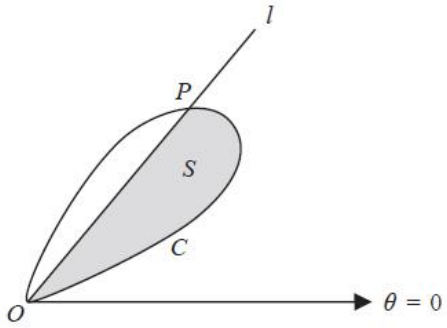
Polar Coordinates Exam Questions (Edexcel)

Q1.



The diagram shows a curve C with polar equation

$r = a\sin 2\theta$, $0 \leq \theta \leq \frac{\pi}{2}$, and a half-line l.

The half-line l meets C at the pole O and at the point P. The tangent to C at P is parallel to the initial line. The polar coordinates of P are (R, ϕ).

(a) Show that $\cos\phi = \frac{1}{\sqrt{3}}$

(6)

(b) Find the exact value of R.

(2)

The region S, shown shaded in Figure 1, is bounded by C and l.

(c) Use calculus to show that the exact area of S is

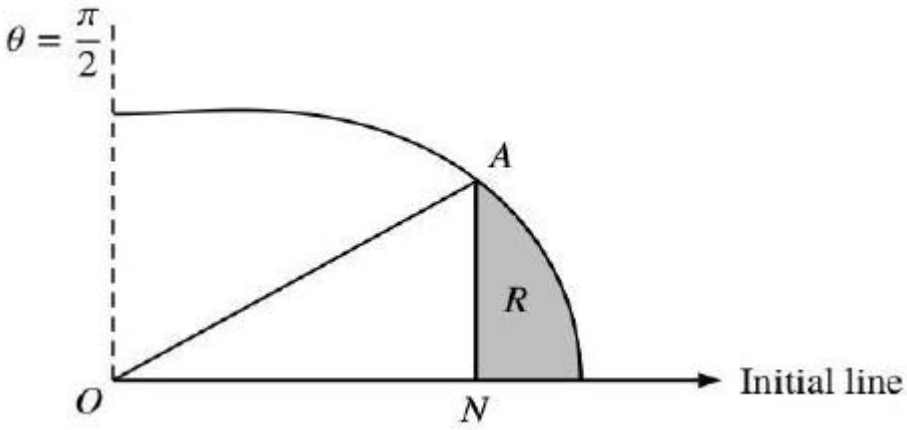
$$\frac{1}{36}a^2\left(9\arccos\left(\frac{1}{\sqrt{3}}\right)+\sqrt{2}\right)$$

(7)

(Total 15 marks)

(Q10 6668/01, June 2013)

Q2.



The curve C shown in the diagram has polar equation

$$r = 4 + \cos 2\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

At the point A on C, the value of r is $\frac{9}{2}$

The point N lies on the initial line and AN is perpendicular to the initial line.

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the initial line and the line AN.

Find the exact area of the shaded region R, giving your answer in the form $p\pi + q\sqrt{3}$ where p and q are rational numbers to be found.

(9)

(Total for question = 9 marks)

(Q04 9FM0/01, Specimen papers)

Q3.

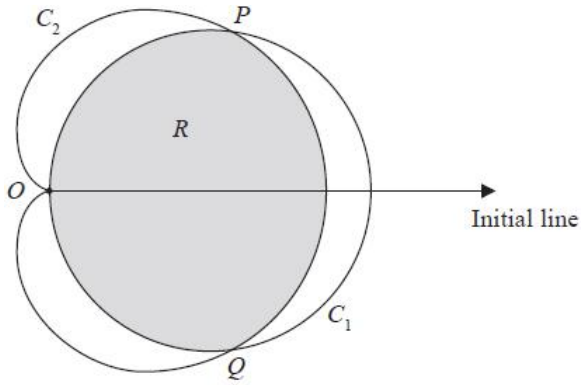


Figure 1

The curve C_1 with equation

$$r = 7\cos\theta, \quad -\frac{\pi}{2} < \theta \leqslant \frac{\pi}{2}$$

and the curve C_2 with equation

$$r = 3(1 + \cos\theta), \quad -\pi < \theta < \pi$$

are shown on Figure 1.

The curves C_1 and C_2 both pass through the pole and intersect at the point P and the point Q.

(a) Find the polar coordinates of P and the polar coordinates of Q.

(3)

The regions enclosed by the curve C_1 and the curve C_2 overlap, and the common region R is shaded in Figure 1.

(b) Find the area of R.

(7)

(Total for question = 10 marks)

(Q12 6668/01, June 2016)

Q4.

The curve C has polar equation

$$r = 1 + 2\cos\theta, \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

At the point P on C, the tangent to C is parallel to the initial line.

