Complex Roots Of Equations Using Exponential And Polar Form Exam Questions (Edexcel)

Q1.

Solve the equation

$$z^3 + 32 + 32i\sqrt{3} = 0$$

giving your answers in the form $re^{i\theta}$ where $r > 0$ and $-\pi < \theta \leq \pi$

(6)

(Total for question = 6 marks)

(Q10 6668/01, June 2017)

Q2.

(a) Find the four roots of the equation $z^4 = 8(\sqrt{3} + i)$ in the form $z = re^{i\theta}$

(5)

(b) Show these roots on an Argand diagram.

(2)

(Total for question = 7 marks)

(Q09 6668/01, June 2016)

Q3.

In an Argand diagram, the points A, B and C are the vertices of an equilateral triangle with its centre at the origin. The point A represents the complex number $6 + 2i$.

(a) Find the complex numbers represented by the points B and C, giving your answers in the form $x + iy$, where x and y are real and exact.

(6)

The points D, E and F are the midpoints of the sides of triangle ABC.

(b) Find the exact area of triangle DEF.

(3)

(Total for question = 9 marks)

(Q06 9FM0/02, June 2019)

Q4.

(a) Determine the roots of the equation

$$z^6 = 1$$

giving your answers in the form $e^{i\theta}$ where $0 \leq \theta < 2\pi$

(2)

(b) Show the roots of the equation in part (a) on a single Argand diagram.

(2)

(c) Show that

$$\left(\sqrt{3} + i\right)^6 = -64$$

(2)

(d) Hence, or otherwise, solve the equation

$$z^6 + 64 = 0$$

giving your answers in the form $re^{i\theta}$ where $0 \leq \theta < 2\pi$

(3)

(Total for question = 9 marks)

(Q07 9FM0/02, June 2024)

Q5.

The points representing the complex numbers $z_1 = 35 - 25i$ and $z_2 = -29 + 39i$ are opposite vertices of a regular hexagon, H, in the complex plane.

The centre of H represents the complex number α

(a) Show that $\alpha = 3 + 7i$

(2)

Given that $\beta = \dfrac{1+i}{64}$

(b) show that

$$\beta(z_1 - \alpha) = 1$$

(2)

The vertices of H are given by the roots of the equation

$$(\beta(z - \alpha))^6 = 1$$

(c) (i) Write down the roots of the equation $w^6 = 1$ in the form $re^{i\theta}$

(1)

(ii) Hence, or otherwise, determine the position of the other four vertices of H, giving your answers as complex numbers in Cartesian form.

(4)

(Total for question = 9 marks)

(Q05 9FM0/02, June 2023)

Q6.

(a) Using the identity $zz^* = |z|^2$, or otherwise, show that if w is any root of unity then

$$|w - 2|^2 = 5 - 2(w + w^*)$$

(3)

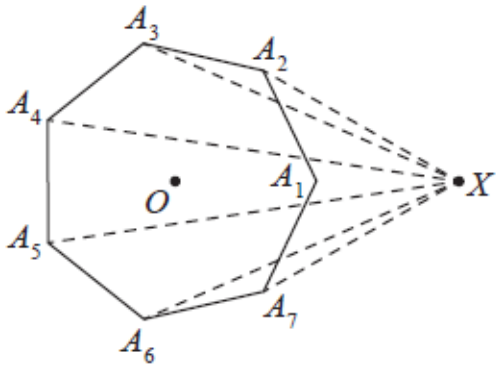

Figure 1

Figure 1 shows a regular heptagon $A_1A_2A_3A_4A_5A_6A_7$ whose vertices all lie on the unit circle with centre at the origin O and A_1 at (1, 0). The point X lies in the same plane as the heptagon and has coordinates (2, 0).

Using the result given in part (a),

(b) find $\sum_{i=1}^{7} (XA_i)^2$

(4)

(Total for question = 7 marks)

(Q04 9FM0/02, Specimen)