Volumes of Revolution Exam Questions (Edexcel)

Q1.

Diagrams not drawn to scale

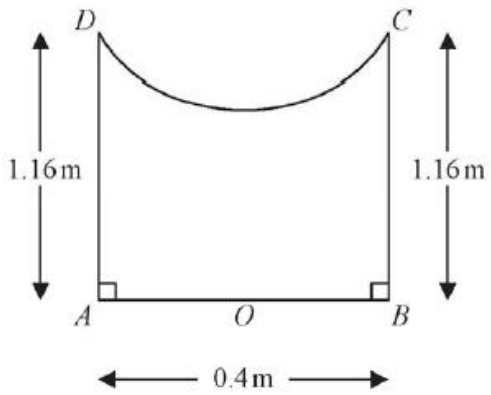

Figure 1

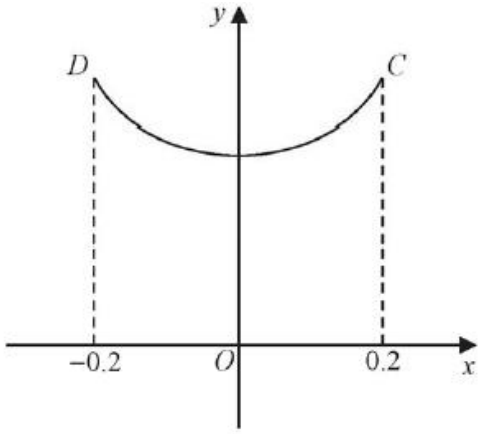

Figure 2

Figure 1 shows the central cross-section *AOBCD* of a circular bird bath, which is made of concrete. Measurements of the height and diameter of the bird bath, and the depth of the bowl of the bird bath have been taken in order to estimate the amount of concrete that was required to make this bird bath.

Using these measurements, the cross-sectional curve *CD*, shown in Figure 2, is modelled as a curve with equation

$$y = 1 + kx^2 \qquad -0.2 \leqslant x \leqslant 0.2$$

where k is a constant and where O is the fixed origin.

The height of the bird bath measured 1.16 m and the diameter, *AB*, of the base of the bird bath measured 0.40 m, as shown in Figure 1.

(a) Suggest the maximum depth of the bird bath.

(1)

(b) Find the value of k.

(2)

(c) Hence find the volume of concrete that was required to make the bird bath according to this model. Give your answer, in m^3, correct to 3 significant figures.

(7)

(d) State a limitation of the model.

(1)

It was later discovered that the volume of concrete used to make the bird bath was 0.127 m^3 correct to 3 significant figures.

(e) Using this information and the answer to part (c), evaluate the model, explaining your reasoning.

(1)

(Total for question = 12 marks)

(Q07 8FM0/01, Specimen papers)

Q2.

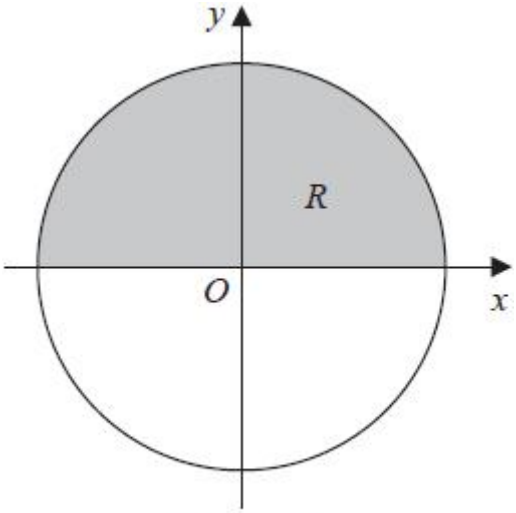

Figure 1

Figure 1 shows a circle with radius r and centre at the origin.
The region R, shown shaded in Figure 1, is bounded by the x-axis and the part of the circle for which $y > 0$
The region R is rotated through 360° about the x-axis to create a sphere with volume V

Use integration to show that $V = \frac{4}{3}\pi r^3$

(Total for question = 5 marks)

(Q03 8FM0/01, Oct 2020)

Q3.

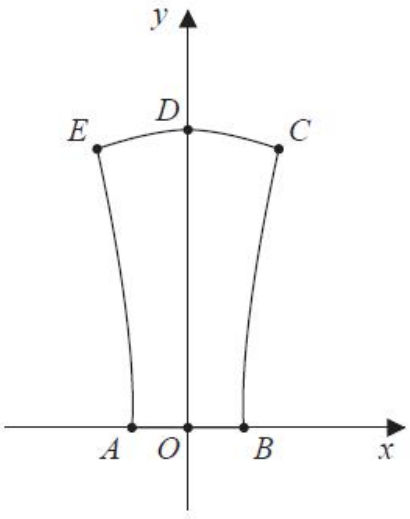

Figure 2

Figure 2 shows the vertical cross-section, *AOBCDE*, through the centre of a wax candle.

In a model, the candle is formed by rotating the region bounded by the *y*-axis, the line *OB*, the curve *BC*, and the curve *CD* through 360° about the *y*-axis.

The point *B* has coordinates (3, 0) and the point *C* has coordinates (5, 15).

The units are in centimetres.

The curve *BC* is represented by the equation

$$y = \frac{\sqrt{225x^2 - 2025}}{a} \qquad 3 \leqslant x < 5$$

where *a* is a constant.

(a) Determine the value of *a* according to this model.

(2)

The curve *CD* is represented by the equation

$$y = 16 - 0.04x^2 \qquad 0 \leqslant x < 5$$

(b) Using algebraic integration, determine, according to the model, the exact volume of wax that would be required to make the candle.