Given that O is the pole, find the exact length of the line OP.

(7)

(Total 7 marks)

(Q08 6668/01, June 2012)

Q5.

(a) Sketch the curve C with polar equation

$$r = 5 + \sqrt{3}\cos\theta, \qquad 0 \leqslant \theta < 2\pi.$$

(2)

(b) Find the polar coordinates of the points where the tangents to C are parallel to the initial line $\theta = 0$. Give your answers to 3 significant figures where appropriate.

(6)

(c) Using integration, find the area enclosed by the curve C, giving your answer in terms of π.

(6)

(Total 14 marks)

(Q06 6674/01, June 2007)

Q6.

Figure 1

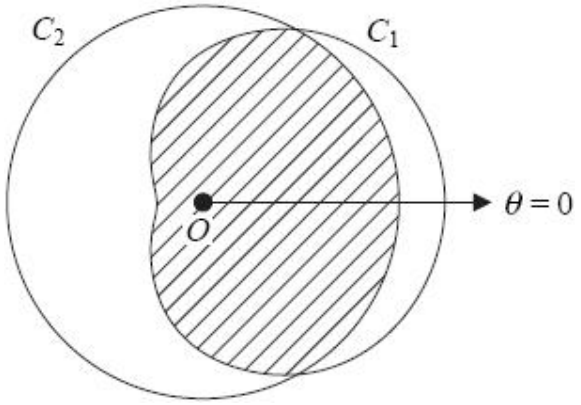


Figure 1 shows the curve C_1 which has polar equation $r = a(3 + 2\cos\theta)$, $0 \leq \theta < 2\pi$, and the circle C_2 with equation $r = 4a$, $0 \leq \theta < 2\pi$, where a is a positive constant.

(a) Find, in terms of a, the polar coordinates of the points where the curve C_1 meets the circle C_2.

(4)

The regions enclosed by the curves C_1 and C_2 overlap and this common region R is shaded in the figure.

(b) Find, in terms of a, an exact expression for the area of the shaded region R.

(8)

(c) In a single diagram, copy the two curves in Figure 1 and also sketch the curve C_3 with polar equation $r = 2a\cos\theta$, $0 \leq \theta < 2\pi$. Show clearly the coordinates of the points of intersection of C_1, C_2 and C_3 with the initial line, $\theta = 0$.

(3)

(Total 15 marks)

(Q08 6674/01, Jan 2008)

Q7.

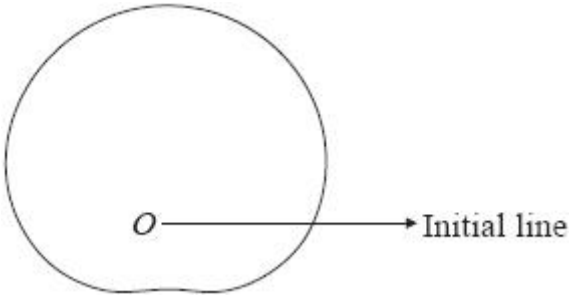


Figure 1

Figure 1 shows a sketch of a curve with polar equation

$$r = 6 + a\sin\theta$$

where $0 < a < 6$ and $0 \leq \theta < 2\pi$

The area enclosed by the curve is $\frac{97\pi}{2}$

Find the value of the constant a.

(8)

(Total for question = 8 marks)

(Q13 6668/01, June 2017)

Q8.

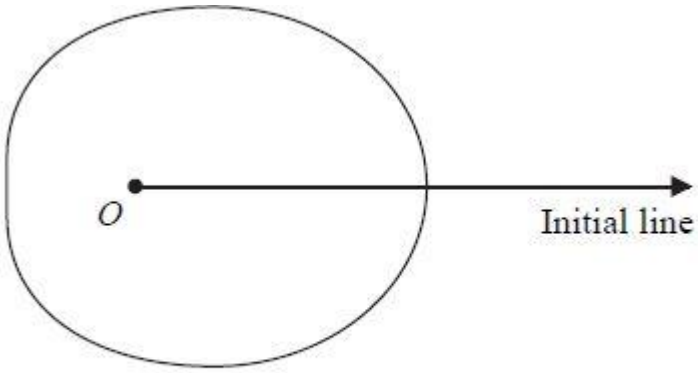


Figure 1

Figure 1 shows a sketch of the curve with polar equation

$$r = a + 3\cos\theta, \quad a > 0, \quad 0 \leqslant \theta < 2\pi$$

The area enclosed by the curve is $\frac{107}{2}\pi$.

Find the value of a.

(8)

(Total 8 marks)

(Q12 6668/01, June 2009)

Q9.

