Coupled First Order Simultaneous Differential Equations Exam Questions (Edexcel)

Q1.

At the start of the year 2000, a survey began of the number of foxes and rabbits on an island.

At time t years after the survey began, the number of foxes, f, and the number of rabbits, r, on the island are modelled by the differential equations

$$\frac{\mathrm{d}f}{\mathrm{d}t} = 0.2f + 0.1r$$

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -0.2f + 0.4r$$

(a) Show that $\frac{\mathrm{d}^2 f}{\mathrm{d}t^2} - 0.6\frac{\mathrm{d}f}{\mathrm{d}t} + 0.1f = 0$

(3)

(b) Find a general solution for the number of foxes on the island at time t years.

(4)

(c) Hence find a general solution for the number of rabbits on the island at time t years.

(3)

At the start of the year 2000 there were 6 foxes and 20 rabbits on the island.

(d) (i) According to this model, in which year are the rabbits predicted to die out?

(ii) According to this model, how many foxes will be on the island when the rabbits die out?

(iii) Use your answers to parts (i) and (ii) to comment on the model.

(7)

(Total for question = 17 marks)

(Q07 9FM0/02, Specimen papers)

Q2.

A scientist is studying the effect of introducing a population of type A bacteria into a population of type B bacteria.

At time t days, the number of type A bacteria, x, and the number of type B bacteria, y, are modelled by the differential equations

$$\frac{dx}{dt} = x + y$$

$$\frac{dy}{dt} = 3y - 2x$$

(a) Show that

$$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 5x = 0$$

(3)

(b) Determine a general solution for the number of type A bacteria at time t days.

(4)

(c) Determine a general solution for the number of type B bacteria at time t days.

(2)

The model predicts that, at time T hours, the number of bacteria in the two populations will be equal.

Given that $x = 100$ and $y = 275$ when $t = 0$

(d) determine the value of T, giving your answer to 2 decimal places.

(5)

(e) Suggest a limitation of the model.

(1)

(Total for question = 15 marks)

(Q08 9FM0/01, June 2024)

Q3.

Two compounds, X and Y, are involved in a chemical reaction. The amounts in grams of these compounds, t minutes after the reaction starts, are x and y respectively and are modelled by the differential equations

$$\frac{dx}{dt} = -5x + 10y - 30$$

$$\frac{dy}{dt} = -2x + 3y - 4$$

(a) Show that

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 50$$

(3)

(b) Find, according to the model, a general solution for the amount in grams of compound X present at time t minutes.

(6)

(c) Find, according to the model, a general solution for the amount in grams of compound Y present at time t minutes.

(3)

Given that $x = 2$ and $y = 5$ when $t = 0$

(d) find

(i) the particular solution for x,
(ii) the particular solution for y.

(4)

A scientist thinks that the chemical reaction will have stopped after 8 minutes.

(e) Explain whether this is supported by the model.

(1)

(Total for question = 17 marks)

(Q05 9FM0/01, Oct 2020)

Q4.

A patient is treated by administering an antibiotic intravenously at a constant rate for some time.

Initially there is none of the antibiotic in the patient.

At time t minutes after treatment began

- the concentration of the antibiotic in the blood of the patient is x mg/ml
- the concentration of the antibiotic in the tissue of the patient is y mg/ml

The concentration of antibiotic in the patient is modelled by the equations

$$\frac{dx}{dt} = 0.025y - 0.045x + 2$$

$$\frac{dy}{dt} = 0.032x - 0.025y$$

(a) Show that

$$40000\frac{d^2y}{dt^2} + 2800\frac{dy}{dt} + 13y = 2560$$

(3)

(b) Determine, according to the model, a general solution for the concentration of the antibiotic in the patient's tissue at time t minutes after treatment began.

(5)

(c) Hence determine a particular solution for the concentration of the antibiotic in the tissue at time t minutes after treatment began.

(4)

To be effective for the patient the concentration of antibiotic in the tissue must eventually reach a level between 185 mg/ml and 200 mg/ml.

(d) Determine whether the rate of administration of the antibiotic is effective for the patient, giving a reason for your answer.

(2)

(Total for question = 14 marks)

(Q09 9FM0/02, June 2023)