Complex Roots Of Equations and Using Exponential and Polar Form (From OCR 4727)

Q1, (Jun 2007, Q1)

(i) By writing z in the form $re^{i\theta}$, show that $zz^* = |z|^2$. [1]

(ii) Given that $zz^* = 9$, describe the locus of z. [2]

Q2, (Jun 2008, Q7)

The roots of the equation $z^3 - 1 = 0$ are denoted by 1, ω and ω^2.

(i) Sketch an Argand diagram to show these roots. [1]

(ii) Show that $1 + \omega + \omega^2 = 0$. [2]

(iii) Hence evaluate

(a) $(2 + \omega)(2 + \omega^2)$, [2]

(b) $\dfrac{1}{2 + \omega} + \dfrac{1}{2 + \omega^2}$. [2]

(iv) Hence find a cubic equation, with integer coefficients, which has roots 2, $\dfrac{1}{2 + \omega}$ and $\dfrac{1}{2 + \omega^2}$. [4]

Q3, (Jan 2009, Q2)

(i) Express $\dfrac{\sqrt{3} + i}{\sqrt{3} - i}$ in the form $re^{i\theta}$, where $r > 0$ and $0 \leqslant \theta < 2\pi$. **[3]**

(ii) Hence find the smallest positive value of n for which $\left(\dfrac{\sqrt{3} + i}{\sqrt{3} - i}\right)^n$ is real and positive. **[2]**

Q4, (Jun 2009, Q1)

Find the cube roots of $\frac{1}{2}\sqrt{3} + \frac{1}{2}i$, giving your answers in the form $\cos\theta + i\sin\theta$, where $0 \leqslant \theta < 2\pi$. **[4]**

Q5, (Jun 2010, Q3)

In this question, w denotes the complex number $\cos\frac{2}{5}\pi + i\sin\frac{2}{5}\pi$.

(i) Express w^2, w^3 and w^* in polar form, with arguments in the interval $0 \leqslant \theta < 2\pi$. **[4]**

(ii) The points in an Argand diagram which represent the numbers

$$1, \quad 1 + w, \quad 1 + w + w^2, \quad 1 + w + w^2 + w^3, \quad 1 + w + w^2 + w^3 + w^4$$

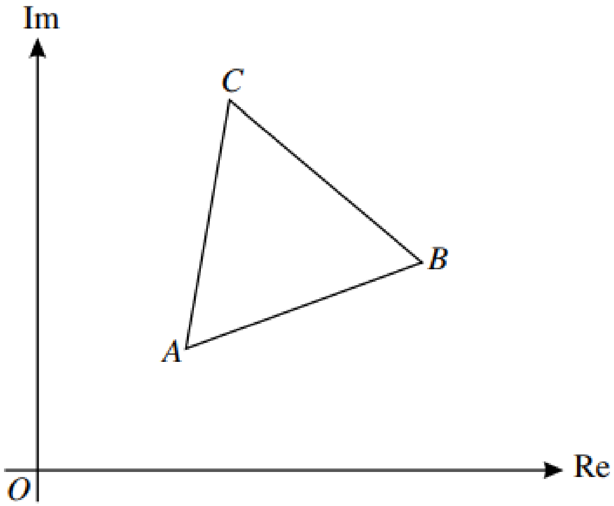
are denoted by A, B, C, D, E respectively. Sketch the Argand diagram to show these points and join them in the order stated. (Your diagram need not be exactly to scale, but it should show the important features.) **[4]**

(iii) Write down a polynomial equation of degree 5 which is satisfied by w. **[1]**

Q6, (Jan 2011, Q4)

The cube roots of 1 are denoted by 1, ω and ω^2, where the imaginary part of ω is positive.

(i) Show that $1 + \omega + \omega^2 = 0$. **[2]**



In the diagram, ABC is an equilateral triangle, labelled anticlockwise. The points A, B and C represent the complex numbers z_1, z_2 and z_3 respectively.

(ii) State the geometrical effect of multiplication by ω and hence explain why $z_1 - z_3 = \omega(z_3 - z_2)$. **[4]**

(iii) Hence show that $z_1 + \omega z_2 + \omega^2 z_3 = 0$. **[2]**

Q7, (Jan 2012, Q2)

(i) Show that $(z^n - e^{i\theta})(z^n - e^{-i\theta}) \equiv z^{2n} - (2\cos\theta)z^n + 1$. **[1]**

(ii) Express $z^4 - z^2 + 1$ as the product of four factors of the form $(z - e^{i\alpha})$, where $0 \leqslant \alpha < 2\pi$. **[6]**

Q8, (Jun 2012, Q2)

(i) Solve the equation $z^4 = 2(1 + i\sqrt{3})$, giving the roots exactly in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $0 \leqslant \theta < 2\pi$. **[5]**

(ii) Sketch an Argand diagram to show the lines from the origin to the point representing $2(1 + i\sqrt{3})$ and from the origin to the points which represent the roots of the equation in part **(i)**. **[3]**

Q9, (Jun 2013, Q4)

The complex numbers 0, 3 and $3e^{\frac{1}{3}\pi i}$ are represented in an Argand diagram by the points O, A and B respectively.

(i) Sketch the triangle OAB and show that it is equilateral. **[3]**

(ii) Hence express $3 - 3e^{\frac{1}{3}\pi i}$ in polar form. **[2]**

(iii) Hence find $(3 - 3e^{\frac{1}{3}\pi i})^5$, giving your answer in the form $a + b\sqrt{3}i$ where a and b are rational numbers. **[3]**

Q10, (Jun 2015, Q4)

In an Argand diagram, the complex numbers $0, z$ and $ze^{\frac{1}{6}i\pi}$ are represented by the points O, A and B respectively.

(i) Sketch a possible Argand diagram showing the triangle OAB. Show that the triangle is isosceles and state the size of angle AOB. **[4]**

The complex numbers $1+i$ and $5+2i$ are represented by the points C and D respectively. The complex number w is represented by the point E, such that $CD = CE$ and angle $DCE = \frac{1}{6}\pi$.

(ii) Calculate the possible values of w, giving your answers exactly in the form $a+bi$. **[5]**

Q11, (Jun 2016, Q1)

In this question, give all non-real numbers in the form $re^{i\theta}$ where $r > 0$ and $0 < \theta < 2\pi$.

(i) Solve $z^5 = 1$. **[2]**

(ii) Hence, or otherwise, solve $z^5 + 32 = 0$. Sketch an Argand diagram showing the roots. **[4]**

Q12, (Jun 2017, Q5)

In an Argand diagram the points O, A and B are represented by the complex numbers 0, z and $2e^{\frac{1}{3}\pi i}z$ respectively, where z is a complex number with modulus 5.

(i) Calculate the exact area of the triangle OAB. **[3]**

The numbers $-1+i$ and $3+3i$ are represented by the points P and Q respectively. The complex number w is represented by the point R, such that $PQ = PR$ and angle $QPR = \frac{1}{4}\pi$.

(ii) Sketch an Argand diagram showing P, Q and the two possible positions of R. Calculate the possible values of w, giving your answers in the form $a+bi$. **[5]**
