Mean Value Of A Function Exam Questions (Edexcel)

Q1.

(a) Write $x^2 + 4x - 5$ in the form $(x + p)^2 + q$ where p and q are integers.

(1)

(b) Hence use a standard integral from the formula book to find

$$\int \frac{1}{\sqrt{x^2 + 4x - 5}}\,dx$$

(2)

(c) Determine the mean value of the function

$$f(x) = \frac{1}{\sqrt{x^2 + 4x - 5}} \qquad 3 \leqslant x \leqslant 13$$

giving your answer in the form $A\ln B$ where A and B are constants in simplest form.

(3)

(Total for question = 6 marks)

(Q02 9FM0/01, June 2023)

Q2.

(i) Evaluate the improper integral

$$\int_1^{\infty} 2e^{-\frac{1}{2}x}\,dx$$

(3)

(ii) The air temperature, θ°C, on a particular day in London is modelled by the equation

$$\theta = 8 - 5\sin\left(\frac{\pi}{12}t\right) - \cos\left(\frac{\pi}{6}t\right) \qquad 0 \leqslant t \leqslant 24$$

where t is the number of hours after midnight.

(a) Use calculus to show that the mean air temperature on this day is 8°C, according to the model.

(3)

Given that the actual mean air temperature recorded on this day was higher than 8°C,

(b) explain how the model could be refined.

(1)

(Total for question = 7 marks)

(Q05 9FM0/01, Oct 2021)

Q3.

(a)

$$y = \tan^{-1}x$$

Assuming the derivative of $\tan x$, prove that

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

(3)

$$f(x) = x\tan^{-1}4x$$

(b) Show that

$$\int f(x)dx = Ax^2\tan^{-1}4x + Bx + C\tan^{-1}4x + k$$

where k is an arbitrary constant and A, B and C are constants to be determined.

(5)

(c) Hence find, in exact form, the mean value of $f(x)$ over the interval $\left[0, \frac{\sqrt{3}}{4}\right]$

(2)

(Total for question = 10 marks)

(Q05 9FM0/02, Oct 2020)

Q4.

$$f(x) = \frac{1}{\sqrt{4x^2+9}}$$

(a) Using a substitution, that should be stated clearly, show that

$$\int f(x)dx = A\sinh^{-1}(Bx) + c$$

where c is an arbitrary constant and A and B are constants to be found.

(4)

(b) Hence find, in exact form in terms of natural logarithms, the mean value of $f(x)$ over the interval [0, 3].

(2)

(Total for question = 6 marks)

(Q03 9FM0/02, June 2019)