8)**

(c) Comment on the suitability of the model for large values of $t$.

**(2)**

**(Total for question = 12 marks)**

**(Q05 9FM0/02, June 2019)**

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**Q9.**

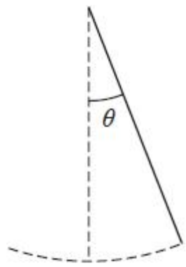


**Figure 3**

The motion of a pendulum, shown in Figure 3, is modelled by the differential equation

$$\frac{d^2\theta}{dt^2} + 9\theta = \frac{1}{2}\cos 3t$$

where $\theta$ is the angle, in radians, that the pendulum makes with the downward vertical, $t$ seconds after it begins to move.

(a) (i) Show that a particular solution of the differential equation is

$$\theta = \frac{1}{12}t\sin 3t$$

**(4)**

(ii) Hence, find the general solution of the differential equation.

**(4)**

Initially, the pendulum

- makes an angle of $\frac{\pi}{3}$ radians with the downward vertical
- is at rest

Given that, 10 seconds after it begins to move, the pendulum makes an angle of $\alpha$ radians with the downward vertical,

(b) determine, according to the model, the value of $\alpha$ to 3 significant figures.

**(4)**

Given that the true value of $\alpha$ is 0.62

(c) evaluate the model.

**(1)**

The differential equation

$$\frac{d^2\theta}{dt^2} + 9\theta = \frac{1}{2}\cos 3t$$

models the motion of the pendulum as moving with forced harmonic motion.

(d) Refine the differential equation so that the motion of the pendulum is simple harmonic motion.

**(1)**

**(Total for question = 14 marks)**

**(Q10 9FM0/01, June 2022)**

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