## *DeMoivre's Theorem And Applications To Series Exam Questions (Edexcel)*

**Q1.**

(a) Given that $|z| < 1$, write down the sum of the infinite series

$$1 + z + z^2 + z^3 + \ldots$$

**(1)**

(b) Given that $z = \frac{1}{2}(\cos\theta + i\sin\theta)$,

(i) use the answer to part (a), and de Moivre's theorem or otherwise, to prove that

$$\frac{1}{2}\sin\theta + \frac{1}{4}\sin 2\theta + \frac{1}{8}\sin 3\theta + \ldots = \frac{2\sin\theta}{5 - 4\cos\theta}$$

**(5)**

(ii) show that the sum of the infinite series $1 + z + z^2 + z^3 + \ldots$ cannot be purely imaginary, giving a reason for your answer.

**(2)**

**(Total for question = 8 marks)**

**(Q09 9FM0/02, Oct 2021)**

**Q2.**

The infinite series C and S are defined by

$$C = \cos\theta + \frac{1}{2}\cos 5\theta + \frac{1}{4}\cos 9\theta + \frac{1}{8}\cos 13\theta + \ldots$$

$$S = \sin\theta + \frac{1}{2}\sin 5\theta + \frac{1}{4}\sin 9\theta + \frac{1}{8}\sin 13\theta + \ldots$$

Given that the series C and S are both convergent,

(a) show that

$$C + iS = \frac{2e^{i\theta}}{2 - e^{4i\theta}}$$

**(4)**

(b) Hence show that

$$S = \frac{4\sin\theta + 2\sin 3\theta}{5 - 4\cos 4\theta}$$

**(4)**

**(Total for question = 8 marks)**

**(Q04 9FM0/02, June 2019)**

**Q3.**

(a) Show that $e^{2i\theta} + e^{-2i\theta} = 2\cos 2\theta$

**(2)**

The convergent infinite series $C$ and $S$ are defined as

$$C = 1 + \frac{1}{4}\cos 2\theta + \frac{1}{16}\cos 4\theta + \frac{1}{64}\cos 6\theta + \ldots$$

$$S = \frac{1}{4}\sin 2\theta + \frac{1}{16}\sin 4\theta + \frac{1}{64}\sin 6\theta + \ldots$$

(b) Show that

$$C + iS = \frac{k}{k - e^{2i\theta}}$$

where $k$ is an integer to be determined.

**(3)**

(c) Show that

$$C = \frac{16 - 4\cos 2\theta}{17 - 8\cos 2\theta}$$

**(3)**

(d) Hence determine the values of $\theta$, in the range $0 \leqslant \theta \leqslant \pi$, for which $C + iS$ is real.

**(3)**

**(Total for question = 11 marks)**