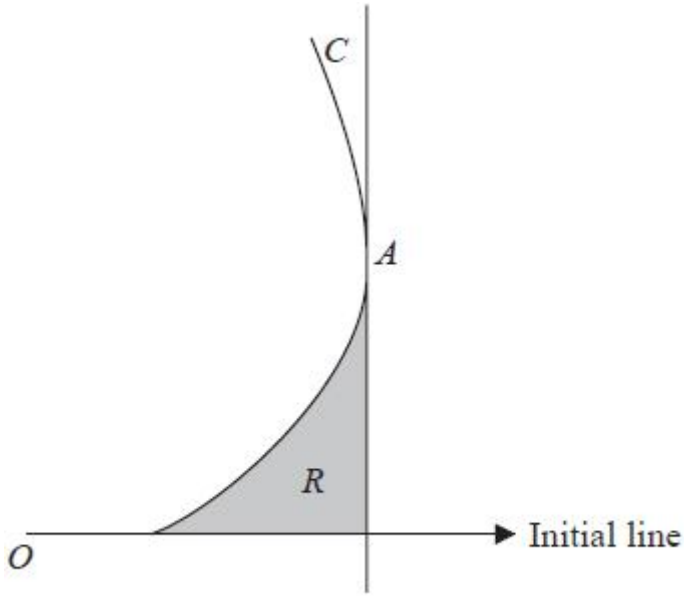


Figure 1

Figure 1 shows a sketch of the curve C with equation

$$r = 1 + \tan\theta \qquad 0 \leq \theta < \frac{\pi}{3}$$

Figure 1 also shows the tangent to C at the point A.
This tangent is perpendicular to the initial line.

(a) Use differentiation to prove that the polar coordinates of A are $\left(2, \frac{\pi}{4}\right)$

(4)

The finite region R, shown shaded in Figure 1, is bounded by C, the tangent at A and the initial line.

(b) Use calculus to show that the exact area of R is $\frac{1}{2}(1 - \ln 2)$

(6)

(Total for question = 10 marks)

(Q07 9FM0/02, June 2022)

Q10.

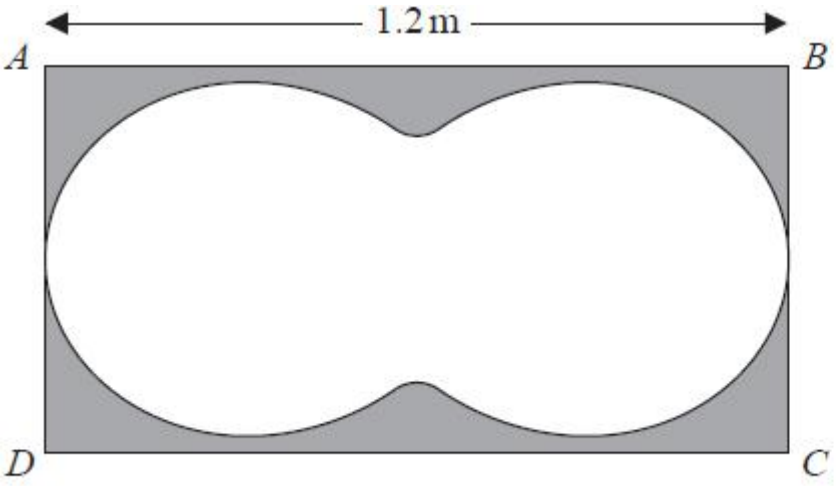


Diagram not to scale

Figure 1

Figure 1 shows the design for a table top in the shape of a rectangle $ABCD$. The length of the table, AB, is 1.2 m. The area inside the closed curve is made of glass and the surrounding area, shown shaded in Figure 1, is made of wood.

The perimeter of the glass is modelled by the curve with polar equation

$$r = 0.4 + a\cos 2\theta \qquad 0 \leqslant \theta < 2\pi$$

where a is a constant.

(a) Show that $a = 0.2$

(2)

Hence, given that $AD = 60$ cm,

(b) find the area of the wooden part of the table top, giving your answer in m^2 to 3 significant figures.

(8)

(Total for question = 10 marks)

(Q03 9FM0/01, June 2019)

Q11.