(9)

(c) State a limitation of the model.

(1)

When the candle was manufactured, 700 cm^3 of wax were required.

(d) Use this information and your answer to part (b) to evaluate the model, explaining your reasoning.

(1)

(Total for question = 13 marks)

(Q09 8FM0/01, Oct 2021)

Q4.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

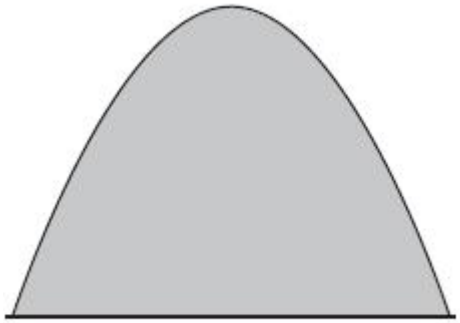

Figure 1

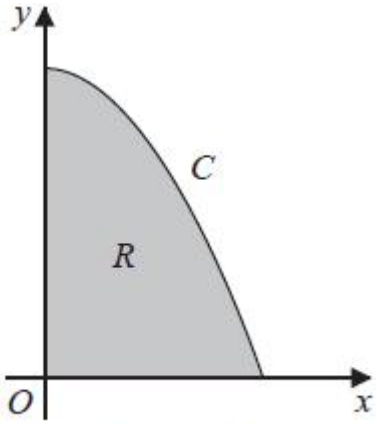

Figure 2

A large pile of concrete waste is created on a building site.

Figure 1 shows a central vertical cross-section of the concrete waste.

The curve C, shown in Figure 2, has equation

$$y + x^2 = 2 \qquad 0 \leqslant x \leqslant \sqrt{2}$$

The region R, shown shaded in Figure 2, is bounded by the y-axis, the x-axis and the curve C.

The volume of concrete waste is modelled by the volume of revolution formed when R is rotated through 360° about the y-axis. The units are metres.

The density of the concrete waste is 900 kgm^{-3}

(a) Use the model to estimate the mass of the concrete waste. Give your answer to 2 significant figures.

(6)

(b) Give a limitation of the model.

(1)

The mass of the concrete waste is approximately 5500 kg.

(c) Use this information and your answer to part (a) to evaluate the model, giving a reason for your answer.

(1)

(Total for question = 8 marks)

(Q05 8FM0/01, June 2023)

Q5.

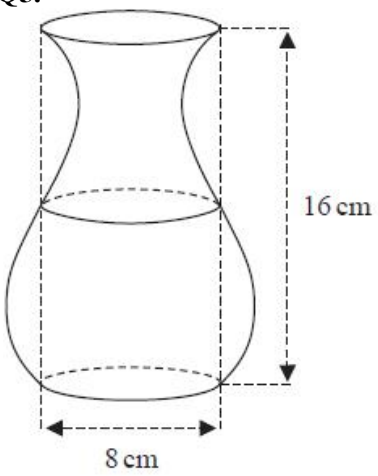

Figure 1

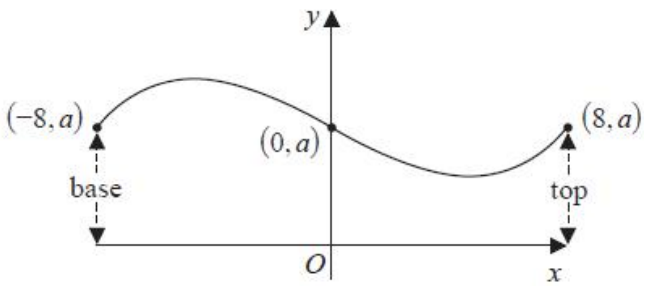

Figure 2

Figure 1 shows a sketch of a 16 cm tall vase which has a flat circular base with diameter 8 cm and a circular opening of diameter 8 cm at the top.

A student measures the circular cross-section halfway up the vase to be 8 cm in diameter.

The student models the shape of the vase by rotating a curve, shown in Figure 2, through 360° about the x-axis.

(a) State the value of a that should be used when setting up the model.

(1)

Two possible equations are suggested for the curve in the model.

Model A $\quad y = a - 2\sin\left(\frac{45}{2}x\right)^\circ$

Model B $\quad y = a + \frac{x(x-8)(x+8)}{100}$

For each model,

(b) (i) find the distance from the base at which the widest part of the vase occurs,

(ii) find the diameter of the vase at this widest point.

(7)

The widest part of the vase has diameter 12 cm and is just over 3 cm from the base.

(c) Using this information and making your reasoning clear, suggest which model is more appropriate.

(1)

(d) Using algebraic integration, find the volume for the vase predicted by Model B.
You must make your method clear.

(5)

The student pours water from a full one litre jug into the vase and finds that there is 100 ml left in the jug when the vase is full.

(e) Comment on the suitability of Model B in light of this information.

(1)

(Total for question = 15 marks)

(Q08 8FM0/01, June 2022)

Q6.

$$f(x) = 2x^{\frac{1}{3}} + x^{-\frac{2}{3}} \qquad x > 0$$

The finite region bounded by the curve $y = f(x)$, the line $x = \frac{1}{8}$, the x-axis and the line $x = 8$ is rotated through θ radians about the x-axis to form a solid of revolution.

Given that the volume of the solid formed is $\frac{461}{2}$ units cubed, use algebraic integration to find the angle θ through which the region is rotated.

(Total for question = 8 marks)

(Q09 8FM0/01, June 2019)