Integration Involving Hyperbolic And Trigonometric Functions Exam Questions (Edexcel)

Q1.

$$f(x) = \frac{1}{\sqrt{4x^2 + 9}}$$

(a) Using a substitution, that should be stated clearly, show that

$$\int f(x)dx = A\sinh^{-1}(Bx) + c$$

where c is an arbitrary constant and A and B are constants to be found.

(4)

(b) Hence find, in exact form in terms of natural logarithms, the mean value of f(x) over the interval [0, 3].

(2)

(Total for question = 6 marks)

(Q03 9FM0/02, June 2019)

Q2.

Using calculus, find the exact value of

(a) $\int_1^2 \frac{1}{\sqrt{(x^2 - 2x + 3)}}\,dx$

(4)

(b) $\int_0^1 e^{2x} \sinh x\,dx$

(4)

(Total 8 marks)

(Q15 6669/01, June 2014)

Q3.

Evaluate $\int_1^3 \frac{1}{\sqrt{(x^2 + 4x - 5)}}\,dx$, giving your answer as an exact logarithm.

(5)

(Total 5 marks)

(Q07 6675/01, June 2007)

Q4.

$$f(x)=\frac{x+2}{x^2+9}$$

(a) Show that

$$\int f(x)\,dx = A\ln\left(x^2+9\right)+B\arctan\left(\frac{x}{3}\right)+c$$

where c is an arbitrary constant and A and B are constants to be found.

(4)

(b) Hence show that the mean value of f(x) over the interval [0, 3] is

$$\frac{1}{6}\ln 2+\frac{1}{18}\pi$$

(3)

(c) Use the answer to part (b) to find the mean value, over the interval [0, 3], of

f(x) + ln$k$

where k is a positive constant, giving your answer in the form $p + \frac{1}{6}\ln q$, p and q are constants and q is in terms of k.

(2)

(Total for question = 9 marks)

(Q06 9FM0/01, Specimen papers)

Q5.

Show that

(a) $$\int_5^8 \frac{1}{x^2-10x+34}\,dx = k\pi,$$ giving the value of the fraction k,

(5)

(b) $$\int_5^8 \frac{1}{\sqrt{(x^2-10x+34)}}\,dx = \ln(A+\sqrt{n}),$$ giving the values of the integers A and n.

(4)

(Total 9 marks)

(Q14 6669/01, June 2011)

Q6.

(a) Find

$$\int \frac{1}{\sqrt{(4x^2+9)}}\,dx$$

(2)

(b) Use your answer to part (a) to find the exact value of

$$\int_{-3}^{3} \frac{1}{\sqrt{(4x^2+9)}}\,dx$$

giving your answer in the form $k \ln(a + b\sqrt{5})$, where a and b are integers and k is a constant.

(3)

(Total 5 marks)

(Q11 6669/01, June 2013)

Q7.

Show that

$$\int_{5}^{6} \frac{3+x}{\sqrt{(x^2-9)}}\,dx = 3\ln\left(\frac{2+\sqrt{3}}{3}\right) + 3\sqrt{3} - 4.$$

(7)

(Total 7 marks)

(Q10 6675/01, June 2008)

Q8.

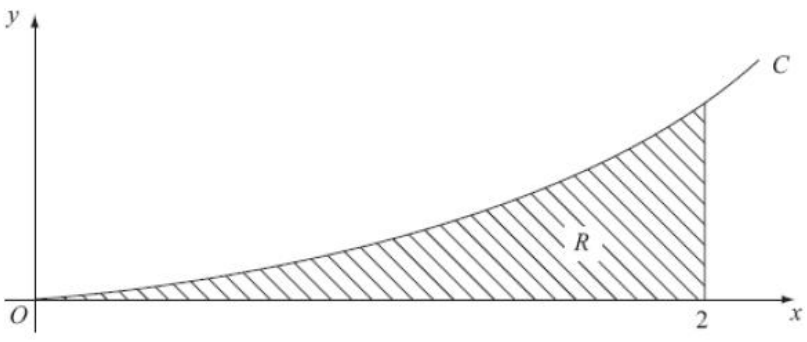

Figure 1

Figure 1 shows the curve C with equation

$y = 0.1\cosh(x) \arctan(\sinh x), \quad x \geq 0.$

The shaded region R is bounded by C, the x-axis and the line $x = 2$.

(a) Find $\int \cosh x \arctan(\sinh x)\,dx$.

(5)

(b) Hence show that, to 2 significant figures, the area of R is 0.34.

(2)

(Total 7 marks)

(Q12 6675/01, June 2008)

Q9.

(i) (a) Explain why $\int_0^\infty \cosh x \, dx$ is an improper integral.

(1)

(b) Show that $\int_0^\infty \cosh x \, dx$ is divergent.

(3)

(ii) $4 \sinh x = p \cosh x$ where p is a real constant

Given that this equation has real solutions, determine the range of possible values for p

(2)

(Total for question = 6 marks)

(Q09 9FM0/01, June 2022)

Q10.

(a) Use a hyperbolic substitution and calculus to show that

$$\int \frac{x^2}{\sqrt{x^2-1}} dx = \frac{1}{2}\left[x\sqrt{x^2-1} + \operatorname{arcosh} x\right] + k$$

where k is an arbitrary constant.

(6)

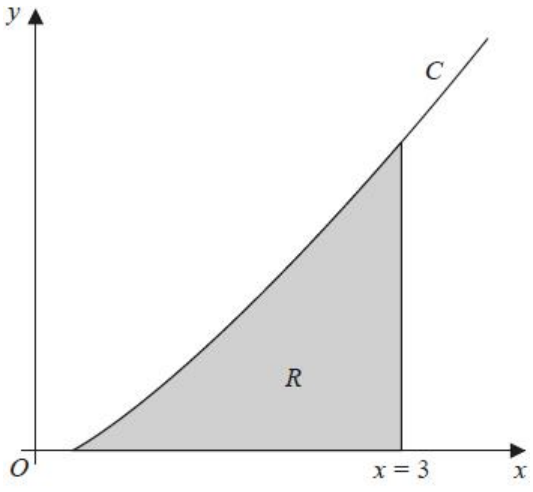

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = \frac{4}{15} x \operatorname{arcosh} x \qquad x \geqslant 1$$

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the x-axis and the line with equation $x = 3$

(b) Using algebraic integration and the result from part (a), show that the area of R is given by

$$\frac{1}{15}\left[17 \ln\left(3 + 2\sqrt{2}\right) - 6\sqrt{2}\right]$$

(5)

(Total for question = 11 marks)

(Q09 9FM0/01, Oct 2021)