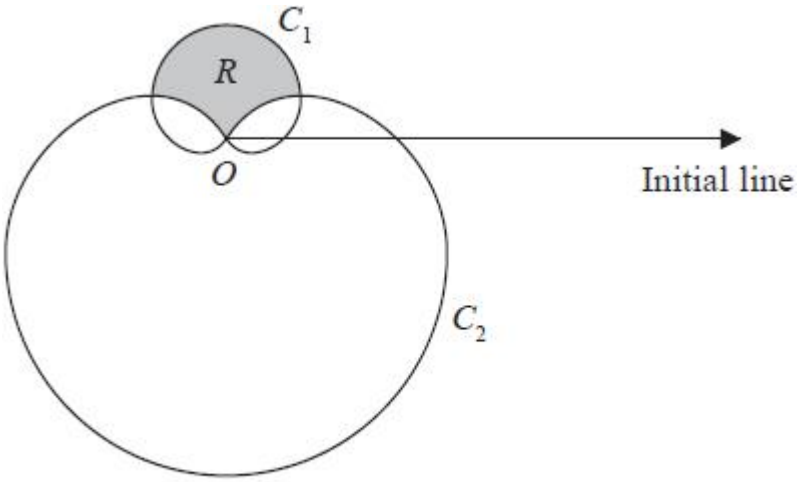


Figure 1

Figure 1 shows a sketch of two curves C_1 and C_2 with polar equations

$$C_1: r = (1 + \sin\theta) \qquad 0 \leqslant \theta < 2\pi$$

$$C_2: r = 3(1 - \sin\theta) \qquad 0 \leqslant \theta < 2\pi$$

The region R lies inside C_1 and outside C_2 and is shown shaded in Figure 1.

Show that the area of R is

$$p\sqrt{3} - q\pi$$

where p and q are integers to be determined.

(Total for question = 9 marks)

(Q03 9FM0/01, Oct 2020)

Q12.

(a) Sketch the polar curve C, with equation

$$r = 3 + \sqrt{5}\cos\theta \qquad 0 \leqslant \theta \leqslant 2\pi$$

On your sketch clearly label the pole, the initial line and the value of r at the point where the curve intersects the initial line.

(2)

The tangent to C at the point A, where $0 < \theta < \pi/2$, is parallel to the initial line.

(b) Use calculus to show that at A

$$\cos\theta = \frac{1}{\sqrt{5}}$$

(4)

(c) Hence determine the value of r at A.

(1)

(Total for question = 7 marks)

(Q04 9FM0/02, June 2023)

Q13.

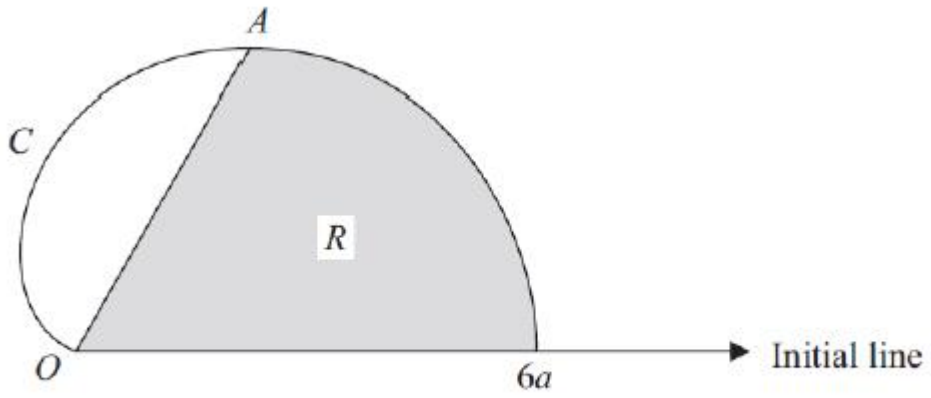


Figure 1

The curve C, shown in Figure 1, has polar equation

$$R = 3a(1 + \cos\theta), \quad 0 \leq \theta < \pi.$$

The tangent to C at the point A is parallel to the initial line.

(a) Find the polar coordinates of A.

(6)

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the initial line and the line OA.

(b) Use calculus to find the area of the shaded region R, giving your answer in the form $a^2(p\pi + q\sqrt{3})$, where p and q are rational numbers.

(5)

(Total for question = 11 marks)

(Q11 6668/01, June 2015)

Q14.

The curve C has equation

$$r = a(p + 2\cos\theta) \qquad 0 \leq \theta < 2\pi$$

where a and p are positive constants and $p > 2$

There are exactly four points on C where the tangent is perpendicular to the initial line.

(a) Show that the range of possible values for p is

$$2 < p < 4$$

(5)

(b) Sketch the curve with equation

$$r = a(3 + 2\cos\theta) \qquad 0 \leq \theta < 2\pi \qquad \text{where } a > 0$$

(1)

John digs a hole in his garden in order to make a pond.

The pond has a uniform horizontal cross section that is modelled by the curve with equation

$$r = 20(3 + 2\cos\theta) \qquad 0 \leq \theta < 2\pi$$

where r is measured in centimetres.

The depth of the pond is 90 centimetres.

Water flows through a hosepipe into the pond at a rate of 50 litres per minute.

Given that the pond is initially empty,

(c) determine how long it will take to completely fill the pond with water using the hosepipe, according to the model. Give your answer to the nearest minute.

(7)

(d) State a limitation of the model.

(1)

(Total for question = 14 marks)

(Q06 9FM0/02, Oct 2021)