Hyperbolic Functions, Identities And Equations Exam Questions (Edexcel)

Q1.

(a) Using the definition for cosh x in terms of exponentials, show that

$$\cosh 2x \equiv 2\cosh^2 x - 1$$

(3)

(b) Find the exact values of x for which

$$29\cosh x - 3\cosh 2x = 38$$

giving your answers in terms of natural logarithms.

(6)

(Total for question = 9 marks)

(Q16 6669/01, June 2017)

Q2.

Solve the equation

$$2\cosh^2 x - 3\sinh x = 1$$

giving your answers in terms of natural logarithms.

(Total for question = 6 marks)

(Q12 6669/01, June 2015)

Q3.

Using the definitions of hyperbolic functions in terms of exponentials,

(a) show that

$$\operatorname{sech}^2 x = 1 - \tanh^2 x$$

(3)

(b) solve the equation

$$4\sinh x - 3\cosh x = 3$$

(4)

(Total 7 marks)

(Q16 6669/01, June 2014)

Q4.

The curve C_1 has equation $y = 3\sinh 2x$, and the curve C_2 has equation $y = 13 - 3e^{2x}$.

(a) Sketch the graph of the curves C_1 and C_2 on one set of axes, giving the equation of any asymptote and the coordinates of points where the curves cross the axes.

(4)

(b) Solve the equation $3\sinh 2x = 13 - 3e^{2x}$, giving your answer in the form $\frac{1}{2} \ln k$, where k is an integer.

(5)

(Total 9 marks)
(Q15 6669/01, June 2011)

Q5.

The curve C_1 has equation $y = 3\sinh 2x$, and the curve C_2 has equation $y = 13 - 3e^{2x}$.

(a) Sketch the graph of the curves C_1 and C_2 on one set of axes, giving the equation of any asymptote and the coordinates of points where the curves cross the axes.

(4)

(b) Solve the equation $3\sinh 2x = 13 - 3e^{2x}$, giving your answer in the form $\frac{1}{2} \ln k$, where k is an integer.

(5)

(Total 9 marks)
(Q15 6669/01, June 2011)

Q6.

(a) Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$\cosh(A - B) = \cosh A \cosh B - \sinh A \sinh B$$

(3)

(b) Hence, or otherwise, given that $\cosh(x - 1) = \sinh x$, show that

$$\tanh x = \frac{e^2 + 1}{e^2 + 2e - 1}.$$

(4)

(Total 7 marks)

(Q08 6675/01, June 2007)

Q7.

Find the values of x for which

$$8\cosh x - 4\sinh x = 13,$$

giving your answers as natural logarithms.

(6)

(Total 6 marks)

(Q09 6675/01, June 2008)

Q8.

(a) Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that

$$\cosh 2x = 1 + 2\sinh^2 x$$

(3)

(b) Solve the equation

$$\cosh 2x - 3\sinh x = 15,$$

giving your answers as exact logarithms.

(5)

(Total 8 marks)
(Q13 6669/01, June 2010)

Q9.

(a) Prove that

$$\tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad -k < x < k$$

stating the value of the constant k.

(5)

(b) Hence, or otherwise, solve the equation

$$2x = \tanh\left(\ln\sqrt{2-3x}\right)$$

(5)

(Total for question = 10 marks)

(Q01 9FM0/02, June 2019)

Q10.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Determine the values of x for which

$$64\cosh^4 x - 64\cosh^2 x - 9 = 0$$

Give your answers in the form $q \ln 2$ where q is rational and in simplest form.

(Total for question = 4 marks)

(Q02 9FM0/01, June 2022)

Q11.

Solve the equation

$$5\tanh x + 7 = 5\operatorname{sech} x$$

Give each answer in the form $\ln k$ where k is a rational number.

(5)

(Total 5 marks)

(Q12 6669/01/R, June 2014)