Hyperbolic Functions Identities Exam Questions (From OCR 4726)

Q1, (Jan 2006, Q9)

(i) Using the definitions of $\cosh x$ and $\sinh x$ in terms of e^x and e^{-x}, prove that

$$\sinh 2x = 2 \sinh x \cosh x. \quad [4]$$

(ii) Show that the curve with equation

$$y = \cosh 2x - 6 \sinh x$$

has just one stationary point, and find its x-coordinate in logarithmic form. Determine the nature of the stationary point. [8]

Q2, (Jun 2006, Q4)

(i) **Using the definition of $\cosh x$ in terms of e^x and e^{-x}, prove that**

$$\cosh 2x = 2 \cosh^2 x - 1. \quad [3]$$

(ii) **Hence solve the equation**

$$\cosh 2x - 7 \cosh x = 3,$$

giving your answer in logarithmic form. **[4]**

Q3, (Jan 2009, Q6)

(i) Using the definitions of $\cosh x$ and $\sinh x$ in terms of e^x and e^{-x}, show that

$$1 + 2 \sinh^2 x \equiv \cosh 2x. \quad [3]$$

(ii) Solve the equation

$$\cosh 2x - 5 \sinh x = 4,$$

giving your answers in logarithmic form. **[5]**

Q4, (Jun 2010, Q8)

(i) Using the definition of $\cosh x$ in terms of e^x and e^{-x}, show that

$$4 \cosh^3 x - 3 \cosh x \equiv \cosh 3x. \quad [4]$$

(ii) Use the substitution $u = \cosh x$ to find, in terms of $5^{\frac{1}{3}}$, the real root of the equation

$$20u^3 - 15u - 13 = 0. \quad [6]$$

Q5, (Jun 2013, Q2)

(i) Using the definitions for $\cosh x$ and $\sinh x$ in terms of e^x and e^{-x}, show that $\cosh^2 x - \sinh^2 x \equiv 1$. **[3]**

(ii) Hence solve the equation $\sinh^2 x = 5 \cosh x - 7$, giving your answers in logarithmic form. **[5]**

Q6, (Jan 2012, Q7)

(i) Given that $y = \sinh^{-1} x$, prove that $y = \ln\left(x + \sqrt{x^2 + 1}\right)$. **[3]**

(ii) It is given that x satisfies the equation $\sinh^{-1} x - \cosh^{-1} x = \ln 2$. Use the logarithmic forms for $\sinh^{-1} x$ and $\cosh^{-1} x$ to show that

$$\sqrt{x^2 + 1} - 2\sqrt{x^2 - 1} = x\,.$$

Hence, by squaring this equation, find the exact value of x. **[5]**

Q8, (Jun 2015, Q1)

By first expressing $\tanh y$ in terms of exponentials, prove that $\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$. **[3]**

Q9, (Jun 2016, Q1)

(i) By first expanding $(e^x + e^{-x})^3$, or otherwise, show that $\cosh 3x \equiv 4\cosh^3 x - 3\cosh x$. **[4]**

(ii) Solve the equation $\cosh 3x = 6\cosh x$, giving your answers in exact logarithmic form. **[5]**
